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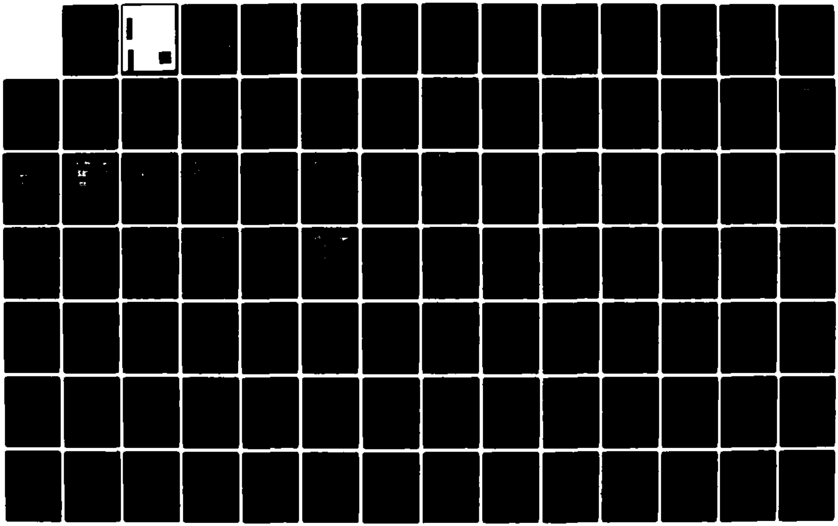

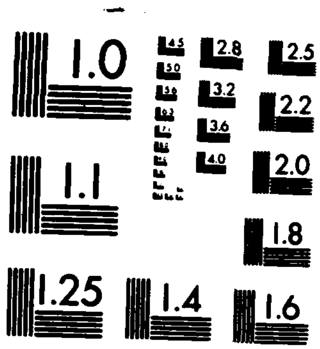

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The performance of convolutionally coded M-ary frequency shift keyed (FSK) signals in the presence of worst case partial band noise jamming is evaluated. A large number of cases are considered including: 1. several alphabet sizes; 2. many values of diversity; 3. several coding schemes; 4. both hard and soft decision receivers; 5. jammer state known or unknown (i.e., in the former case the receiver knows whether or not each received symbol has been jammed); 6. non-fading and Rayleigh fading channels.

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FINAL REPORT

to

NAVAL RESEARCH LABORATORY

(Contract Award No. N00014-80-K-0935)

for

HF COMMUNICATION NETWORK SIGNALS
USING CHANNEL EVALUATION DATA

Principal Investigator

Jim K. Omura
Professor
System Science Department
University of California
Los Angeles, California

November, 1981

I. Introduction

This final report to the Naval Research Laboratory for work performed under Grant No. N00014-80-K-0935 consists of a summary, eight technical notes, and the Ph.D. thesis of Dan Avidor.

The purpose of this research was to analyze anti-jamming waveforms and receivers for the High Frequency Intra Task Force (HF ITF) network. Our main focus was on point-to-point communication between a fixed pair of HF terminals with either a groundwave or skywave propagation path. Since there is little difference in degradation due to worst case partial band noise and multitone jamming, we limited our analysis to worst case partial band noise jamming. Also if the jammer propagation path has Rayleigh fading then a jamming tone appears as narrowband Gaussian noise at the receiver.

Because of limited bandwidth at HF frequencies and random variations across this band (i.e., frequency selectivity) the natural choice of waveforms is noncoherent MFSK with frequency hopping (FH). Rather than consider several other alternative waveforms, we chose to examine MFSK/FH in greater detail in this research. In addition, to avoid repeat-back jamming, hop rates should be chosen as fast as the coherence bandwidth of the HF channel will allow. Here 2400 hops per second is a natural choice. With this choice of hop rate multipath problems are not important with either skywave or groundwave (whichever is stronger) can be used. In our analysis we treat these two cases separately as channels with and without Rayleigh fading.

Before summarizing the work done on this contract we briefly review the essential features of the HF ITF network in the next section. This is followed by a general discussion motivating our choice of the MFSK/FH waveform. This is next followed by a tutorial on the generation of pseudo random (PN) sequences that are used in spread spectrum systems. A summary of the work done on this grant is then presented,followed by recommendations for further studies.

II. The HF ITF Network [1]

At a particular time, the HF ITF network would have a topological structure akin to the simplified example given in Figure 1. The nodes represent task force units (surface ships, submarines and aircraft), while the branches represent one or more HF channels for communication between the connected nodes. There may be up to 100 nodes covering an area of 300 nautical miles. Individual HF channels are represented by lowercase letters on the branches; for instance, there are four HF channels available between nodes B and C. These channels may be coded spread spectrum signals using code division multiple access together with an overall TDMA format. Each of these channels could, for example, be a 2400 bps HF channel. The overall HF communications facility must efficiently handle vast amounts of information and be adaptable to a wide variety of applications including serving as backup for long haul UHF/SHF satellite links.

Aircraft and submarines will typically have no network control responsibilities such as the relaying of messages. Submarines may also use low probability of intercept (LPI) waveforms to avoid being detected. In periods of stress anti-jamming requirements will also be imposed. Our approach to the HF ITF network waveform design is to <u>optimize for stress conditions, not for normal (unstressed) conditions</u>. This principle, which should be a bedrock for the design of tactical military systems is too often eroded by the "natural" tendency of designers to fashion a system that performs efficiently under normal conditions.

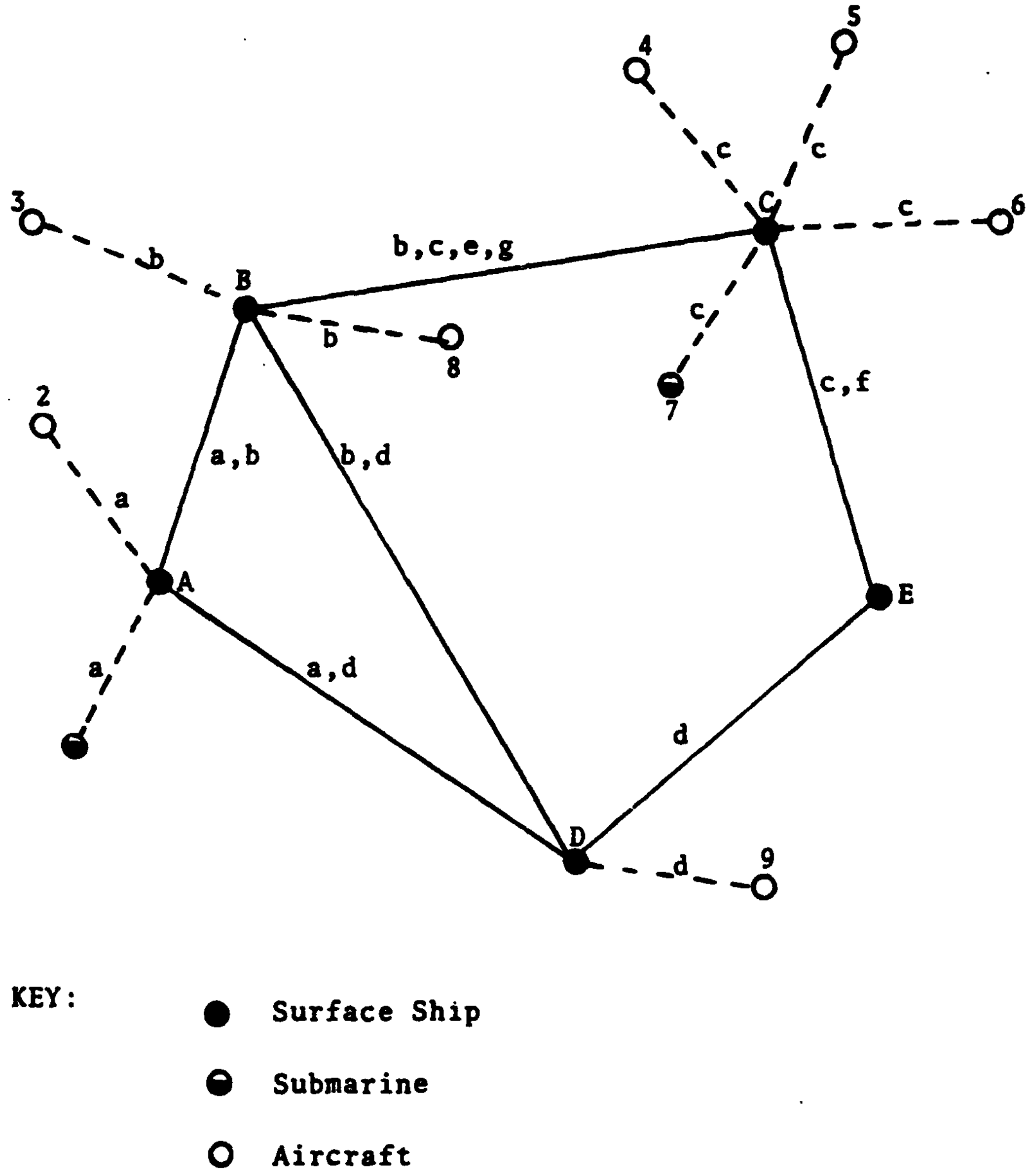


Figure 1: Simplified Example Showing the Possible HF ITF Network Topology at Some Instant.

The HF communications network will function as the primary Extended Line of Site (ELOS) (20 t0 600 miles) communications in the intra-task force environment where platforms move about at different speeds creating a time-varying network topology. Propagation conditions, jamming, and node disappearance (equipment failures or combat losses) are changing with time in this network which forms a subnet of the total HF communication system for the Task Force. Within the HF ITF network there may be several smaller subnets where often each subnet serves a common function such as submarine support. A Node may belong to more than one subnet at a time.

The primary goal of the signal waveform design for the HF ITF network is robust performance in the presence of severe jamming. Modes of operation are

- point-to-point
- broadcast
- conferencing (up to 10 users).

There are also severe precedence levels for recording traffic including:

- Flash
- Immediate
- Priority
- Routine

A mix of voice (digitized) and data traffic with variable maximum allowable bit error rate requirements* must be handled by the network. As an example, the total ELOS traffic for a

* $p_b = 10^{-3}$ to 10^{-5} for voice while computer data requires less than 10^{-6}. Some coding is obviously required here.

a Task Force of 50 ships is estimated [2-4] for 1985 at 200 Kbps where 70% is voice traffic and the remaining 30% data traffic. Each voice circuit requires 2400 bps rates. During times of stress the traffic tends to increase while jamming and possible loss of nodes will cause overall network degradation. The network should degrade gracefully under stress conditions with high priority messages suffering less degradation than low priority messages.

There are several constraints on the communication network imposed by the HF ITF platforms and the HF propagation channel. Besides jamming, locally generated interference is common and with limited platform space, large directive array antennas are not practical. In the case of submarines low probability of intercept (LPI) operation is required. Half duplex operation and adaptive interference cancellation techniques (AIC) will likely be required to lower the level of the interference in the colocated wideband receiver. Care must be taken to avoid interference due to intermodulation terms caused by multiple signal in nonlinear components.

The HF radio channel can occur via both ground and skywaves. An effective signal waveform design must be able to handle the special characteristics of this channel over the 300 nautical mile diameter of the HF ITF network. Daytime skywave signals are often negligible while at nighttime they can exceed the groundwave signal in power for relatively short distances (180 miles or more). Much of the research proposed here will investigate the detailed nature of the HF radio channel and the inter-

ference/jamming environment. Generally man-made interference such as jamming will be handled by assuming the worst type of jammer waveform. Because of limited bandwidth and poor spatial discrimination of antennas jamming is difficult to combat at HF frequencies.

For the HF ITF network the HF radio band (2MHz to 30 MHz) is divided into sub-bands with bandwidths from 1 MHz to 5 MHz. Hence there are anywhere from 6 to 28 possible non-overlapping spread spectrum channels available in the HF ITF network. The lower frequency channels will tend to use groundwave propagation, while for longer distances and higher frequency channels sky-wave propagation may be more effective. The relative strength between groundwave and skywave propagation between two points depend on distance, frequency, and time of day.

Suppose we define the terms

W = spread spectrum channel bandwidth in Hz

R = data rate in bits per second

J = jammer power at the receiver

S = signal power at the receiver

Here typically W ranges from 1 MHz to 5 MHz while R ranges from 75 bps to 2400 bps. The "processing gain" (PG) is defined here as

$$PG = \frac{W}{R}$$

Thus we have the range of processing gains from a minimum of

- W = 1 MHz, R = 2400 bps

$$\begin{aligned} PG &= 416.67 \\ &= 26.20 \text{ dB} \end{aligned}$$

to a maximum of

- $W = 5$ MHz, $R = 75$ bps

$$PG = 66{,}666.67$$
$$= 48.24 \text{ dB}$$

Throughout this research we define the effective energy-per-bit to noise ratio, E_b/N_o, as

$$\frac{E_b}{N_o} = \frac{PG}{J/S}$$

This definition is used regardless of the particular form of the jammer and signal waveform as long as J and S are long term average power values. This is discussed in detail in Note No.2 titled "Conventional Jamming Analysis". If to obtain acceptable bit error probabilities we require

$$\frac{E_b}{N_o} = 15 \text{ dB}$$

then we can operate effectively for J/S ratios up to 11.20 dB in the smallest processing gain case and up to 33.24 dB in the best case with $W = 5$ MHz and $R = 75$ bps. The actual picture is considerably more complex and is described in detail in the attached notes. This shows roughly the amount of jamming we can tolerate in the HF ITF network.

III. Choice of Anti-Jamming Waveform

There are two basic techniques for spreading a signal bandwidth to combat intentional Jamming. These are direct sequencing (DS) and frequency hopping(FH). Although there are hybrid spread spectrum signals that use both spreading techniques, most spread spectrum signals use either DS or FH alone. DS is typically used with BPSK and QPSK modulations with coherent receivers. FH is usually used with orthorgonal MFSK modulation with noncoherent receivers. Coding, interleaving, and diversity are important additional signal design techniques that should be used in any spread spectrum system.

Here we compare DS and FH techniques from several viewpoints. These include performance, bandwidth, synchronization, sensitivity to propagation, robustness, hardware complexity, and network aspects where many spread spectrum signals must share the total bandwidth available. These different aspects of spread spectrum signal design have some interdependencies and are generally difficult to quantify. Complexity, for example, is a time-varying notion that strongly depends on the rapidly changing solid state technology. Here we compare DS and FH techniques in a qualitative manner from these several viewpoints.

A. Performance Against Worst Case Jamming

Since there are many possible types of jammers we compare DS and FH against the worst possible type of jammer waveform against a DS system and against an FH system. In addition, we assume that each system, whether DS or FH, takes full advantage of coding, interleaving, and diversity.

B. Bandwidth

The DS systems have total bandwidth proportional to the chip rate of the pseudo random (PN) code generator. Today this typically means the DS signal bandwidth is limited to below 100MHz. This, of course, is a soft limit that depends on technology. By contrast, FH systems have total bandwidth that does not strongly depend on the PN code chip rate. The PN code chip rate is a function of primarily the signal hopping rate which does not depend on bandwidth. Today FH signal bandwidths greater than 2 GHz are possible. The hop rate, however, is limited by complexity and frequency synthesizer design. The hop rate of 20 K hops or more per second is possible today. If the signal bandwidth is not required to be above 100 MHz then both DS and FH techniques can be used. Beyond 100 MHz, some form of FH is required.

At HF the primary advantage of FH is that it does not require a contiguous frequency band. Unlike the DS signal an FH signal can skip over parts of the channel that are corrupted by excessive noise or have poor propagation conditions. By carefully selecting the hopping bands the FH technique can allow considerably more total effective signal energy to reach the receiver.

C. Propagation Path

Multipath can cause problems with most communication systems. Spread spectrum signals, however, have a natural anti-multipath capability. DS receivers, for example, do cross correlation at the receiver and hence at the correlation output multipath components can be resolved as distinct signals. The resolution for DS signals is roughly the chip duration which is inversely proportional to the total spread bandwidth.

For FH systems if the multipath delay exceeds the hop duration then multipath has no harmful effect at the receiver.

Since the DS signal continuously occupies the total spread bandwidth dispersive channels with frequency selective fading can cause severe degradation in performance. To combat such distortions due to dispersive channels complex RAKE-like receivers have been proposed. These, however, are too complex for current applications.

By contrast, FH systems occupy a narrow band at any instant of time and thus the FH signals are not distorted by frequency selective fading, at least compared to DS signals. Indeed, frequency selective fading helps FH systems by providing independent fading in each hopped time duration.

D. Synchronization of PN Code

Acquisition of the PN code at the receiver, is the most critical problem for spread spectrum systems. The difficulty in acquisition is mostly tied to the PN code rate. For DS signals this rate is proportional to the total spread bandwidth

whereas, for FH signals, it is primarily proportional to the hop rate. Since PN code rates are much higher for DS signals than for FH signals, the acquisition for DS systems is generally much more difficult.

E. Robustness

FH systems are generally much more robust than DS systems. For FH systems PN code acquisition is easier, there is less sensitivity to dispersive channels, a contiguous band is not required, and wider bandwidths can be achieved. Also,since FH systems are noncoherent there are no carrier acquisiton problems. Oscillator frequency drift and doppler shifts can also be handled using wider spacing between tones and wider tone filter bandwidths at some loss in performance.

F. Network Aspects

In the HF ITF network there will be many spread spectrum signals simultaneously transmitting in a spread spectrum channel. Since each FH signal would instantaneously be a narrow band signal only interference in this narrow signal bandwidth would cause degradation. Thus FH systems can handle better the "near-far" problem associated with several mobile transmitters using the same total spread bandwidth. Overall network timing requirements are also easier to maintain with FH waveforms due to its easier PN code synchronization. Finally as described in Note No.1 titled, "Impact of Spread Spectrum Signals on Multiple Access Design" there is a natural way to maintain separate overhead, data and network restructuring channels using an FH system.

G. FH/MFSK Waveform Choice

For most applications and particularly for the HF ITF network the spreading technique ought to be frequency hopping. Although there are many advantages of FH over DS techniques, the most important one for the HF ITF network is the fact that the FH technique can hop with non-uniform probability across the spread spectrum channel. Indeed it can altogether avoid certain narrow signal bands that have poor propagation and/or interference conditions.

To avoid repeat back jamming the frequency hopping rate should be as large as the coherence band of the HF skywave will allow. Also as is shown in this research, the hopping rate must be chosen to provide enough diversity at the receiver. Since the highest data rates are most vulnerable to jamming (assuming fixed signal power S), the hop rate must be high enough to provide some diversity at the high data rates. The natural choice for the HF ITF is the hop rate

$$R_h = 2400 \text{ hops/second}$$

Thus with the highest data rate of $R = 2400$ bps we have

$$L = \frac{R_h}{R} = 1 \text{ hop/data bit}$$

which is barely enough diversity. (See Note No.8 showing performance). At the low rate $R = 75$ bps there is

$$L = 32 \text{ hops/ data bit}$$

which is more diversity than necessary. However, this excessive diversity at the lower data rates is compensated for by the

increased energy per data bit available or equivalently more processing gain.

When we have hop rate greater than or equal to the data rate, we must use noncoherent receivers. This is primarily due to the fact that maintaining phase between hops is difficult particularly in HF skywave paths. Also, channel characteristics result in the loss of phase information. This leads us to the natural choice of noncoherent MFSK signals as the basic waveform.

IV . Results of This Research

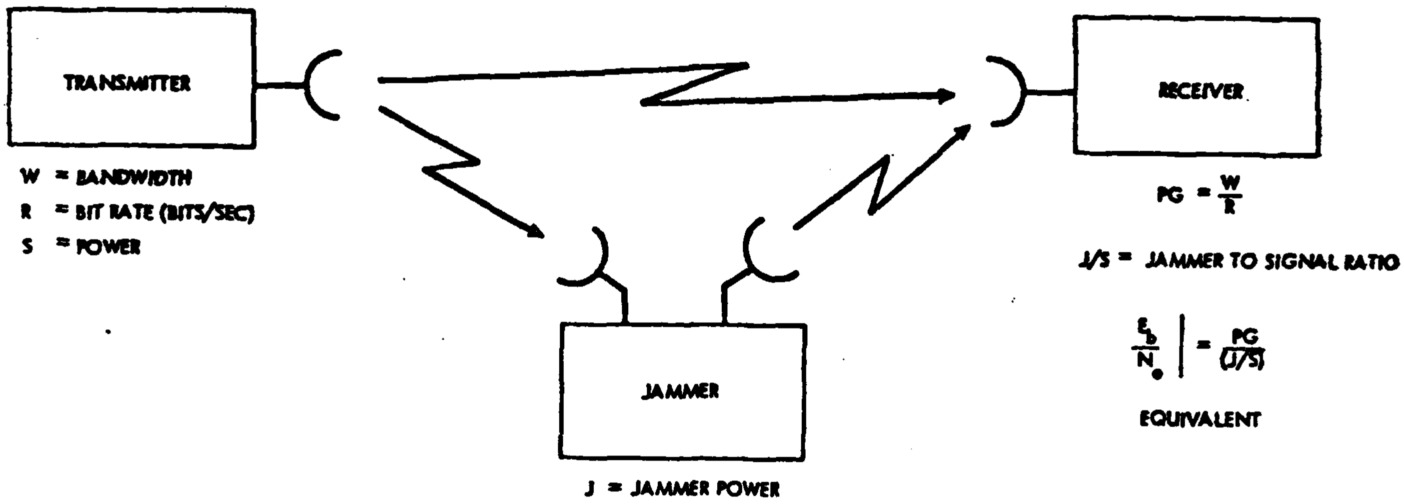
A detailed description of the work conducted on this grant is reported in the eight technical notes and the Ph.D. thesis of Dan Avidor. In this section, we summarize the main results and relate them to the HF ITF network. These technical notes are:

Note No. 1.	IMPACT OF SPREAD SPECTRUM SIGNALS ON MULTIPLE ACCESS DESIGN
Note No. 2.	CONVENTIONAL JAMMING ANALYSIS
Note No. 3.	FACTOR OF ONE HALF IN ERROR BOUNDS
Note No. 4.	A GENERAL ANALYSIS OF ANTI-JAM COMMUNICATION SYSTEMS
Note No. 5.	FADING DISPERSIVE CHANNELS
Note No. 6.	ANTI-JAM ANALYSIS OF FH/MFSK SYSTEMS IN RAYLEIGH FADING CHANNELS
Note No. 7.	VARIABLE DATA BIT RATES WITH A FIXED HOP RATE NONCOHERENT FH/MFSK SYSTEM
Note No. 8.	PERFORMANCE OF CONVOLUTIONALLY CODED NON-COHERENT FH/MFSK SYSTEMS
Avidor's Ph.D. Thesis:	ANTI-JAM ANALYSIS OF FREQUENCY HOPPING M-ARY FREQUENCY SHIFT KEYING COMMUNICATION SYSTEMS IN HIGH FREQUENCY RAYLEIGH FADING CHANNELS

An overview of the two basic spread spectrum techniques, coherent DS/BPSK and noncoherent FH/MFSK, are discussed in Note No.2 titled, "Conventional Jamming Analysis". Here we motivate the definition of an equivalent energy-per-bit to noise ratio

$$\frac{E_b}{N_o} = \frac{PG}{J/S}$$

which we use regardless of the signal or jammer waveform used. This definition is useful since it allows comparison of different types of anti-jamming systems. Figure 2 illustrates the over-



Signals

- MFSK/FH (Noncoherent)
- BPSK/DS (Coherent)
- Coding/Interleaving/Diversity

Jammers

- Types
  --Noise
  --Multitone or CW
- Modes
  --Broadband
  --Partial Band
  --Pulsed

Figure 2: System Overview

all system parameters. Here we also emphasize our design approach of always assuming the worst possible type of jamming waveform. For coherent DS/BPSK and DS/QPSK the worst type of jammer is a pulse jammer with the worst case duty cycle chosen. Partial band jamming is worst for FH/MFSK systems where the fraction of the band jammed is chosen to do the most damage. For FH/MFSK systems with no diversity, Figure 3 shows how much this worst case partial band jammer can degrade performance compared to broadband noise jamming. Note that at 10^{-6} bit error rates, there is about a 50 dB difference in effective jammer to signal power ratio. This is shown for the no fading or groundwave propagation case.

In Note No. 2 we also introduce the use of Chernoff bounds to simplify the generally complex form of the bit error probability to the point of being able to numerically evaluate them. Such bounds must, however, be reasonably accurate. Figure 4 compares the Chernoff bound with exact bit error probability for FH/MFSK where M=2 and broadband noise jamming. There is about a 1 dB difference. Later in Note No.3 titled, "Factor of One Half in Error Bounds", we show that the Chernoff bound can be tightened by a factor of one half. Throughout the rest of these notes we used the Chernoff bounds to evaluate bit error probabilities since exact expressions are difficult to derive.

Although diversity and coding are often treated separately, diversity is a special case of coding. One of the key points we make in Note No.2 is that the huge 50 dB loss of worst case partial jamming compared to broadband jamming can be recovered

Figure 3 No Diversity FH/MFSK

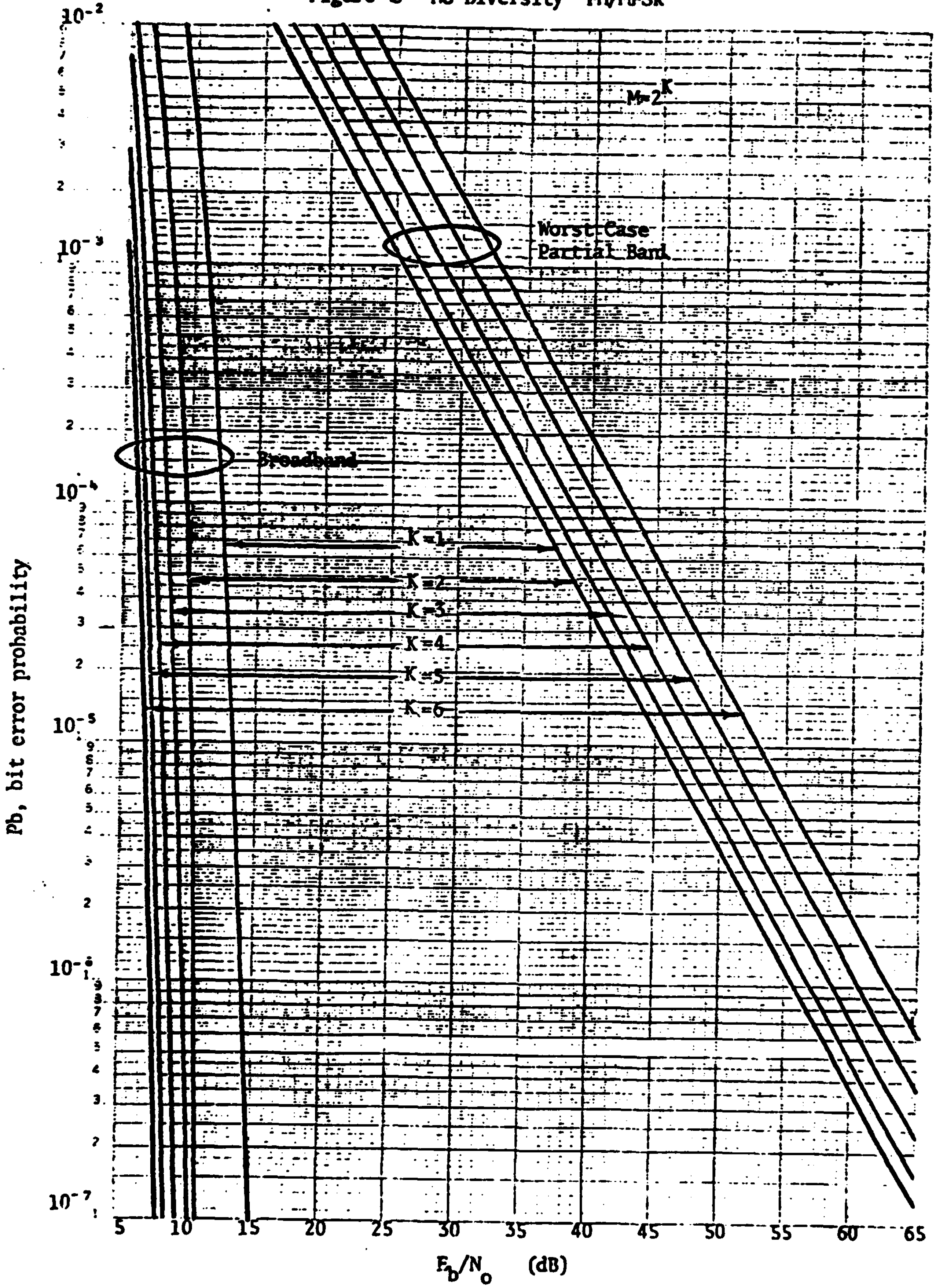
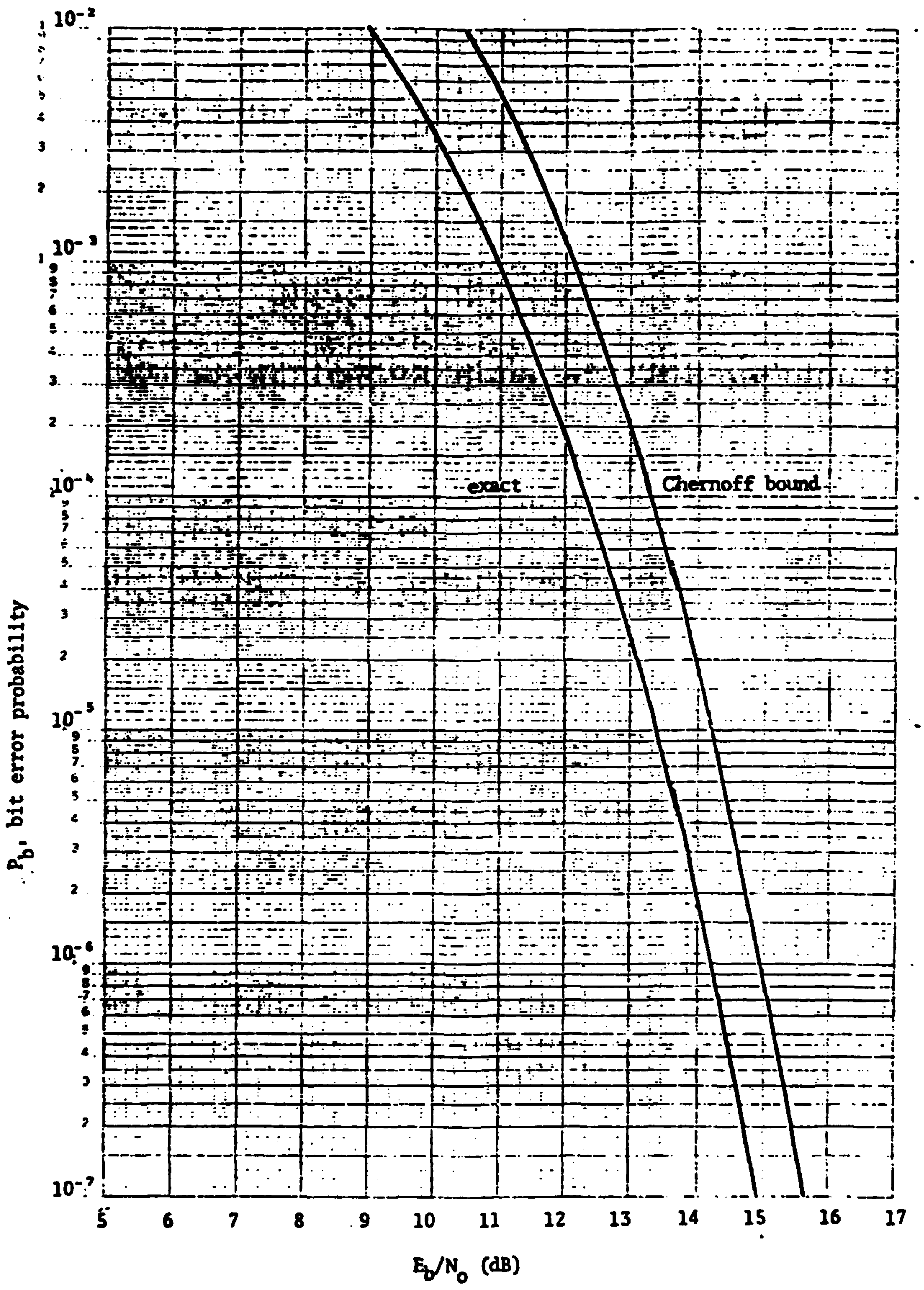


Figure 4: Broadband Jammer FH/BFSK



with diversity and coding. Figure 5 shows this with just conventional diversity chosen to minimize the bit error bound. Here we assume the worst case partial band jammer and the optimum diversity which depends on the value of E_b/N_o. Figure 6 illustrates further improvement when coding and optimum diversity are both used against worst case partial band jamming. These results are all for groundwave propagation and an ideal receiver that non-coherently combines chip energies to make decisions.

In the HF ITF network it is likely that the hop rate will be fixed at say $R_h = 2400$ hops per second and various data rate sources will all be using this same hop rate. Thus, rather than use some optimum diversity, each source will have to live with the diversity characterized by the parameter

$$L = \frac{R_h}{R}$$

where R is the data rate. For the same conditions discussed above we have Figures 7 - 9 showing uncoded FH/MFSK for M=2,4,8 and a range of values for L. These results are discussed in Note No.7 titled, "Variable Data Rate with a Fixed Hop Rate Noncoherent FH/MFSK System". Recall that for R=75 bps and $R_h = 2400$ hops per second we get L=32 which is the maximum diversity parameter value for the HF ITF network. In Note No.8 titled, "Performance of Convolutionally Coded Noncoherent FH/MFSK Systems" we examined constraint length 7 optimum convolutional codes and obtained the coded cases shown in Figures 10-13.

All the results shown above in Figures11-13 are for an ideal receiver that knows exactly when a signal MFSK chip hops into the part of the band where the jammer's partial band noise exists.

Figure 5: Optimum Diversity FH/MFSK

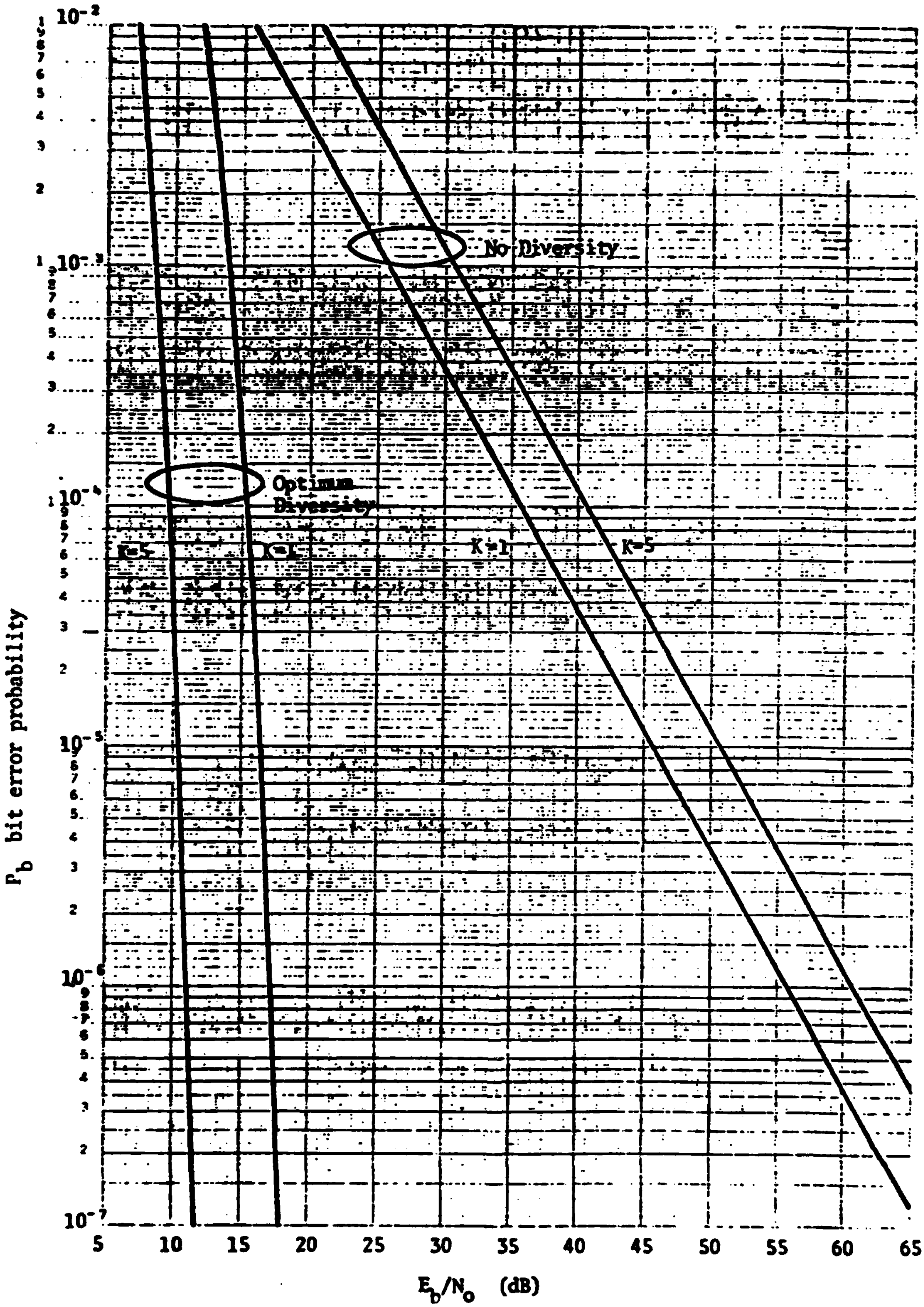


Figure 6 : Optimum Diversity FH/MFSK

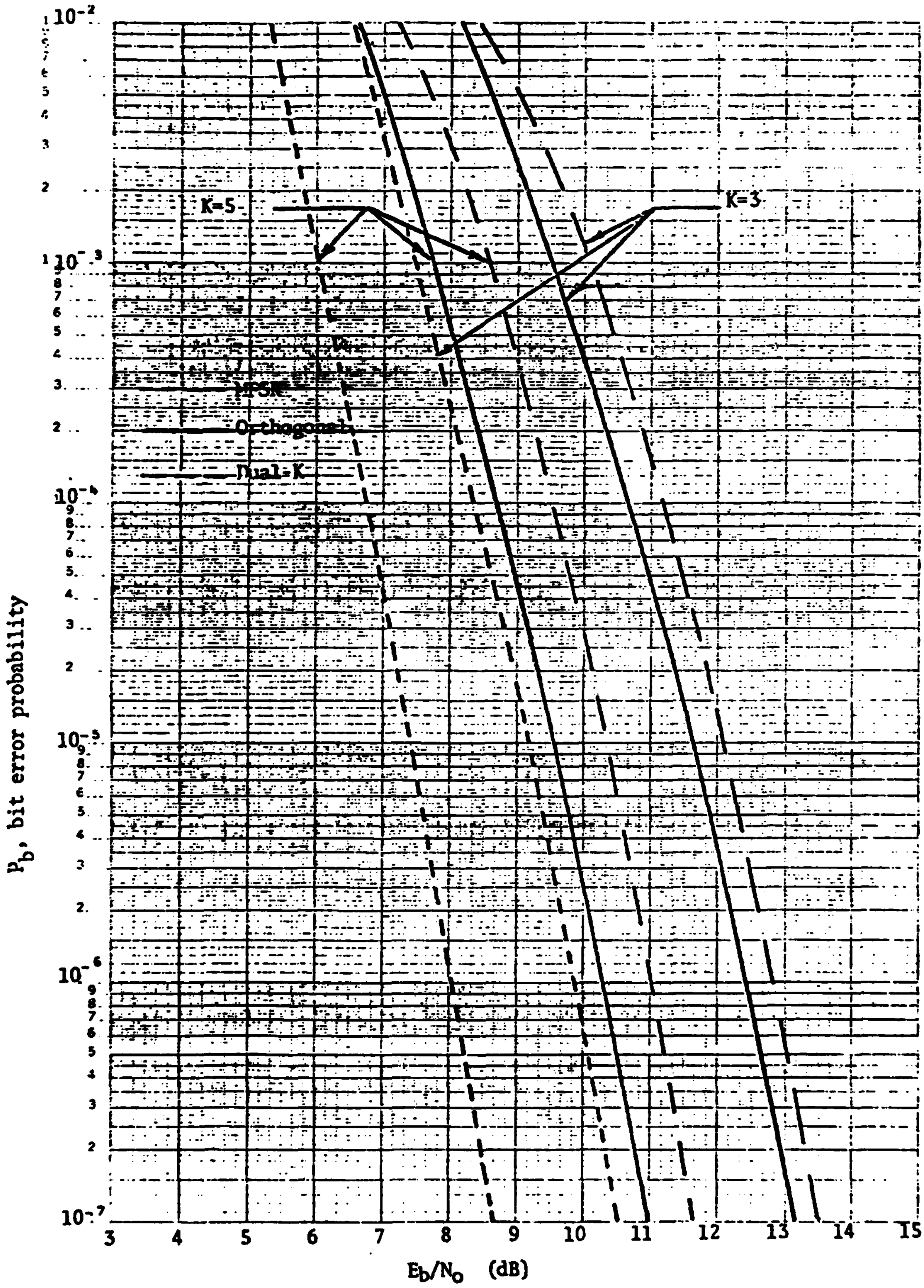


Figure 7 : No Fading M=2

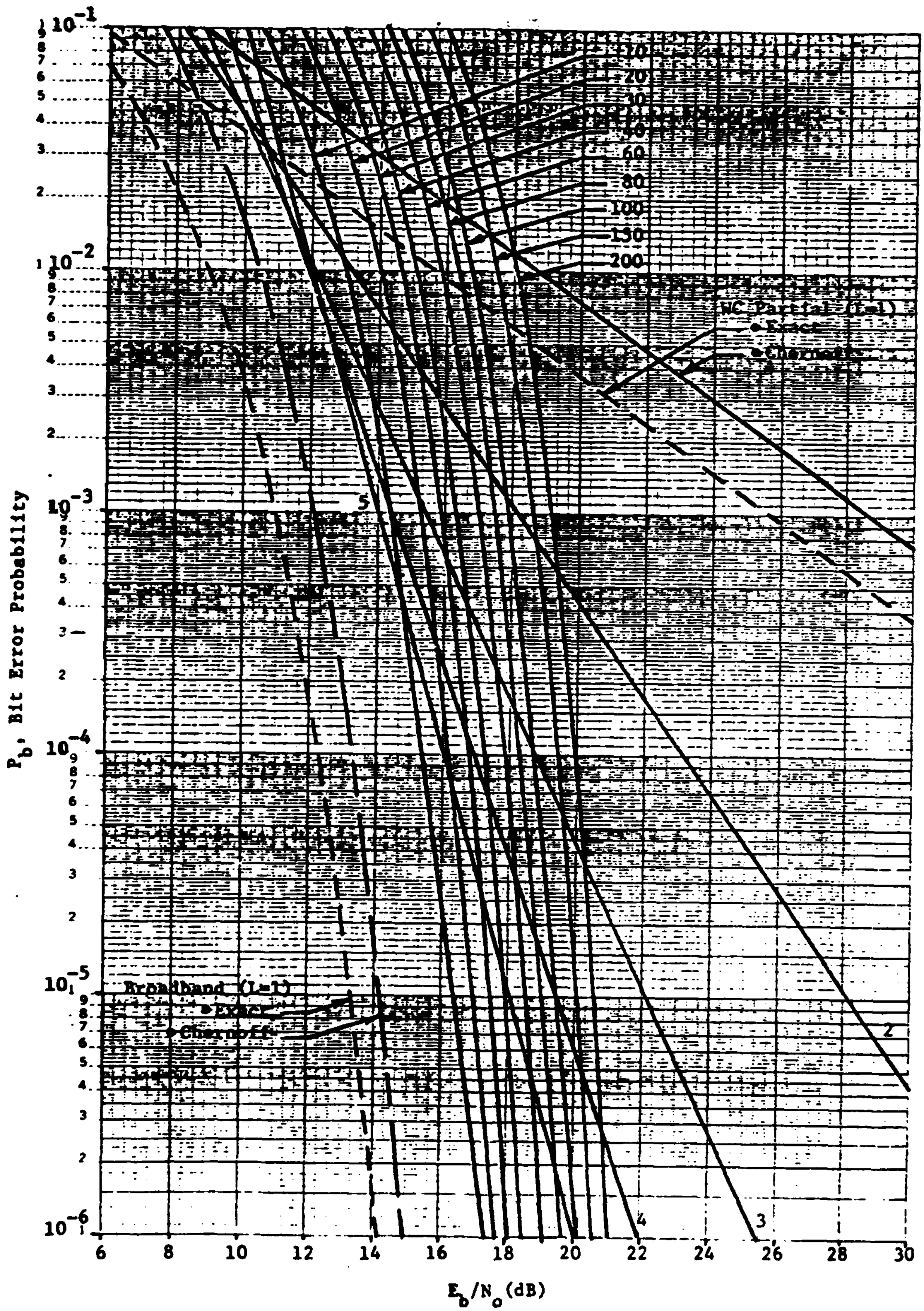


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Figure 8 : No Fading M=4

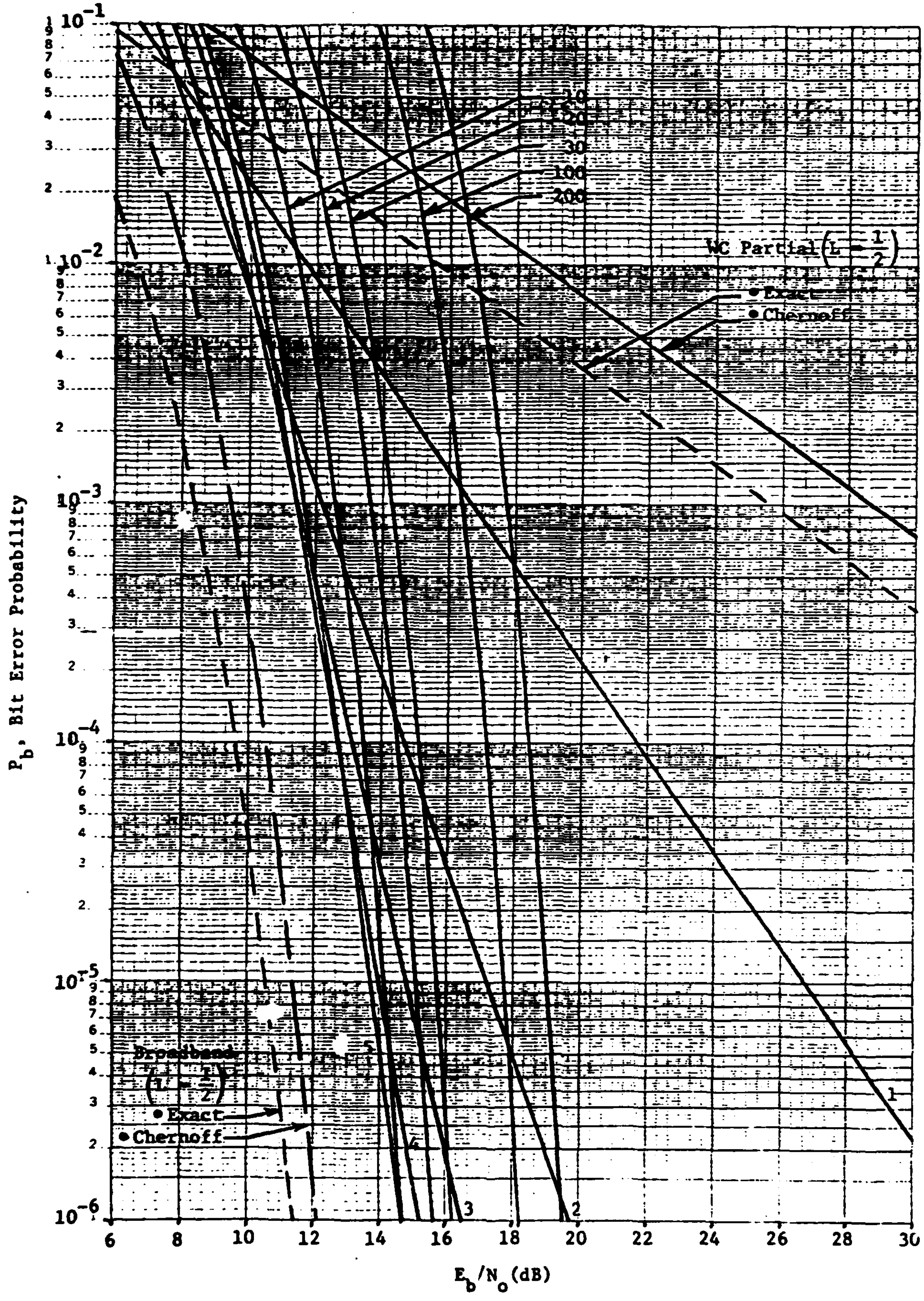

Figure 9 : No Fading M=8

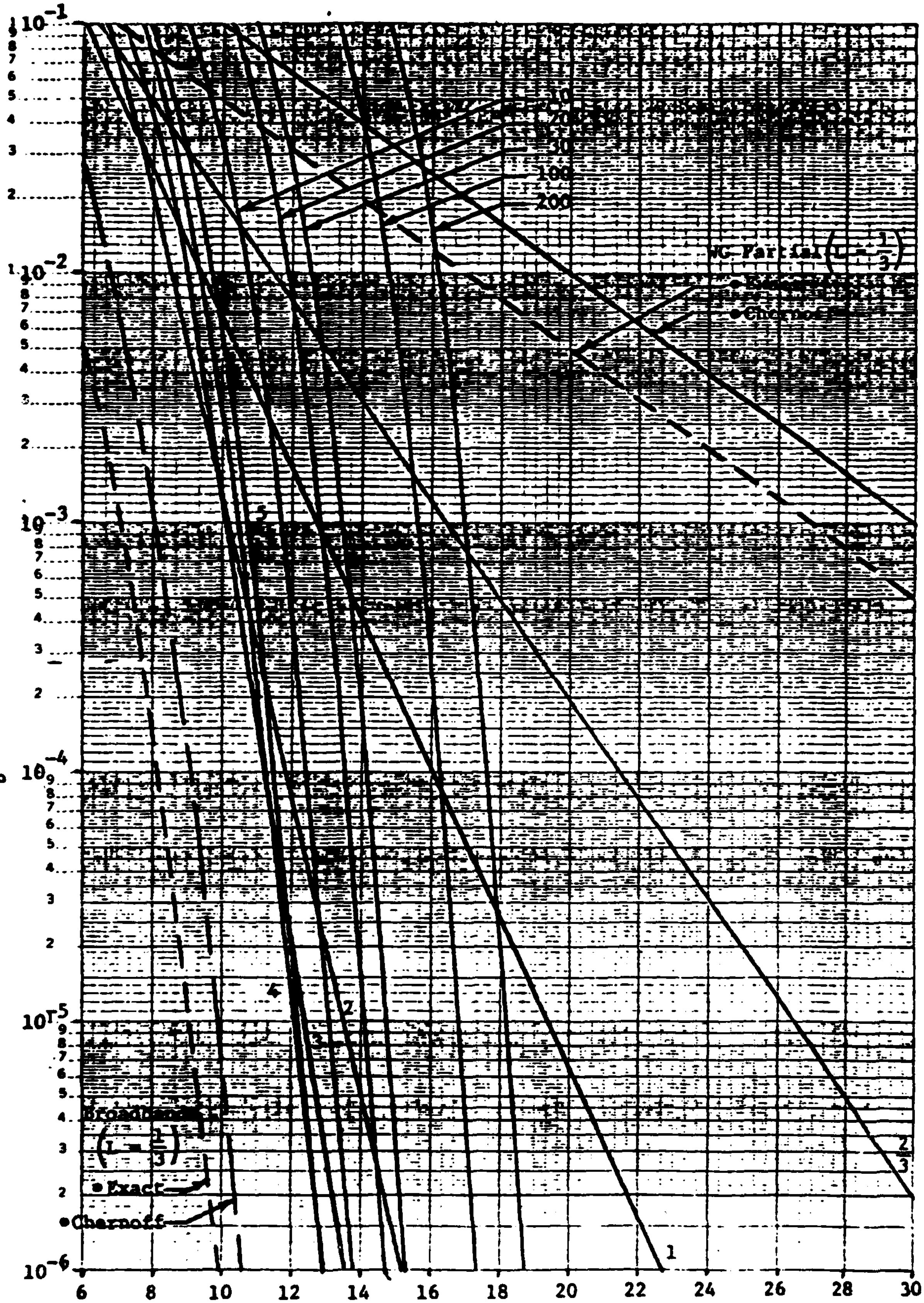

Figure 10 $M = 2,\ r = \frac{1}{2}$

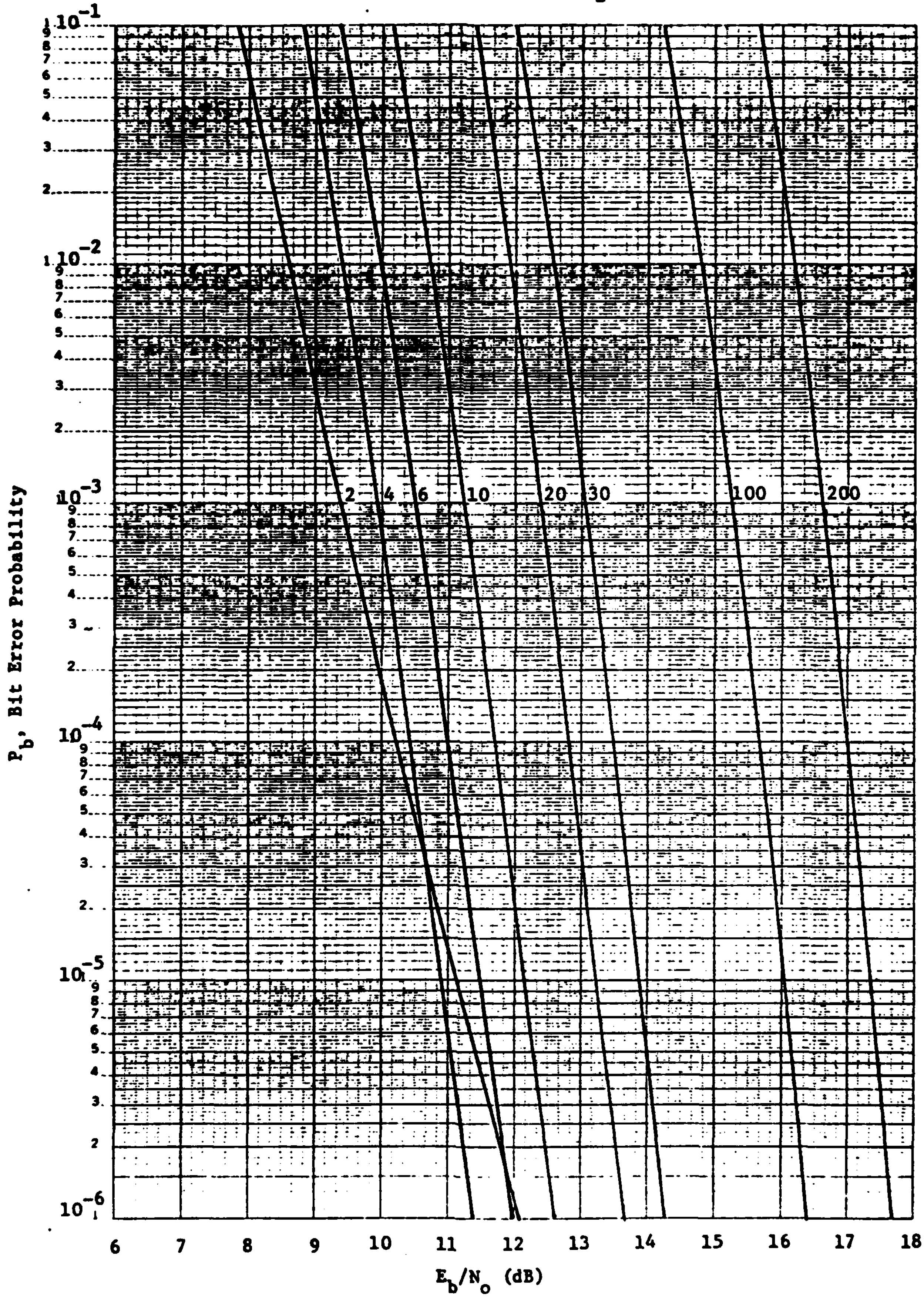

Figure 11 $M = 2,\ r = \frac{1}{3}$

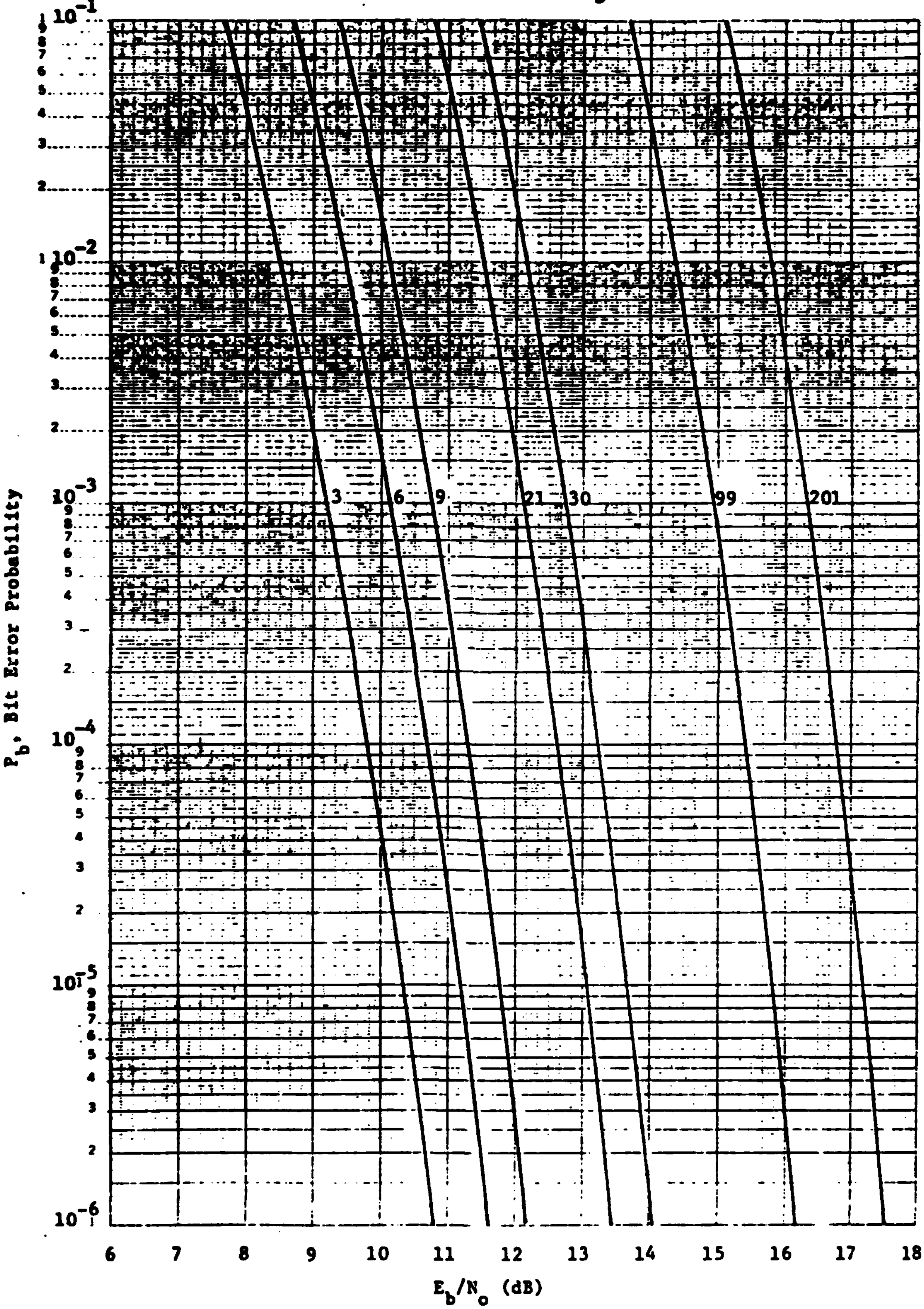

Figure 12 M = 4, r = 1

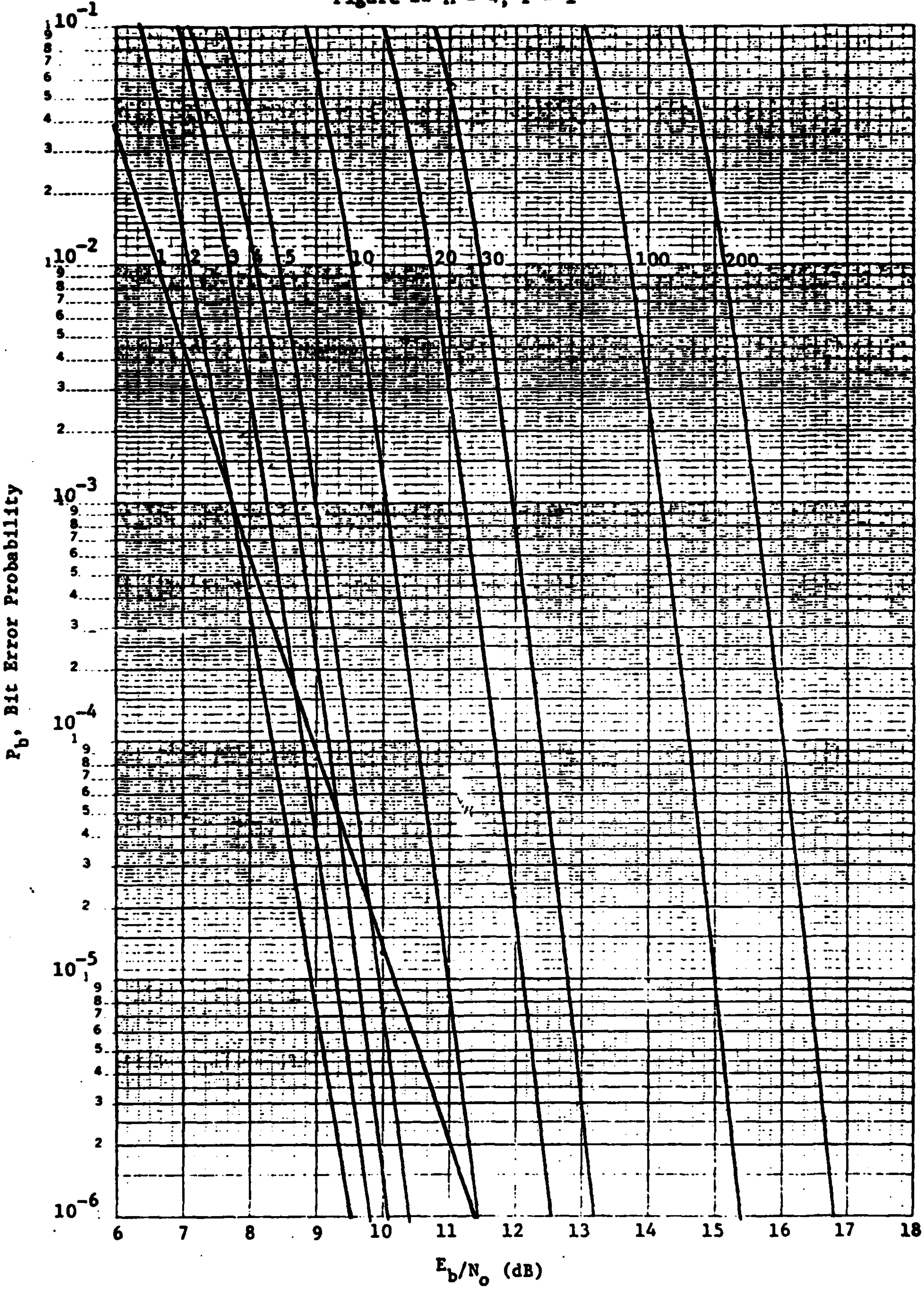

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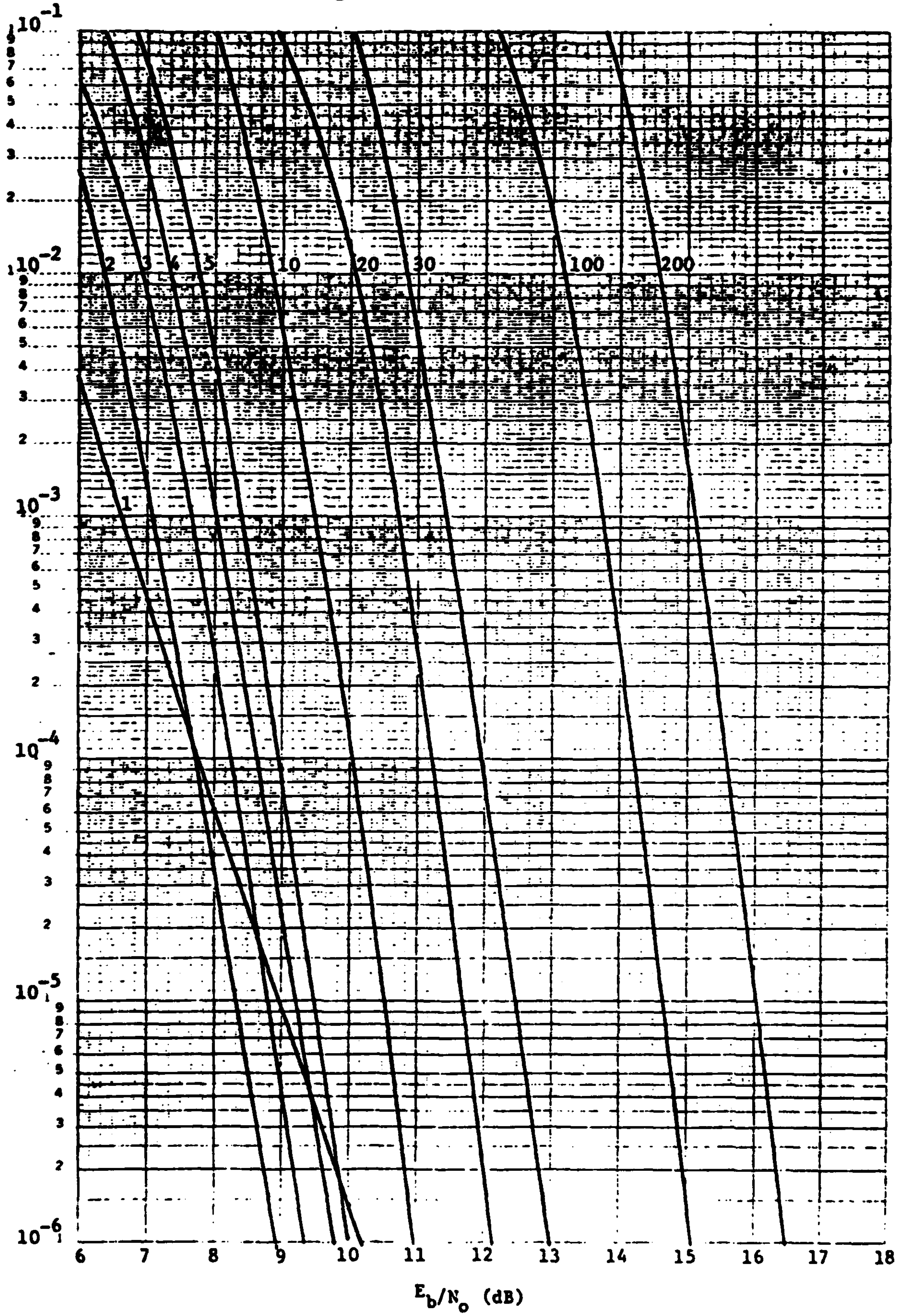
Figure 13 M = 8, r = 1



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This receiver also non-coherently combines the chip energies to make any decisions. Since the chip energy terms are not quantized we refer to this as a soft decision receiver with channel state knowledge. A fundamental question now arises: How do we evaluate the degradation from the results shown in Figure 3-13 for other receivers such as hard decision receivers without channel state knowledge? In Note No. 4 titled, "A General Analysis of Anti-Jam Communication Systems" we present an answer using the cutoff rate parameter.

Figures 14-16 show curves of the cutoff rate versus E_c/N_o, the energy-per-chip to noise ratio, for cases given by:

(1) soft decision and known jammer state
(2) hard decision and known jammer state
(3) soft decision and unknown jammer state
(4) hard decision and unknown jammer state.

All these curves are for the worst case partial band jammer. Two cases with the same value of the cutoff rate will also have the same coded or uncoded bit error bound. Thus using case (1) as a base line with bit error probabilities shown in Figures 3-13 we can see how much degradation we encounter with other cases. To illustrate this further consider the following examples:

Example: Uncoded, M=2, L=5, E_b/N_o=17dB

For case (1) we see from Figure 7 that the bit error is $P_b=3\times10^{-5}$. Here, however, $E_b=5E_c$, and thus E_c/N_o=10dB. From Figure 14 we see the cutoff rate is R_o=.8. For case (2) to achieve the same bit error probability we keep R_o=.8 fixed and find that E_c/N_o=11.3 is required which is a degradation of 1.3dB from the ideal case (1). Similarly for the same bit error bound

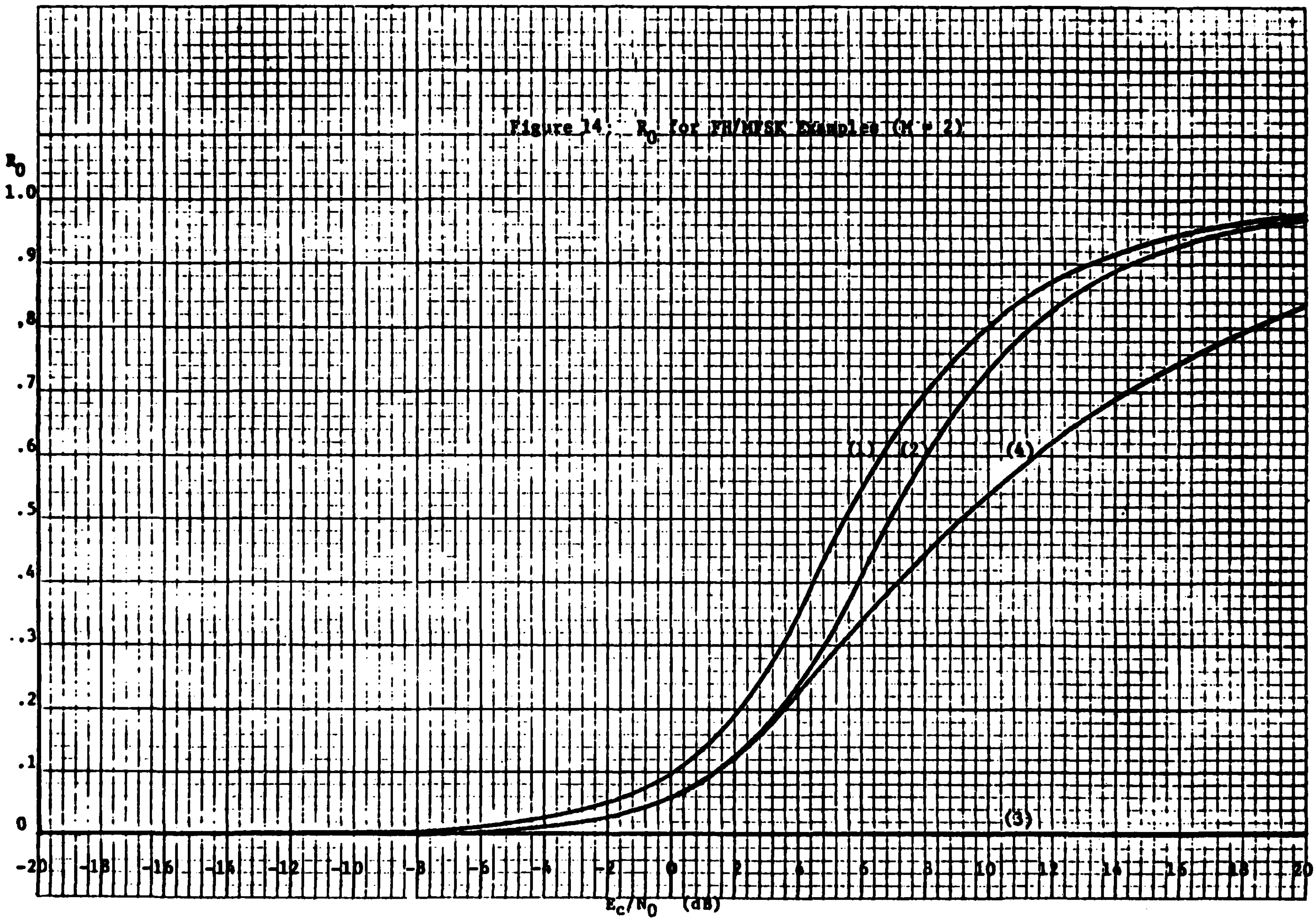

Figure 14: R_0 for FH/MFSK Examples (M = 2)

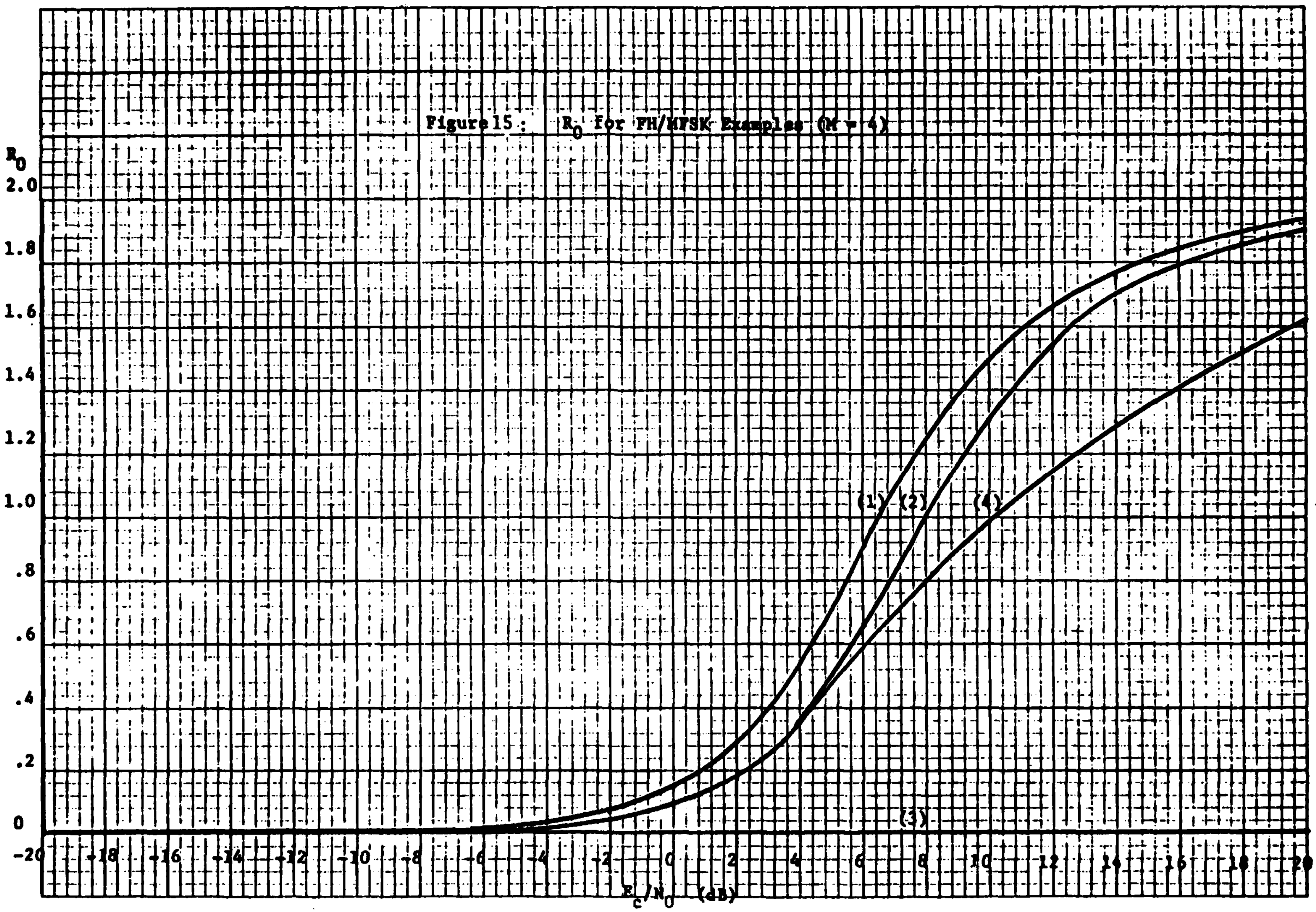

Figure 15: R_0 for FH/MFSK Examples ($M = 4$)

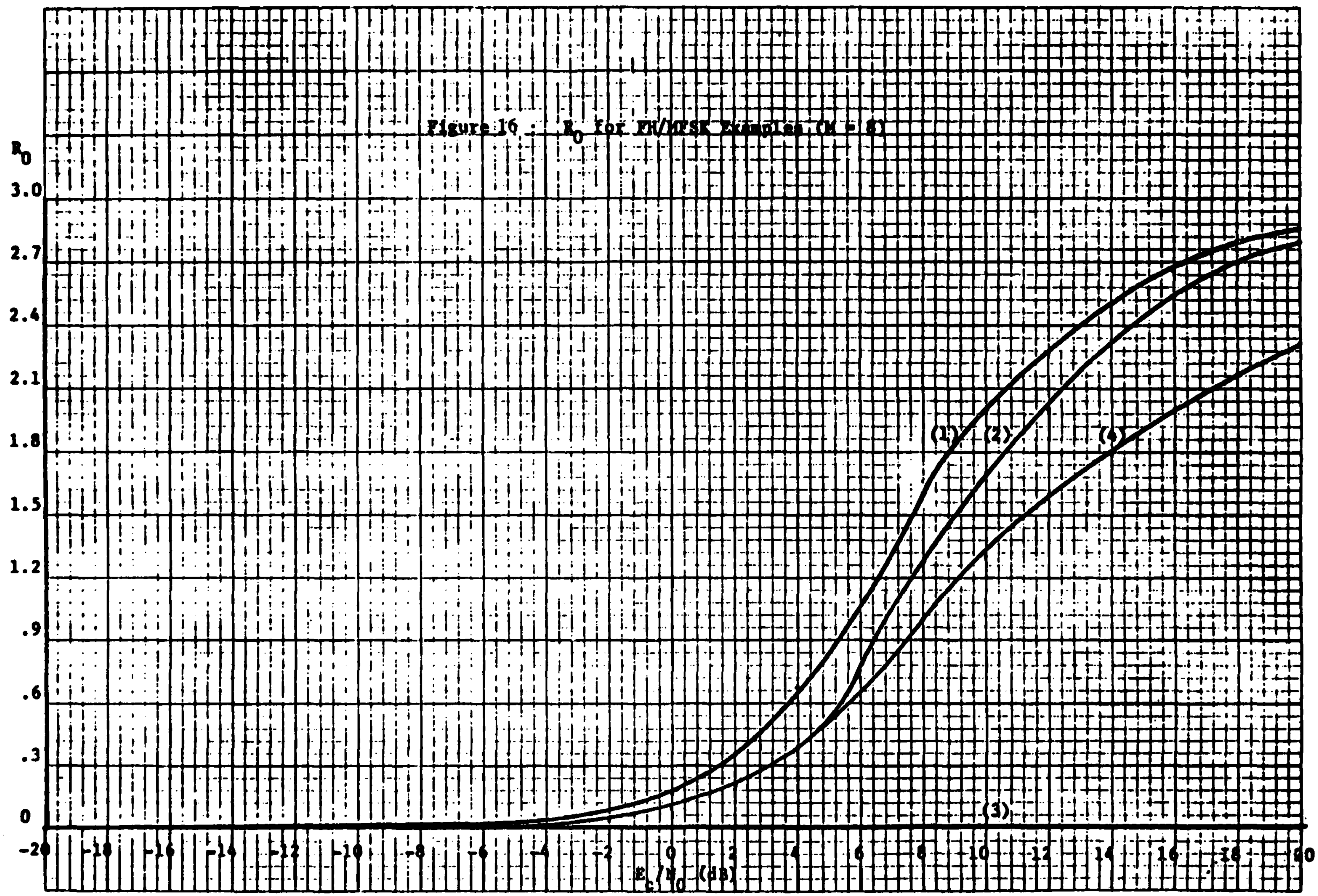

Figure 16 : R_0 for FH/MFSK Examples (M = 8)

case (4) requires 18.2dB and has a degradation of 8.2dB relative to case (1). For hard decision receivers the jammer state knowledge is worth 6.9dB in performance.

$M=4$, $L=10$, $E_b/N_o=11dB$

For case (1) we see from Figure 14 that the bit error is $P_b=2x10^{-4}$. Since $E_b=10E_c$ we have $E_c/N_o=1dB$. From Figure 15 we see that $R_o=.2$ and the corresponding values for case (2) and (4) are the same $E_c/N_o=2.3dB$. This is a degradation of 1.3dB for both case (2) and case (4) relative to case (1). For hard decisions at this operating point the jammer state knowledge is unimportant. Also there is only a 1.3dB loss due to hard decisions.

Note that case (3) has $R_o=0$ for all E_c/N_o. This is because without jammer state knowledge the soft decision receiver against worst case jamming has a bit error probability that cannot decrease exponentially with any code block or constraint length. For cases where $R_o > 0$, there exists codes where the bit error bound can be made to decrease exponentially fast with increasing code lengths.

By using cutoff rates,which we can numerically evaluate,we can now determine the relative degradation various receiver structures have relative to the baseline receiver that assumes jammer state knowledge and noncoherently combines unquantized chip energies to make decisions. In Note No.4 we considered only the four examples described above. Other possibilities include receivers that use three bit quantization on chip energies

before combining or various clipped energy detector outputs. Some sort of list decoding scheme is another possibility. Varying degree of channel state knowledge may be available at the receiver as well as only partial jammer state knowledge. Channel knowledge may include propagation conditions and interference levels across the HF band measured every 30 minutes using sounders. This low data rate information can be provided throughout the HF ITF network using an overhead channel.

Up to this point we have only discussed ground wave propagation where there is no fading assumed. Also we assumed uniform propagation conditions across the spread spectrum channel band. For sky wave propagation we now encounter Rayleigh fading and non-uniform propagation conditions across the spread spectrum channel band. In Note No. 5 titled, "Fading Dispersive Channels", we present a somewhat tutorial discussion of models for fading channels and some basic performance analysis. An important concept introduced here is the tradeoff one can have between hop rates and interleaving. If, for example, the hop rate is too slow to provide adequate diversity then the desired diversity can be achieved by making several chips per hop and interleaving these among different hop intervals.

A complete study of the fading skywave HF channel was done by Ph.D. candidate Dan Avidor. His Ph.D thesis is attached here along with partial results presented in Note No.6 titled, "Anti-Jam Analysis of FH/MFS Systems in Rayleigh Fading Channels". Here we consider a general model where each MFSK slot in the spread

spectrum band can have different fading and noise parameters and the jammer is allowed to distribute his power in any manner across the spread spectrum band. Hence each MFSK slot is characterized by an average chip energy, noise spectral density, and jammer noise spectral density. We allow, however, the frequency hopping pattern to be non-uniform across the available MFSK slots.

For the special case of no noise and uniform propagation condition across the spread spectrum channel, with Rayleigh fading the worst case jammer is the uniform or broadband jammer. Here the hopping pattern is also uniform. For the general case we assumed a minimax strategy where the frequency hopping pattern or probability distribution for the MFSK slots was chosen to minimize the error bound that is first maximized by the worst case jammer power distribution.

Huristically, the worst case jammer power distribution is such that the jammer places its jammer power primarily into the MFSK slots that have good energy-per-chip to noise ratios. In this way it attempts to make all MFSK slots uniformly bad. Of course when there are lots of good MFSK slots available to the anti-jam system then all the jammer can do is degrade all these slots uniformly. Basically it attempts to use its power to degrade the channel slots with the highest E_c/N_o first until many of the better channel slots have the same degraded E_c/N_o values. This means some of the worst slots may never have jamming power since it already has smaller E_c/N_o than the jammed slots.

Taking into account the above worst case jammer strategy, the minimax frequency hopping probability distribution for the

MFSK slots is based on the expected E_c/N_o on each slot. This strategy is conservative since any other jammer power distribution cannot degrade performance further.

The skywave HF channel is considerably more complex due to the non-uniform conditions across each spread spectrum band. For the uniform condition special case the only difference from the ground wave results is the addition of Rayleigh fading. These cases are covered in Notes No.7 and No.8. Note No.6 examines the evaluation of the cutoff rate parameters for a variety of receiver types when we have a Rayleigh fading channel. Although we assume a uniform channel here, it can be easily generalized to include non-uniform channels. Most of the non-uniform channel results are in Avidor's thesis. For the case of identical independent Rayleigh fading in each MFSK band, Figures 17-23 are the results corresponding to the non fading results of Figures 7-13.

Figure 17 Fading M=2

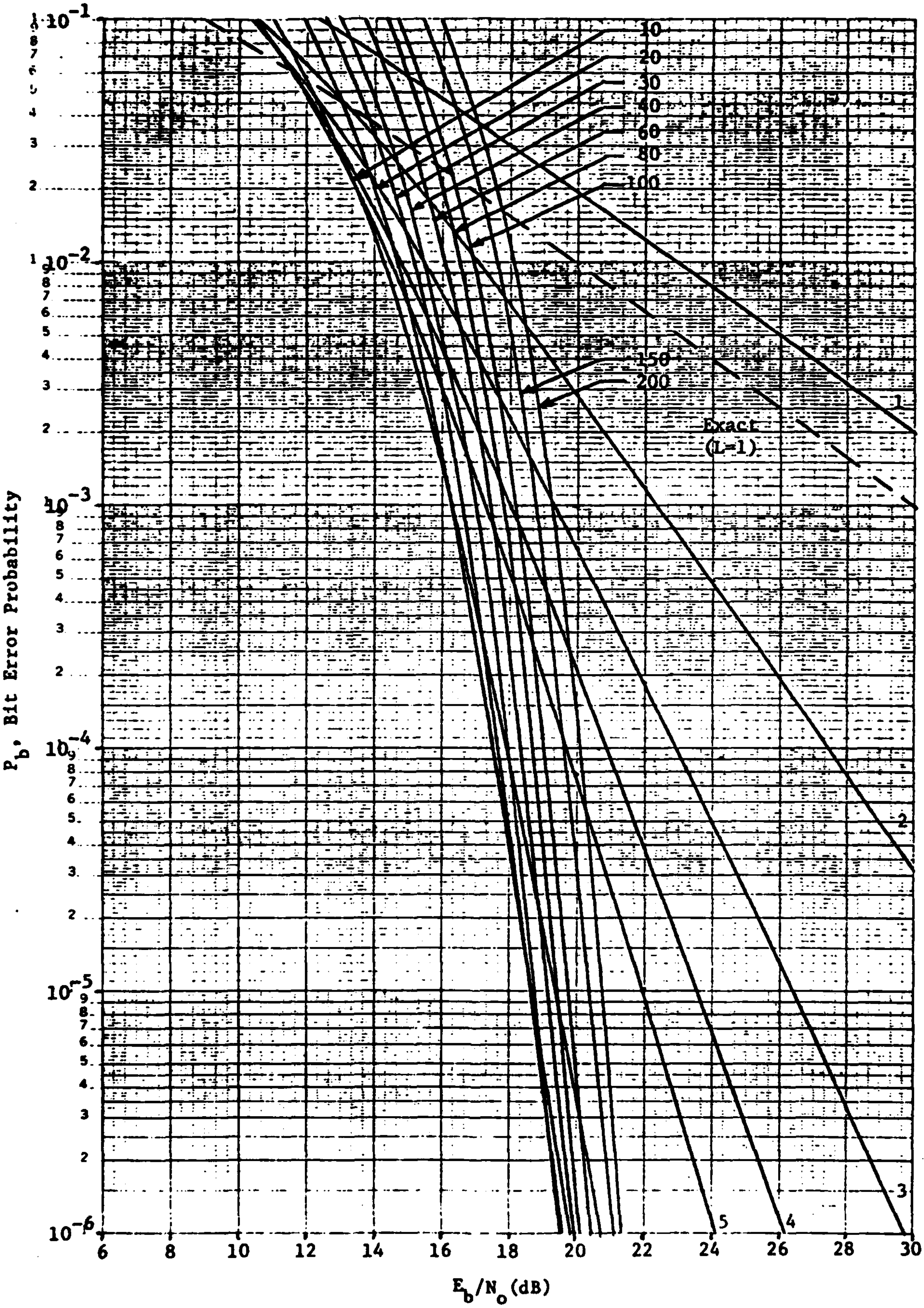

Figure 18 Fading M=4

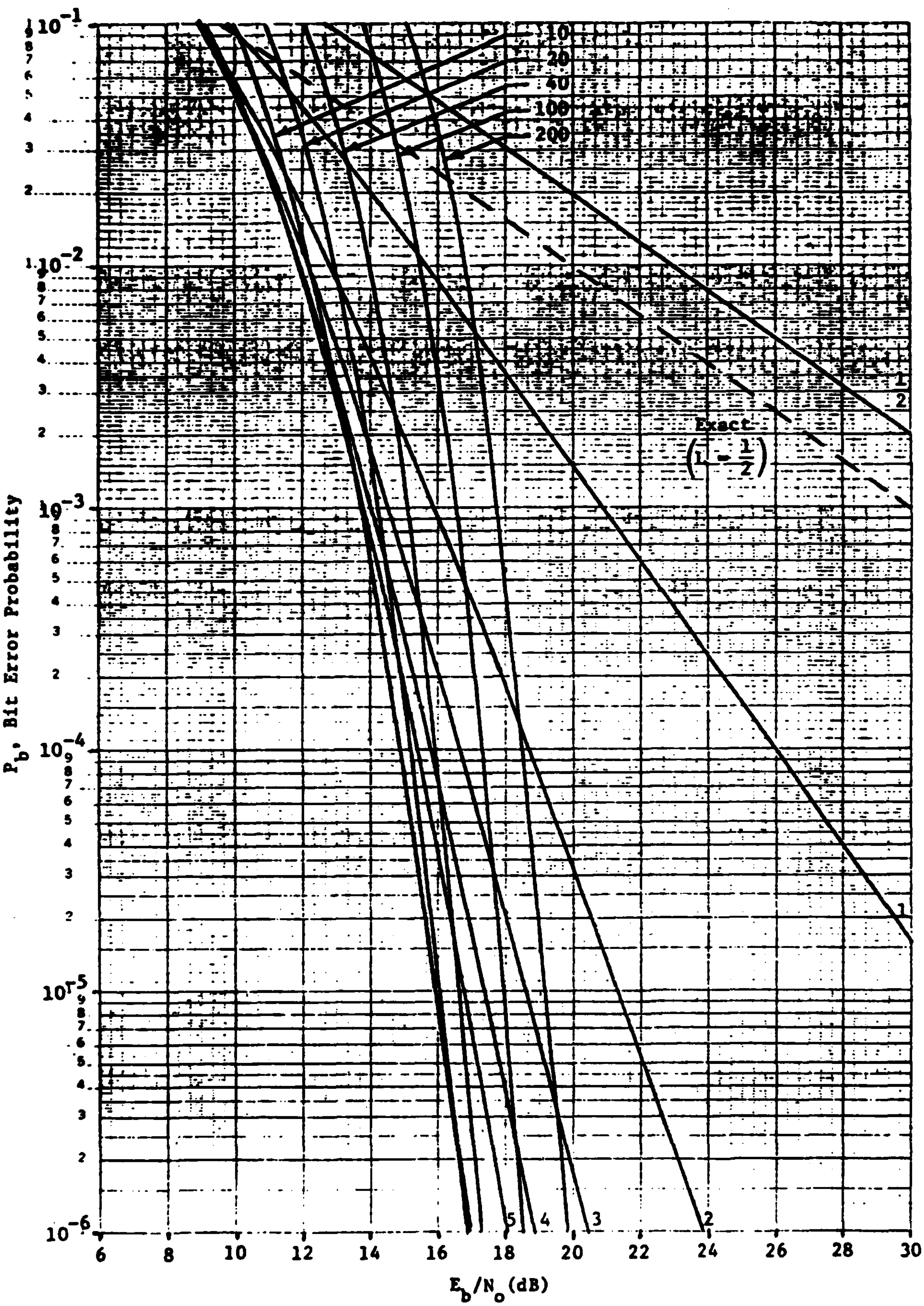

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Figure 19 Fading M=8

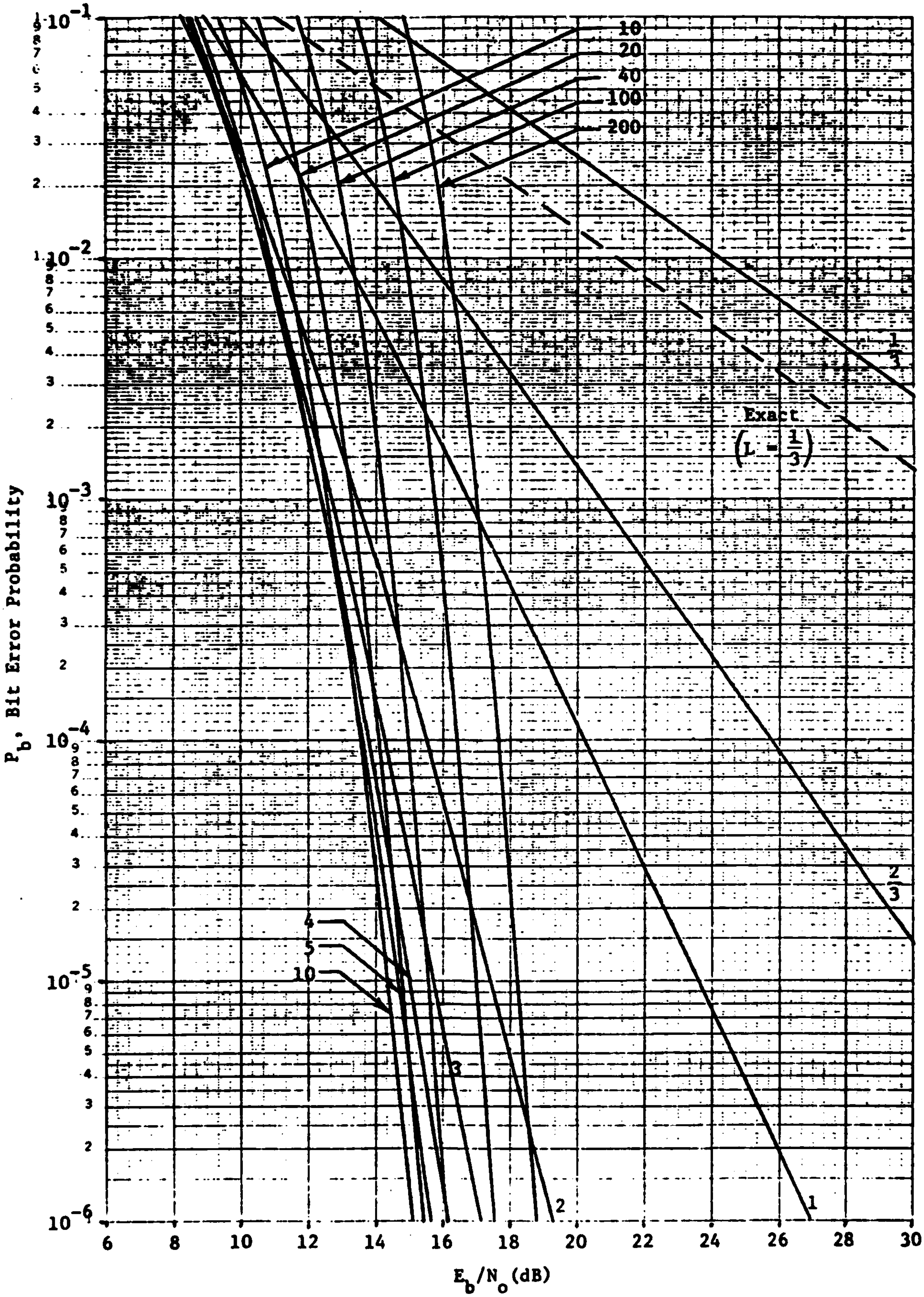

Figure 20 $M = 2$, $r = \frac{1}{2}$, Fading

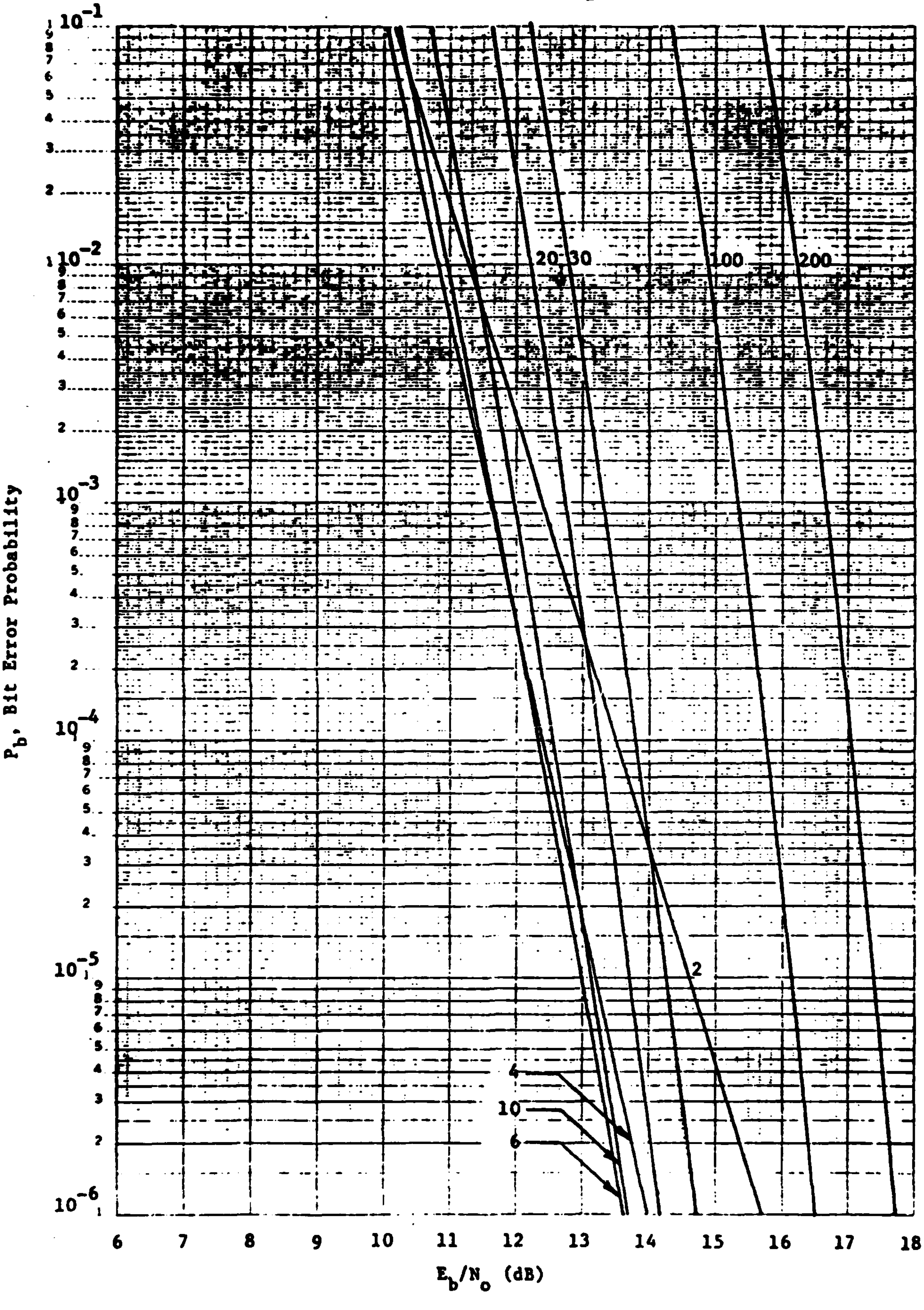

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Figure 21 $M = 2,\ r = \frac{1}{3}$, Fading

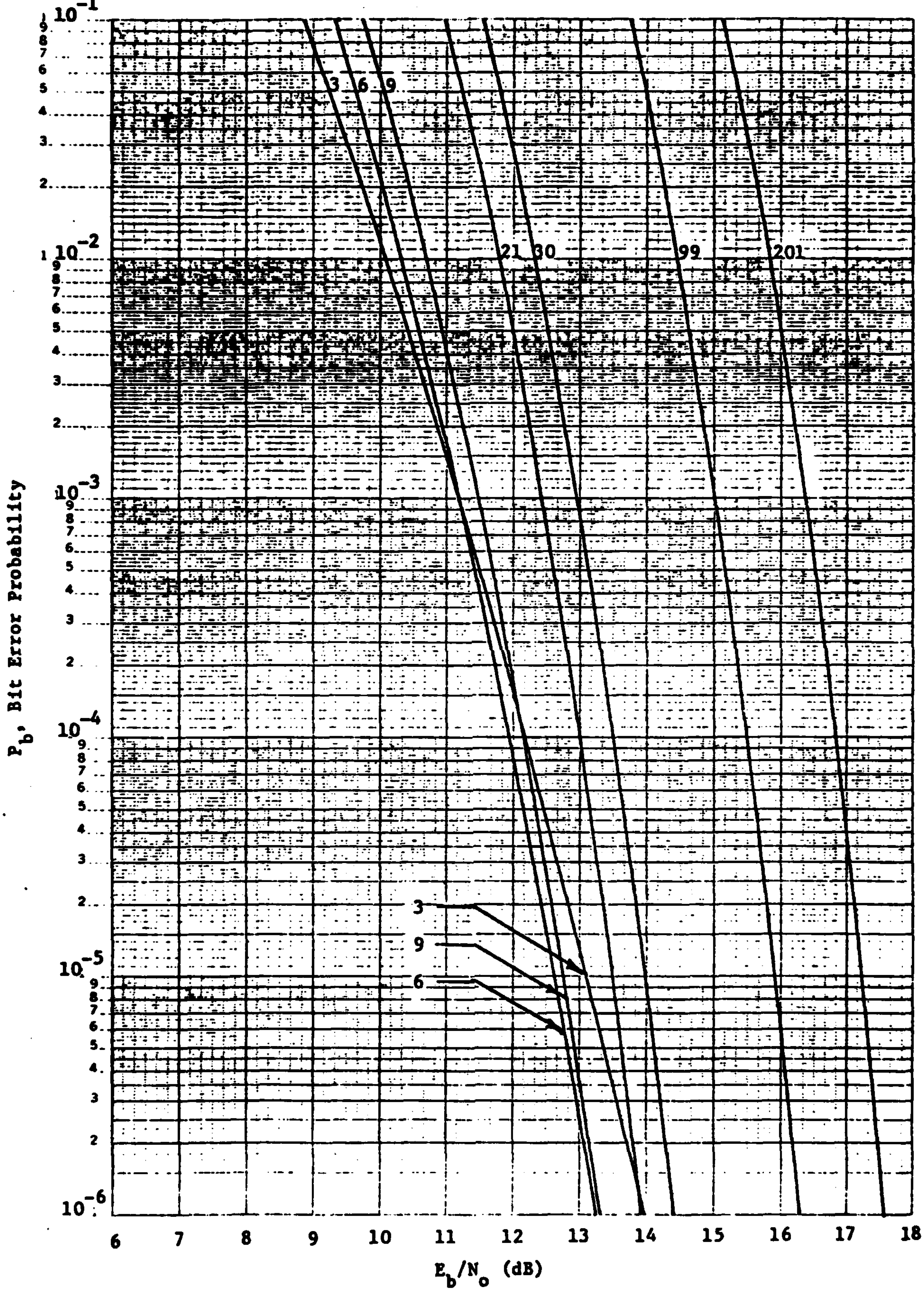

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Figure 22 M = 4, r = 1, Fading

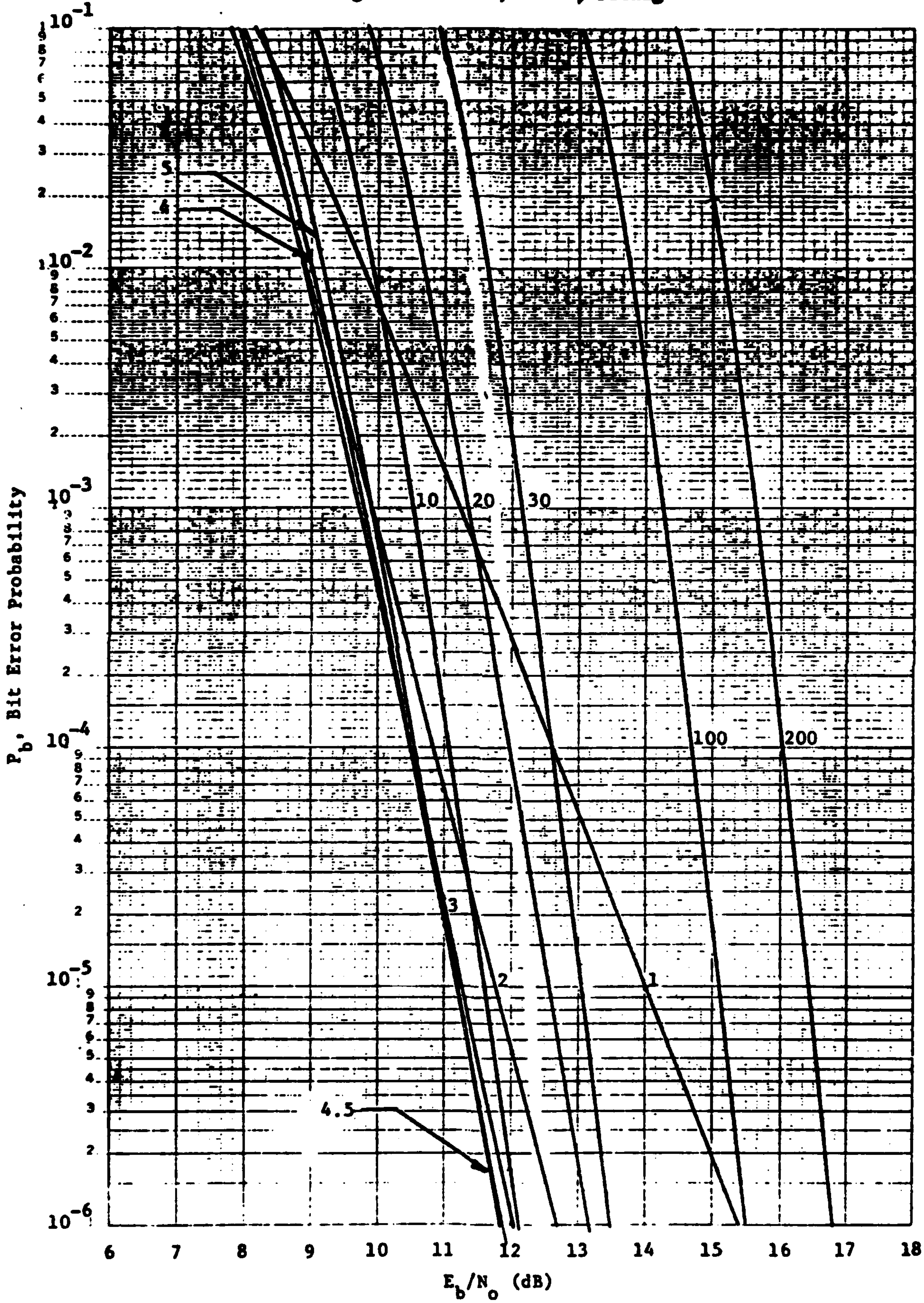
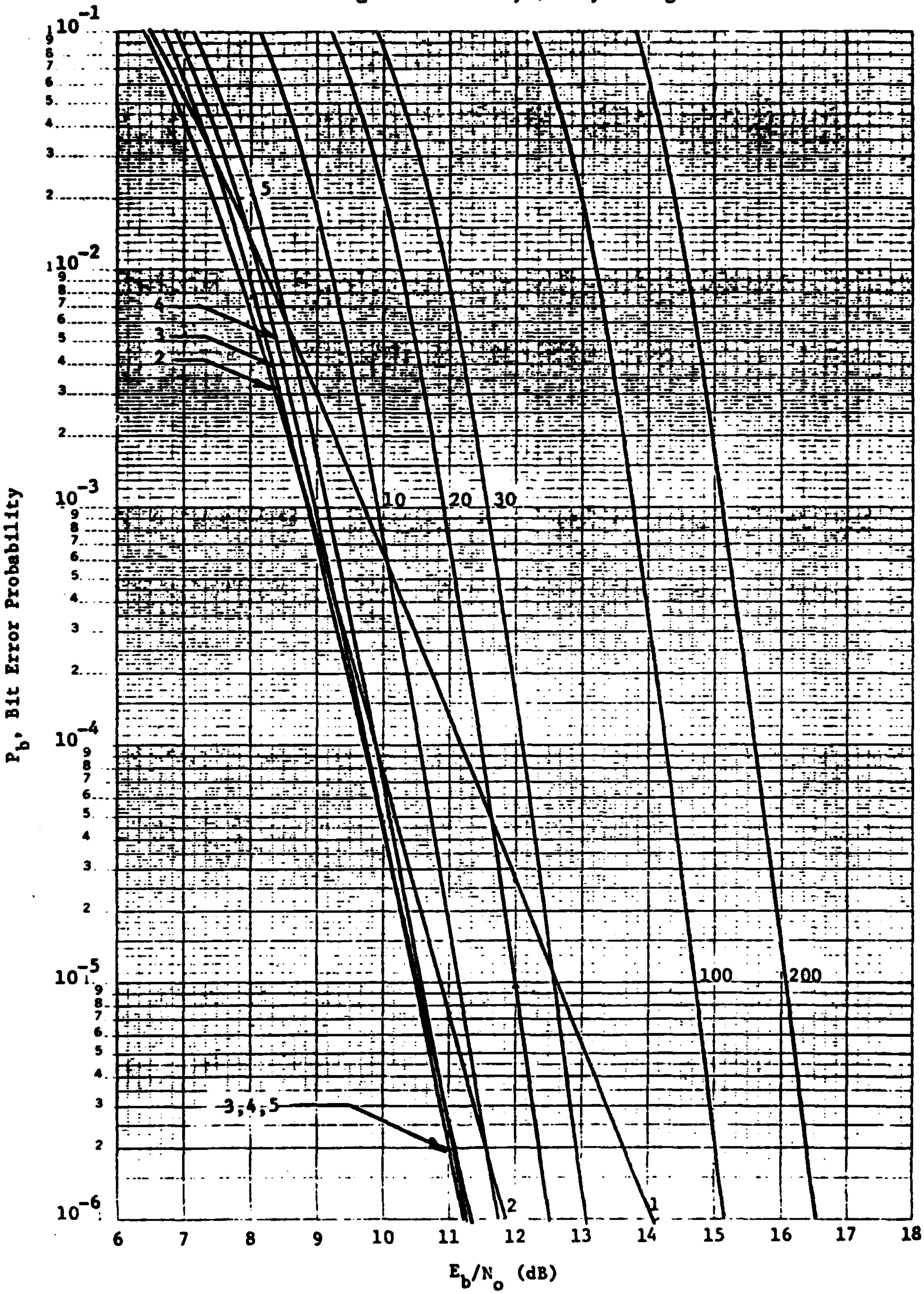


Figure 23 M = 8, r = 1, Fading



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V. Recommendations

The HF channel is a complex channel with ground waves and fading skywaves. There are also strong interferences due to distant thunderstorms and man-made radio signals which have both short term and long term random variations. These natural channel conditions together with intentional jamming and other local user simultaneously transmitting in the same frequency band makes the task of evaluating performance difficult. In addition, there are no obviously good receiver designs and so we must evaluate a variety of different types of detectors and decision metrics of a receiver.

In the current NRL supported grant we examined the performance of various coded frequency hopped MFSK waveforms for HF channels with jamming. Here we developed a useful analysis technique based on a generalized cutoff rate parameter which allows us to easily compare the performance of a bariety of point-to-point coded HF communication systems. This included various types of jammers and receiver structures but not the problem of other users simultaneously transmitting in the same frequency band. We recommend that this work be extended to the network environment where there are many users simultaneously using the same spread spectrum HF frequency band. In addition, one should examine new forms of receiver detectors including those with erasures, lists, nonlinear weighting, and any quantization together with a variety of associated metrics for making decisions. Groundwave and skywave (Rayleigh fading) channels should be assumed along with the partial band Gaussian noise jamming and multitone jamming.

APPENDIX: Generation of PN Sequences for Frequency Hopping Patterns

We briefly sketch a standard way of generating pseudo noise (PN) binary sequences using linear feedback shift registers (LFSR). Consider the three stage LFSR shown in Figure A-1. Here the adder denoted $\oplus$ is a modulo-2 addition. For this LFSR we have two cycles where if we start with any non-zero sequence of 3 bits in the register and we periodically cycle through all possible seven non-zero sequences of three bits. The other cycle is the trivial all zero cycle that always exists.

Figure A-2 illustrates another LFSR example but with four stages. This example has the trivial all zero cycle and three non-zero cycles. In general for n stages there are several LFSR's with only two cycles, the trivial all zero cycle and a non-zero cycle. Since the non-zero cycle must include all possible non-zero sequences of length n once and only once this cycle has period

$$\text{Period} = 2^n - 1$$

These LSFR's are call maximal length LFSR's and their non-trivial binary sequences are call maximal *length sequences*. Note that Figure A-1 shows a three stage maximal length LFSR while the four stage LFSR in Figure A-2 is not a maximal length LFSR.

Note that during a single period of a maximal length sequence there are 2^{n-1} one's out of 2^{n-1} binary symbols. For n large this is approximately half the bits. Indeed, for large n the maximal length sequence looks like a purely random binary sequence. Naturally, if two such identical LFSR's are started with the initial n register bits and synchronized in time then they would generate identical "random like" binary sequences. For n = 64 for example, the period would be 1.89×10^{19} binary symbols. Suppose these LFSR's are clocked at 10 MHz rate, then one period would last 1.89×10^{12} seconds or 58,409 years.

In a spread spectrum application it is important to have PN sequences with a long period, easy implementation, and be difficult for anyone to predict future parts of the PN sequence with observations of the past of the sequence. The maximal length LFSR's are easy to implement and can have long periods. However,

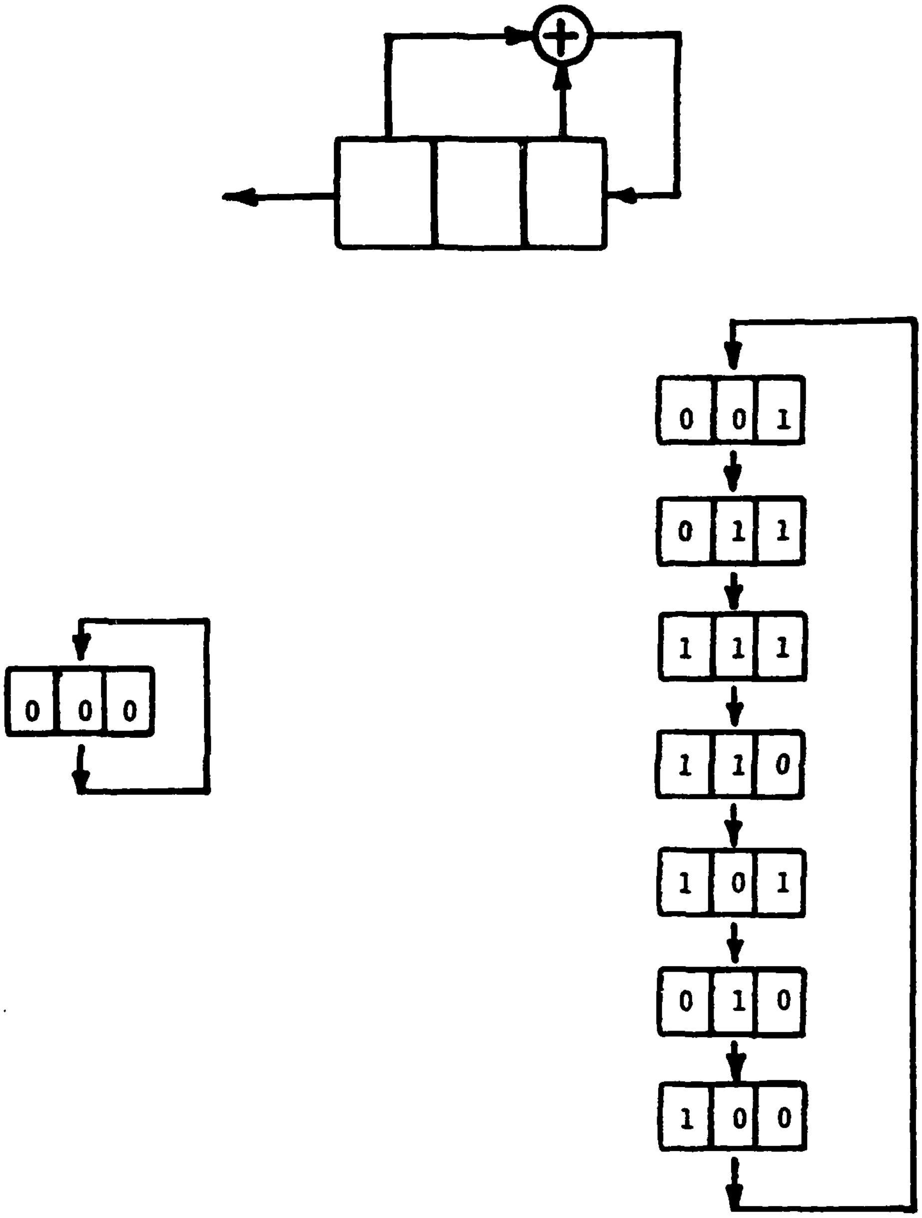

Figure A-1:3 Stage LFSR

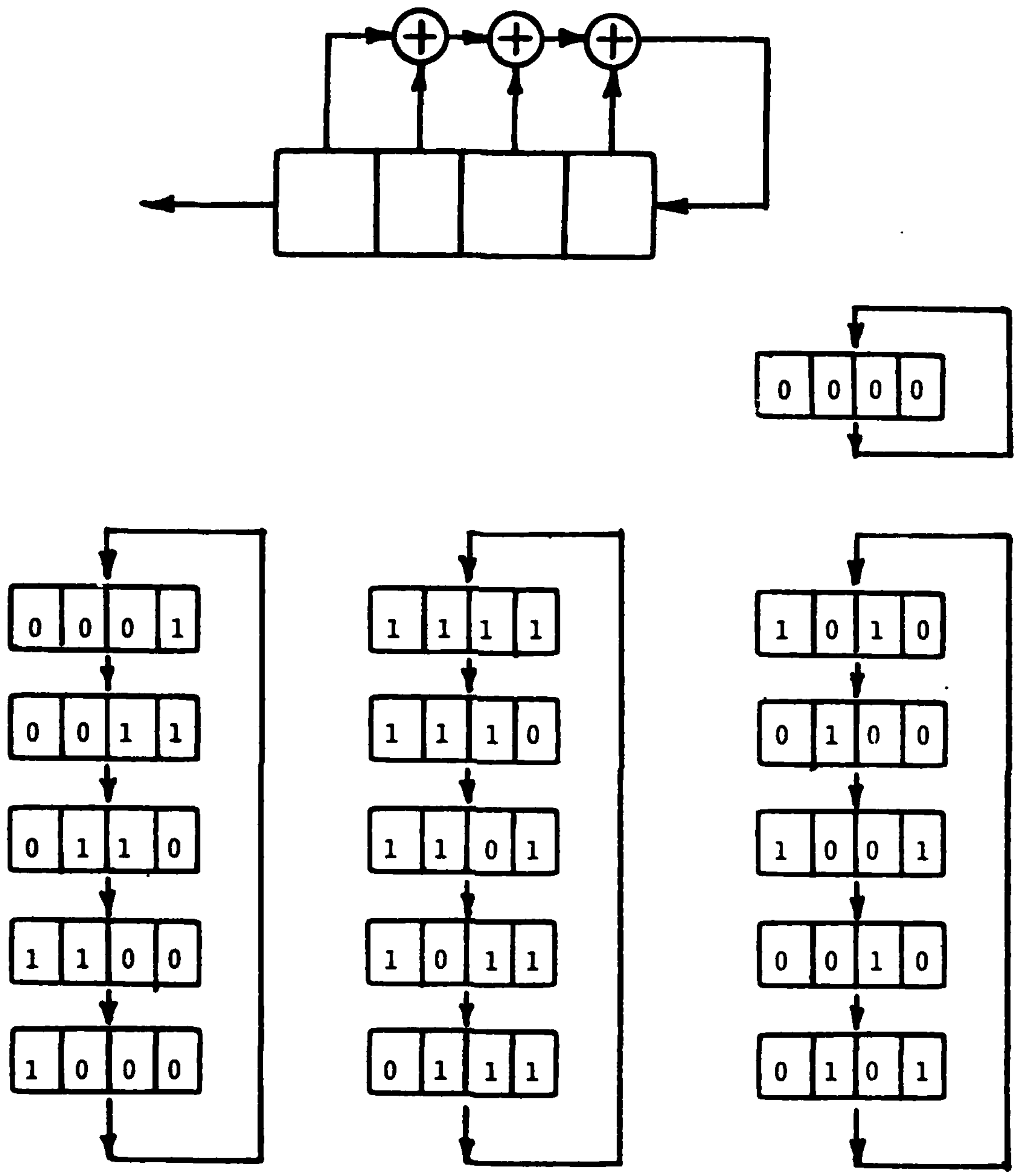

Figure A-2: 4 Stage LFSR

by observing only 2n bits of the PN sequence and using the Berlekamp-Massey algorithm [5], the LFSR can be known exactly and all future parts of the PN sequence can then be generated or predicted. Hence LFSR's cannot be used without some nonlinear modifications. An example of a popular nonlinear modification of a maximal length LFSR is shown in Figure A-3 where nonlinear output logic (NOL) is used. The NOL makes the sequence difficult to predict [6].

Let us now examine how a PN sequence is used in the FH/MFSK waveform proposed for the HF ITF network. Figure A-4 shows a hypothetical 3 MHz spread spectrum channel (10 MHz to 13 MHz) where we have divided this sub-band into many narrow MFSK signal slots. Each MFSK signal slot is roughly

$$B = \frac{M}{T_c} \text{ Hz}$$

where T_c is the MFSK "chip" duration or equivalently

$$T_c = \frac{1}{R_h}$$

where R_h is the hop rate. Here we assume the hop rate R_h is always greater or equal to the MFSK symbol rate. Each hopped MFSK signal is referred to as a "chip" A single MFSK signal may be hopped several times and thus consist of several chips. In Figure A-4 we show how the energy-per-chip to noise ratio (E_c/N_o) might vary across the 3 MHz spread spectrum sub-band.

If E_c/N_o is uniform or constant across the sub-band then a natural frequency hopping strategy would be to hop with equal probability into any slot. Hence if there were $N = 2^{K_o}$ slots then every K_o bits of the PN sequence can be used to select the slots that are hopped into in a time sequence. The PN generator

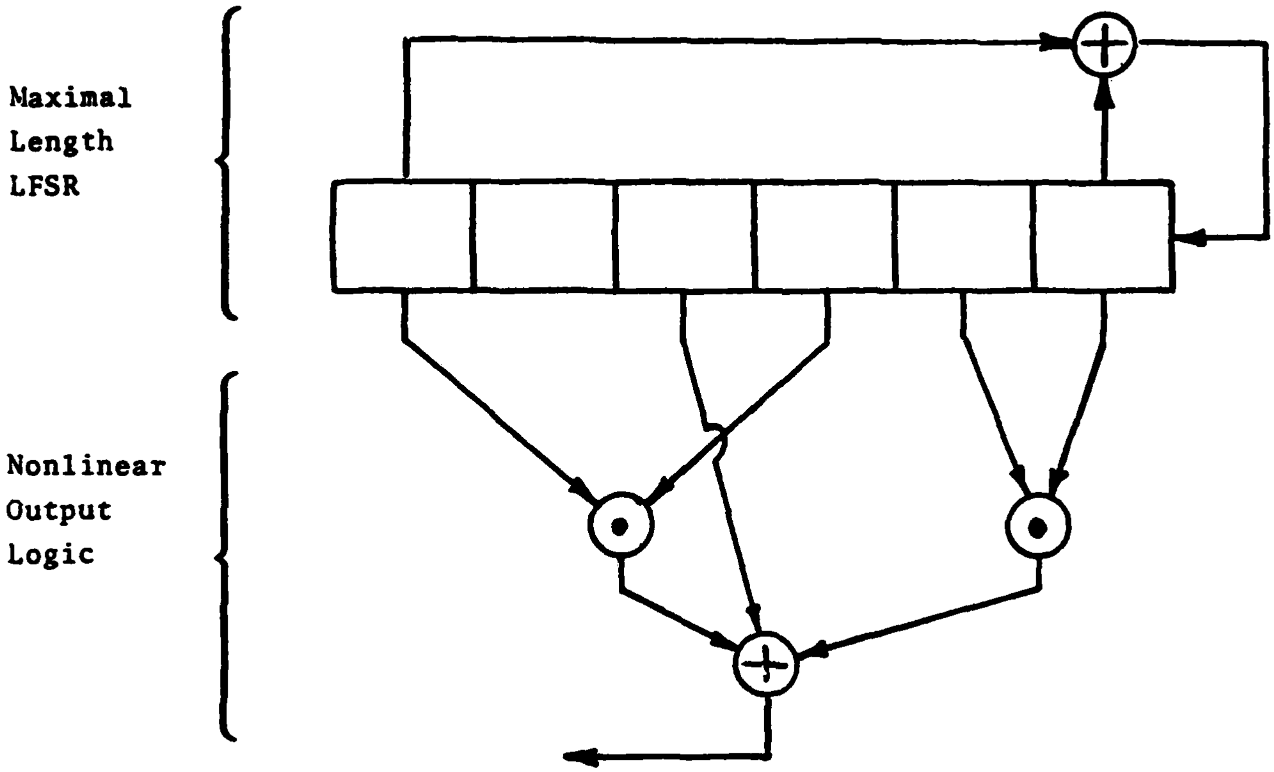

Figure A-3: Nonlinear Modified LFSR

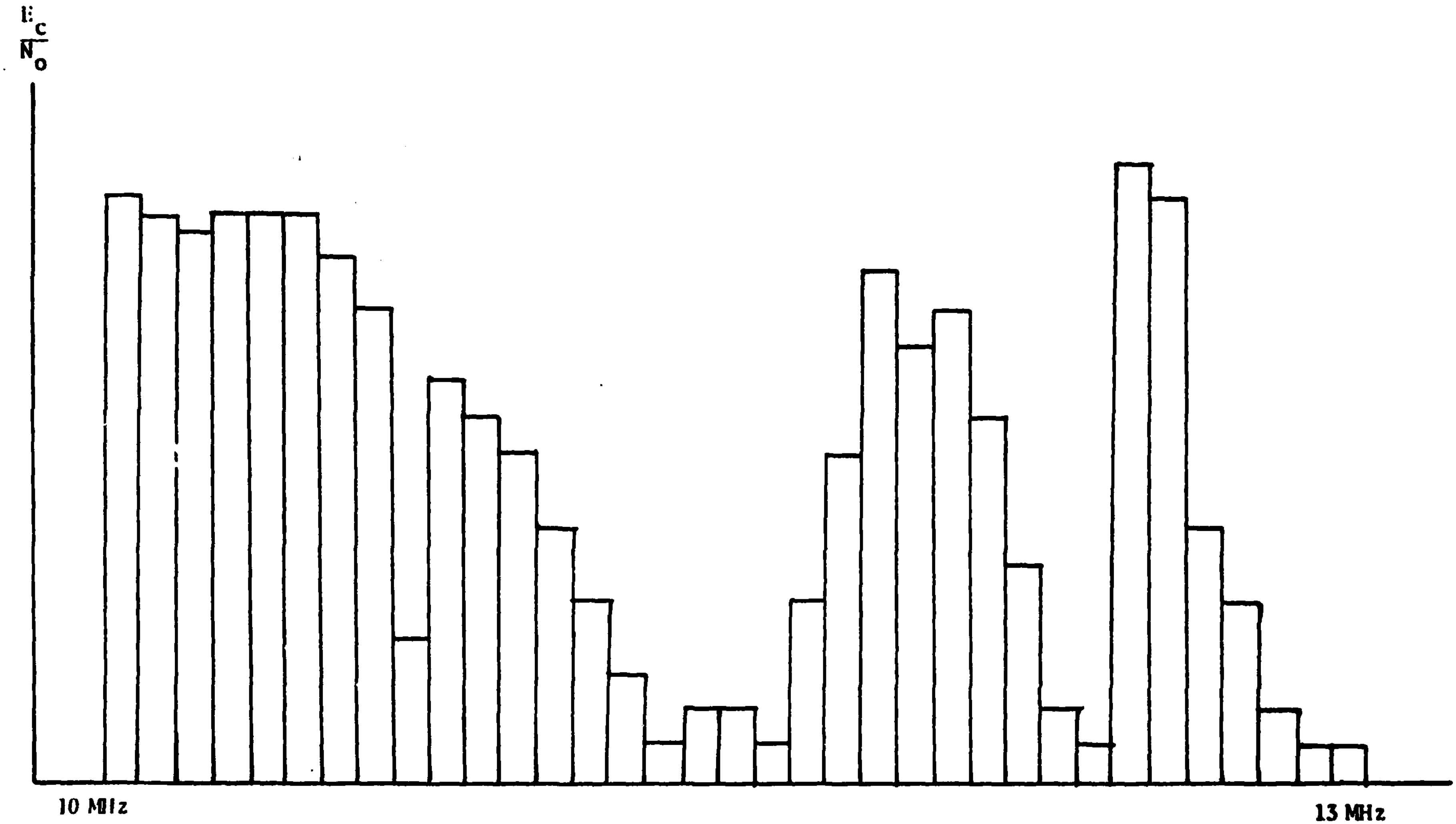

Figure A-4: A Spread Spectrum Channel

would generate bits at K_oR_h bits/seconds.

If, on the other hand, E_c/N_o is not uniform as sketched in Figure A-4,then we would require some sort of non-uniform frequency hopping strategy. This can be accomplished by choosing $Q = 2^q$ to be much larger than the number of available slots. Then every q bits of the PN sequence can be used to select one of the Q values. By assigning the Q values in varying numbers to the $N = 2^{K_o}$ slots you can achieve a non-uniform hopping strategy.

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[1] D. Baker, "HF Intra Task Force Network Study Guidelines", Technical NFL Memorandum, NRL Prob.75R01-98A, 10 September 1979.

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[4] (S) Naval Telecommunications System Architecture - 1985(U).

[5] J.L.Massey, "Shift-Register Synthesis and BCH Decoding", IEEE Trans. Info. Thy., vol.IT-15, pp.122-127, Jan.1969.

[6] E.L.Key, "An Analysis of the Structure and Complexity of Nonlinear Binary Sequence Generators", IEEE Tran. Info. Thy., vol. IT-22,pp.732-736, Sept. 1976.

Note No. 1

IMPACT OF SPREAD SPECTRUM SIGNALS ON MULTIPLE ACCESS DESIGN

to

NAVAL RESEARCH LABORATORY

(Contract Award No. N00014-80-K-0935)

for

HF COMMUNICATION NETWORK SIGNALS

USING CHANNEL EVALUATION DATA

Principal Investigator

Jim K. Omura
Professor
System Science Department
University of California
Los Angeles, California

November, 1980

IMPACT OF SPREAD SPECTRUM SIGNALS ON MULTIPLE ACCESS DESIGN

In this note, we comment on some impact spread spectrum signals can have on multiple access design for a network of users such as the HF ITF Network.

I. Spread Spectrum Signals

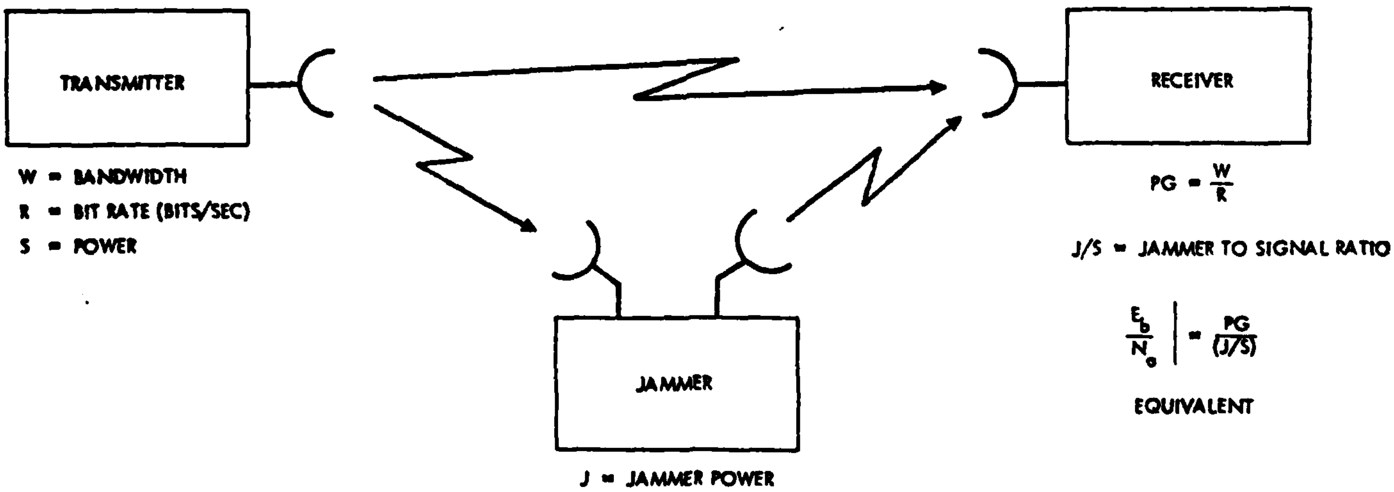
Spread spectrum signals are designed to combat intentional jamming. Suppose the point-to-point single user to single receiver requirements are to transmit R bits per second with total available bandwidth of W Hz The "processing gain" is thus defined as

$$PG = \frac{W}{R}$$

If the signal power is S and a jammer of power J exists, then the jammer-to-signal power ratio is denoted (J/S). The equivalent energy per bit-to-noise ratio is

$$\left.\frac{E_b}{N_o}\right|_{\text{equivalent}} = \frac{PG}{(J/S)}$$

This is sketched in the following figure:



The bit error probability depends on the type of spread spectrum signals, coding and interleaving, and the jammer signal waveform. For the worst case jammer waveform of average power J the bit error probability has the form

$$P_b = F\left(\left.\frac{E_b}{N_o}\right|_{\text{equivalent}}\right)$$

Here F(·) depends on spreading technique, interleaving, and coding. In general

there is at most a 3dB difference between the two basic spreading techniques

- Coherent direct sequencing (DS)
- Noncoherent frequency hopping (FH)

There is, however, a big difference in pseudo random code synchronization requirements. The FH approach generally requires much less accurate time references and lower speed pseudo random (PN) sequence generation than the DS approach.

In a network of many bursty users timing and PN code syncronization requirements will heavily favor the use of frequency hopping as the primary spread spectrum technique. We shall assume here that FH is used throughout the network to provide anti-jamming protection.

II. CDMA, TDMA, and FDMA

Spread spectrum signals are designed to combat intentional jamming. For this same reason, they are also designed to perform well with many other signals in the same signal band. For example, if a spread spectrum signal is designed to perform well against a jammer-to-signal power ratio of

$$\frac{J}{S} = 30\text{dB} = 1000$$

then having L = 100 other users in the same signal band with equal power S results in an increase of equivalent jammer-to-signal power ratio of

$$\left.\frac{J}{S}\right|_{\text{equivalent}} = 1000 + 100 = 1100 = 30.41\ \text{dB}$$

This additional .41 dB would have small impact of the performance of each user and yet allow many users to simultaneously use the total spread bandwidth.

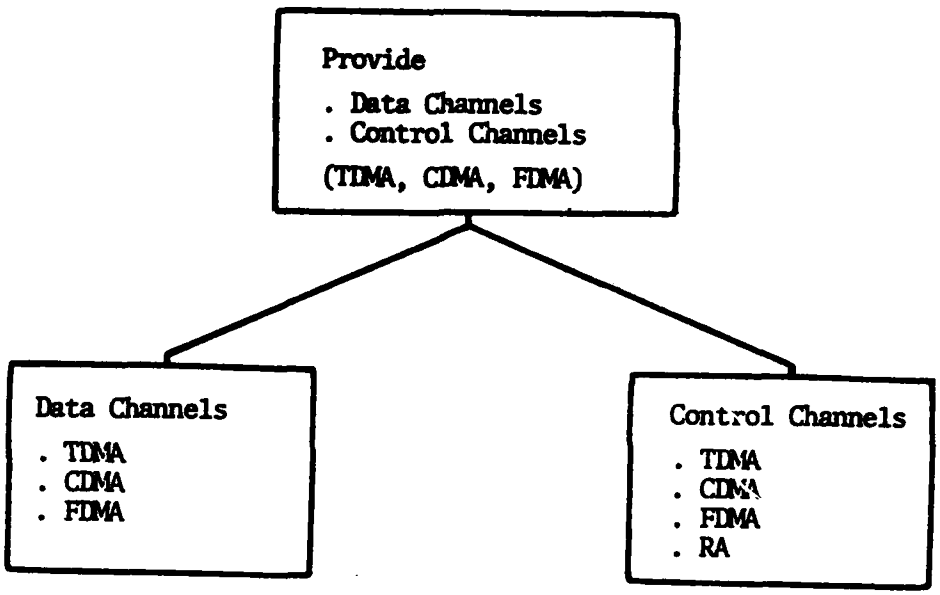
This code division multiple access (CDMA) is a natural bye product of spread spectrum signalling designed to combat jamming. From a practical viewpoint, however, if all L users wish to communicate to one central receiver then that receiver must have L pseudo random sequence generators to extract each of the L transmitted signals. Hence at first it seems that conventional TDMA would be the natural multiple access technique. Conventional FDMA reduces the effective spread spectrum signal bandwidth of each user and thus reduces the anti-jamming effectiveness. Hence for spread spectrum, CDMA and TDMA are the primary candidates for multiple access. An exception to this is a nonconventional FDMA technique we discuss next.

III. Control and Data Channels

In a network with a mix of many types of users including some having bursty

traffic, dedicated channels on a fixed assignment basis is impractical. Hence there is a need for control signals between users and various control centers or nodes. How should this additional communication requirement fit together with the regular data transmission requirements?

We now have a hierarchy of multiple access systems as shown in the following figure. First we need to provide separate data channels and control channels. This might be done using any of the multiple access techniques.

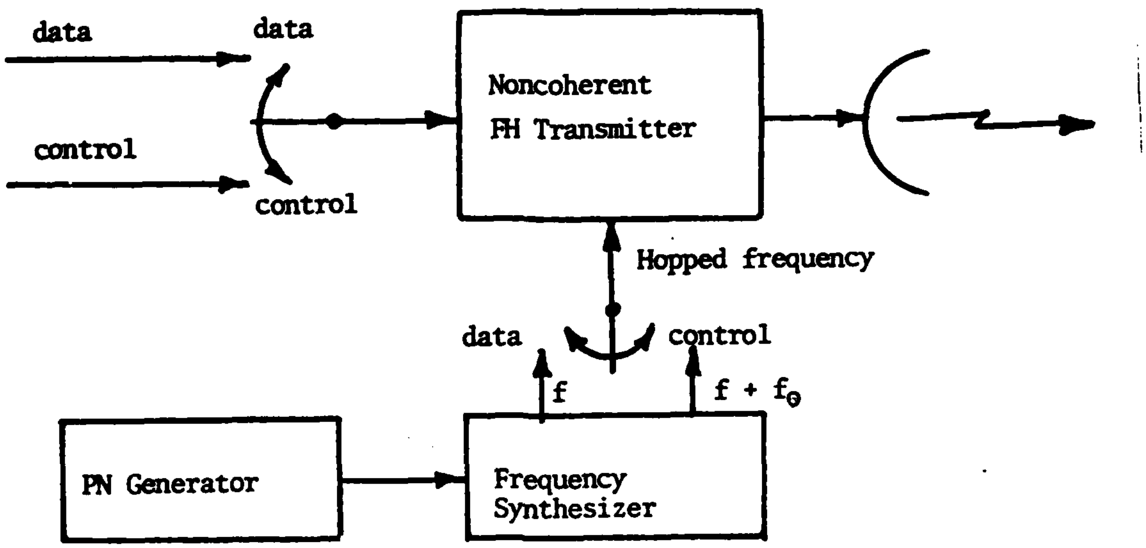


Next for each of the separate channels (Data or Control) another multiple access scheme is needed to handle the many users of these channels. Typically control signals have lower data rate requirements and request for data channels from users arrive as low duty cycle single bursts. For this reason, random access(RA) is also a possible technique here.

A. Providing Control and Data Channels

Control signals require as much or more antijamming protection as data channels. Thus we should not set aside a separate narrow band channel for control signals as is often done in commercial systems. Control channels must use the total available signal bandwidth to maintain protection against jamming.

An obvious approach is to use TDMA to separate the total channel into control channels and data channels. This is simple but reduces the channels that might be available for data. CDMA allows for no reduction of data and control channels with only negligible degredation in performance. Unfortunately, separate pseudo random codes would be required to simultaneously receive both data and control signals at a receiver. One approach that has the advantage of both TDMA and CDMA without any of their disadvantages is a modified FDMA technique shown below.



Here each transmitter uses the same frequency hopping sequence for both data and control information transmission except for a fixed frequency off-set f_0. In a noncoherent FH system f_0 can be a very small shift relative to the total bandwidth W and still allow for no mutual interference between data and control signals. This is essentially an FDMA technique. The receiver needs to have only one PN generator to simultaneously receive both data and control signals.

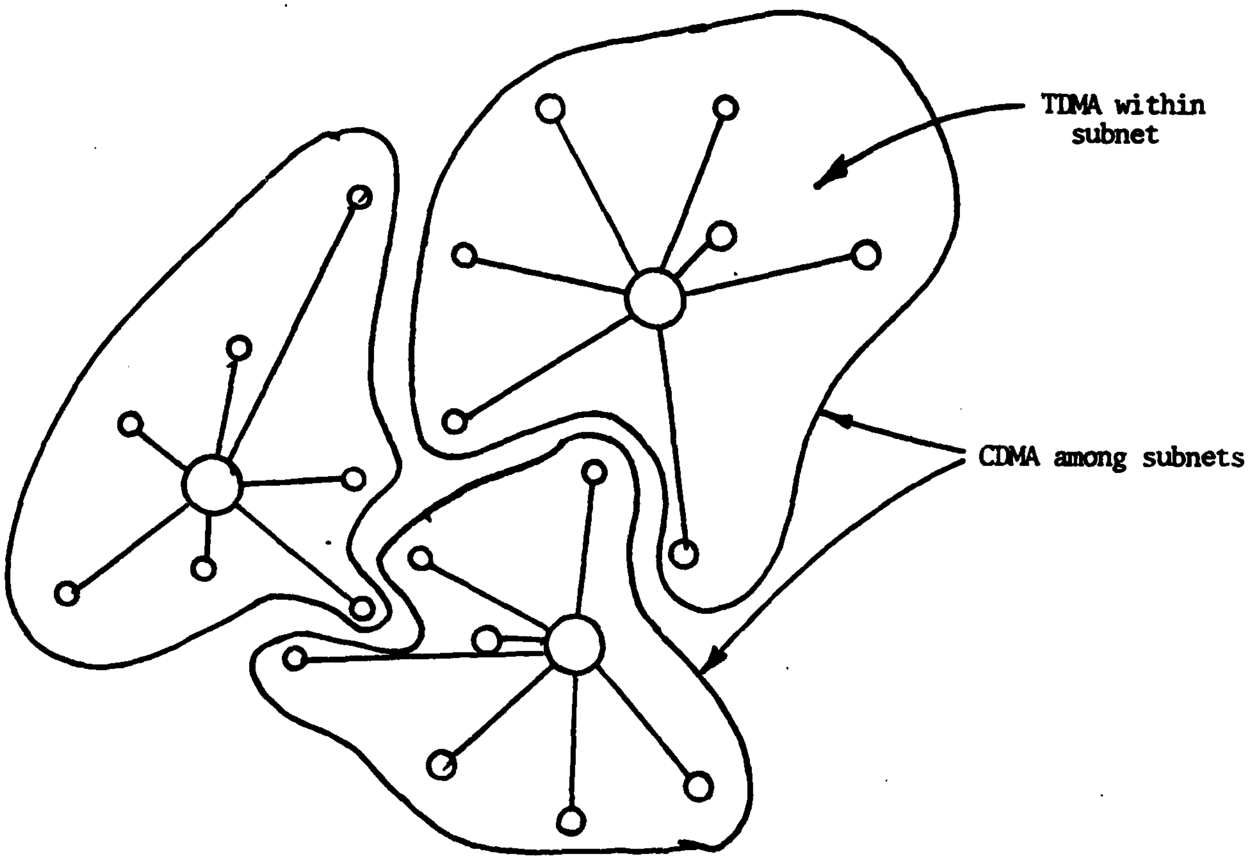
B. Data Channels

The simplest multiple access scheme for the data channels is TDMA. In a network, however, where users are grouped into separate subnets, each subnet can use different PN sequence generators and operate in a CDMA mode relative to other subnets. Within a subnet the users might typically use TDMA on a demand assignment basis.

C. Control Channels

If a user wants to join a particular subnet he can transmit in the control channel mode using the particular PN generator for that subnet. If he receives no response or is not given any data channel assignment due to saturation of that subnet data channel then he can attempt to access another subnet. This he does by transmitting with the particular PN generator of this next subnet.

Since control signals are typically low duty cycle short bursts of transmission, slotted random access for these channels seems the best. Also since any subnet control node may not apriori know which users may enter its subnet, fixed assigned TDMA slots are impractical for control channels.



Note No. 2

CONVENTIONAL JAMMING ANALYSIS

to

NAVAL RESEARCH LABORATORY

(Contract Award No. N00014-80-K-0935)

for

HF COMMUNICATION NETWORK SIGNALS

USING CHANNEL EVALUATION DATA

Principal Investigator

Jim K. Omura
Professor
System Science Department
University of California
Los Angeles, California

November, 1980

Conventional Jamming Analysis

This note summarizes the main results of conventional jamming analysis. Although the details may differ for each spread spectrum system against each type of jammer signal, the overall behavior illustrated here is essentially unchanged. Our purpose here is to illustrate techniques for combatting intentional jamming which will serve as background for the specific application to the HF ITF Network.

I. Introduction

How can we overcome the effects of intentional jamming particularly when the jammer has a lot more power than the transmitted signal? Classical communication theory is based on the additive white Gaussian noise channel where this interference is spread over all frequencies with infinite power. Here good performance is achieved whenever the signal-to-noise ratio "in the signal coordinates" is large. Following this example we arrive at the basic key to combatting intentional jamming.

SELECT SIGNAL COORDINATES SUCH THAT THE JAMMER CANNOT ACHIEVE LARGE JAMMER-TO-SIGNAL POWER RATIO IN THESE COORDINATES.

If there are lots of signal coordinates available and only a small subset of them are used at any time which the jammer does not know, then the jammer is forced to jam all coordinates with little power in each coordinate or jam a few coordinates with more power in each of the jammed coordinates.

The signal coordinates used are determined by a pseudo random (PN) sequence which is assumed to be unknown to the jammer but known at the intended receiver.

The more signal coordinates available the better the protection against jamming. For signals of bandwidth W and duration T the number of coordinates is roughly

$$N \cong \begin{cases} 2WT & \text{coherent signals} \\ WT & \text{noncoherent signals} \end{cases}$$

To make N large W is most commonly made large by (See References)

- Direct Sequence Spreading (DS)
- Frequency Hopping (FH)

Hence the term "Spread Spectrum" signals. Various hybrids of these two spreading techniques are possible but their performance does not significantly differ from these two basic ones. (Low probability of intercept (LPI) requirements and vulnerability to repeat back jamming often lead to hybrid designs.) Therefore, this note is limited to these two systems.

Throughout this note we assume the basic system of Figure 1 where the following system parameters are fixed:

W = total spread spectrum signal bandwidth
R = data rate in bits per second
S = signal power
J = jammer power } at input to intended receiver

Regardless of the signal and jammer waveforms we define an equivalent bit energy-to-noise ratio as

$$\frac{E_b}{N_o} = \frac{WS}{RJ}$$

where typically the names are given to

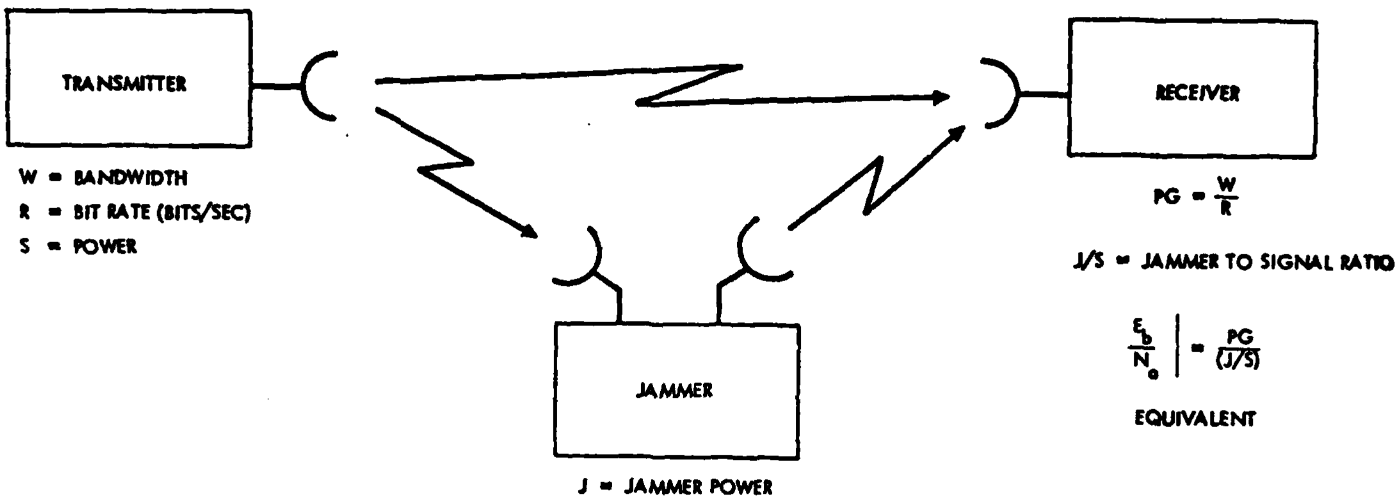
$$\frac{W}{R} = \text{processing gain}$$

and

$$\frac{J}{S} = \text{jammer-to-signal power ratio}$$

In terms of dB we have

$$\frac{E_b}{N_o}\ (\text{dB}) = [PG]_{dB} - [J/S]_{dB}$$



Signals

- MFSK/FH (Noncoherent)
- BPSK/DS (Coherent)
- Coding/Interleaving/Diversity

Jammers

- Types
  - --Noise
  - --Multitone or CW
- Modes
  - --Broadband
  - --Partial Band
  - --Pulsed

Figure 1. System Overview

where

$$[PG]_{dB} = 10 \log_{10} \frac{W}{R}$$

$$[J/S]_{dB} = 10 \log_{10} \left(\frac{J}{S}\right)$$

In this note we present bit error bounds as a function of this equivalent E_b/N_o for various DS and FH waveforms and various types of jamming waveforms.

II. Jamming Waveforms

A. Broadband and Partial Band Noise Jammers

A broadband noise jammer spreads noise of total power J evenly over the total spread bandwidth W as shown in Figure 2a. This results in an equivalent single-sided noise power density.

$$N_o = \frac{J}{W}$$

Since the signal energy per bit is ST_b where $T_b = 1/R$ we have

$$E_b = \frac{S}{R}$$

Thus in this case

$$\frac{E_b}{N_o} = \frac{WS}{RJ}$$

is exactly the bit energy-to-noise ratio.

A partial band noise jammer shown in Figure 2b spreads noise of total power J evenly over some bandwidth W_J, which is a subset of the total spread bandwidth W. We define ρ as the ratio

$$\rho = \frac{W_J}{W} \leq 1$$

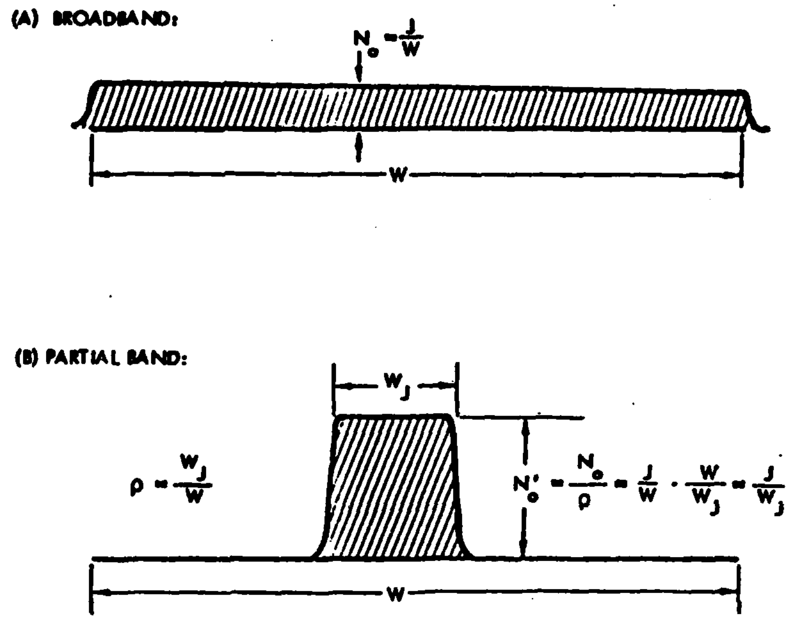


Figure 2. Noise Jammer

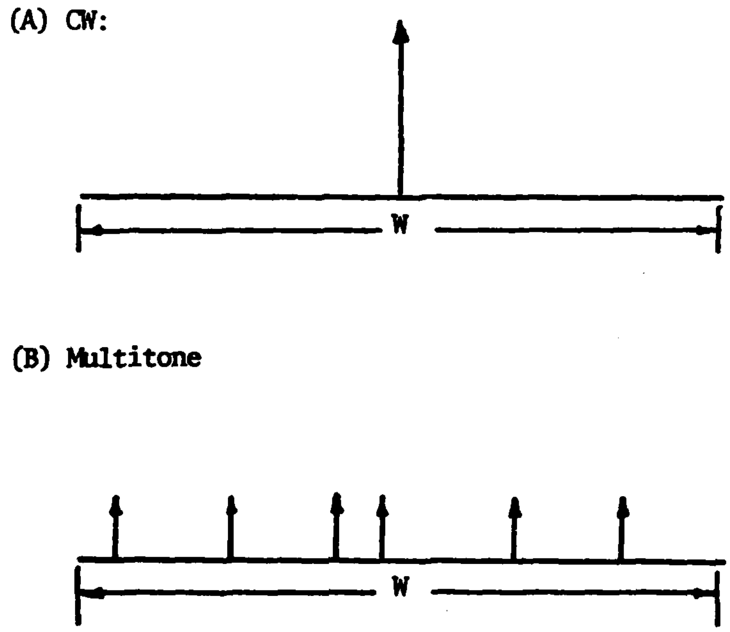


Figure 3. Tone Jammer

which is the fraction of the total spread spectrum band that has noise of power density

$$\frac{J}{W_J} = \frac{J}{W} \cdot \frac{W}{W_J}$$

$$= N_o/\rho$$

B. <u>CW and Multitone Jammers</u>

A CW jammer has the form

$$J(t) = \sqrt{2J} \cos [wt + \theta]$$

while multitone jammers using L equal power tones can be described by

$$J(t) = \sum_{\ell=1}^{L} \sqrt{2J/L} \cos [w t_\ell + \theta_\ell]$$

These are shown in figure 3a and 3b.

C. <u>Pulse Jammer</u>

Pulse jamming occurs when a jammer transmits with power

$$J_p = \frac{J}{\rho}$$

for a fraction ρ of the time, and nothing for the remaining fraction $1-\rho$ of the time. During the pulse transmission noise or tones can be transmitted.

D. <u>Repeat-Back Jammers</u>

A repeat back jammer first estimates parameters from the intercepted spread spectrum signal and then transmits jamming waveforms that use this information. Such jammers are primarily effective against FH systems when the hop rate is slow enough for the repeat back jammer to respond within

the hop duration. In this summary we assume hop rates are fast enough to resist repeat-back jamming.

III. Direct Sequence System

Direct sequence spreading is used with coherent BPSK and QPSK modulations. The direct sequence spreading at the transmitter and despreading at the receiver reduces any jamming signal to equivalent white Gaussian noise. The specific form of the jamming waveform does not change the accuracy of the equivalent white Gaussian noise model. However, to be most effective the jammer should concentrate most of its power at the carrier frequency. After despreading at the receiver this maximizes the jammer signal power in the signal band. (There may be up to 3dB additional jammer advantage for ideal CW jamming).

A. Continuous Jamming/Uncoded

With continuous jamming the bit error probability of both BPSK and QPSK direct sequence spread signals without coding is given by

$$P_b = Q\left(\sqrt{2\left(\frac{E_b}{N_o}\right)}\right)$$

where

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\, dt$$

$$< \tfrac{1}{2} e^{-\frac{x^2}{2}}$$

This is the usual BPSK and QPSK bit error probability except here

$$\frac{E_b}{N_o}\ (\text{dB}) = [PG]_{dB} - [J/S]_{dB}$$

B. Continuous Jamming/Coded

Coding can be used here just as any conventional BPSK or QPSK modulation with a white Gaussian noise channel. Standard constraint length K convoluitonal codes of rate r has the error bound

$$P_b \le \sum_{k = d_{min}}^{\infty} N_k \; Q\left(\sqrt{2r\left(\frac{E_b}{N_o}\right)k}\right)$$

where N_k is the number of data bits corresponding to paths at Hamming distance k from the transmitted path. Two common examples are as follows:

Example: $K = 7$, $r = \frac{1}{2}$, $d_{min} = 10$

$N_{10} = 36$	$N_{14} = 1{,}404$
$N_{11} = 0$	$N_{15} = 0$
$N_{12} =$ ~~20~~ 211	$N_{16} = 11{,}633$
$N_{13} = 0$	$N_{17} = 0$

Example: $K = 7$, $r = 1/3$, $d_{min} = 14$

$N_{14} = 1$	$N_{18} = 53$
$N_{15} = 0$	$N_{19} = 0$
$N_{16} = 20$	$N_{20} = 184$
$N_{17} = 0$	$N_{21} = 0$

Figure 4 shows these standard coding curvers along with the uncoded case.

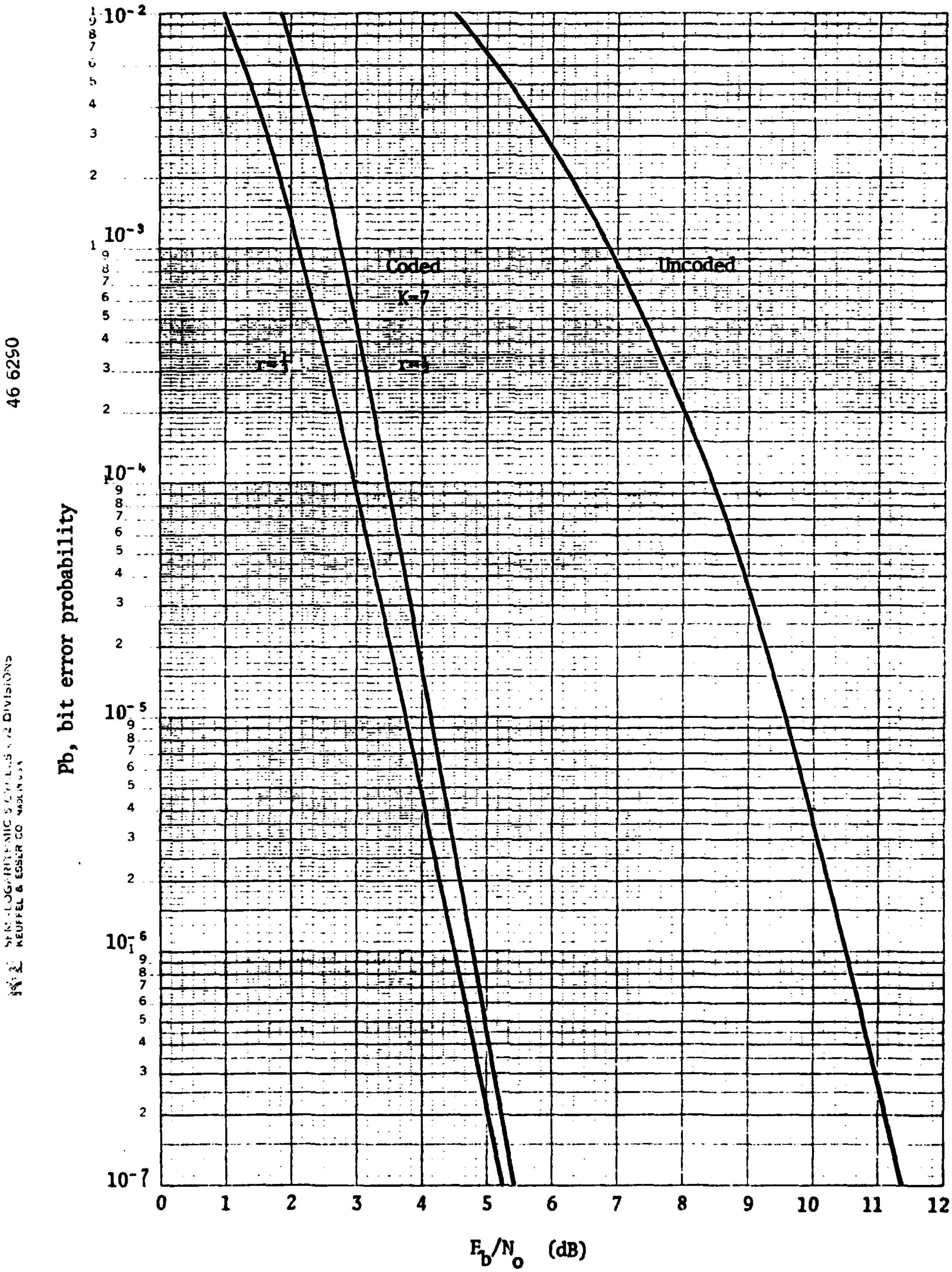
C. Pulse Jamming/Uncoded

At first it appears that jamming signals can do no worst than a white Gaussian noise channel of the equivalent E_b/N_o. Pulse jamming, however, can degrade performance by much more than expected. Indeed pulse jamming can make the channel look like a Rayleigh fading channel.

Assume a pulse jammer with parameter $o \le \rho \le 1$. Then the uncoded bit error probability is

$$P_b(\rho) = \rho\, Q\left(\sqrt{2\rho\left(\frac{E_b}{N_o}\right)}\right)$$

Figure 4. Continuous Jamming DS/BPSK



The worst case choice of ρ is [6]

$$\rho^* = \begin{cases} \frac{0.709}{(E_b/N_o)} & , \frac{E_b}{N_o} > 0.709 \\ 1 & , \frac{E_b}{N_o} \le 0.709 \end{cases}$$

This worst case yields (7)

$$P_b = \frac{(0.709)Q(1.19)}{(E_b/N_o)}$$

$$= \frac{0.08285}{(E_b/N_o)}$$

for $E_b/N_o > 0.709$. Note that this error probability behaves much like the performance in a Rayleigh fading channel (see Ref. [1]).

It will be computationally convenient to use upper bounds on the error probabilities and then choose the pulse parameter that minimizes the bound. For example using

$$Q(x) < \tfrac{1}{2} e^{-\frac{x^2}{2}}$$

we have the bound

$$P_b\ (p) < \tfrac{1}{2}\rho\, e^{-\rho \frac{E_b}{N_o}}$$

By direct differentiation we see that this bound is maximized by the choice

$$\rho = \begin{cases} \frac{1}{(E_b/N_o)} & , \frac{E_b}{N_o} > 1 \\ 1 & , \frac{E_b}{N_o} \le 1 \end{cases}$$

---

* When there is no jamming pulse we assume negliable bit error probability. Also the pulse duration is assumed larger than the data bit duration.

This yields the bit error bound

$$P_b \le \frac{e^{-1}}{2(E_b/N_o)}$$

$$= \frac{0.18394}{(E_b/N_o)}$$

In figure 5 we compare the exact expressions and the bounds for $\rho = 1$ (no pulse jamming) and the worst case pulse jamming. We see that the bound is tight in both cases and we are justified is working with its simpler form. Also note the tremendous impact worst case pulse jamming can have on the direct sequence uncoded system. At $P_b = 10^{-6}$ we have almost 40 dB of degredation due to worst case pulse jamming.

D. Pulse Jamming/Coding

The combination of coding and interleaving can effectively neutralize the impact of pulse jamming.

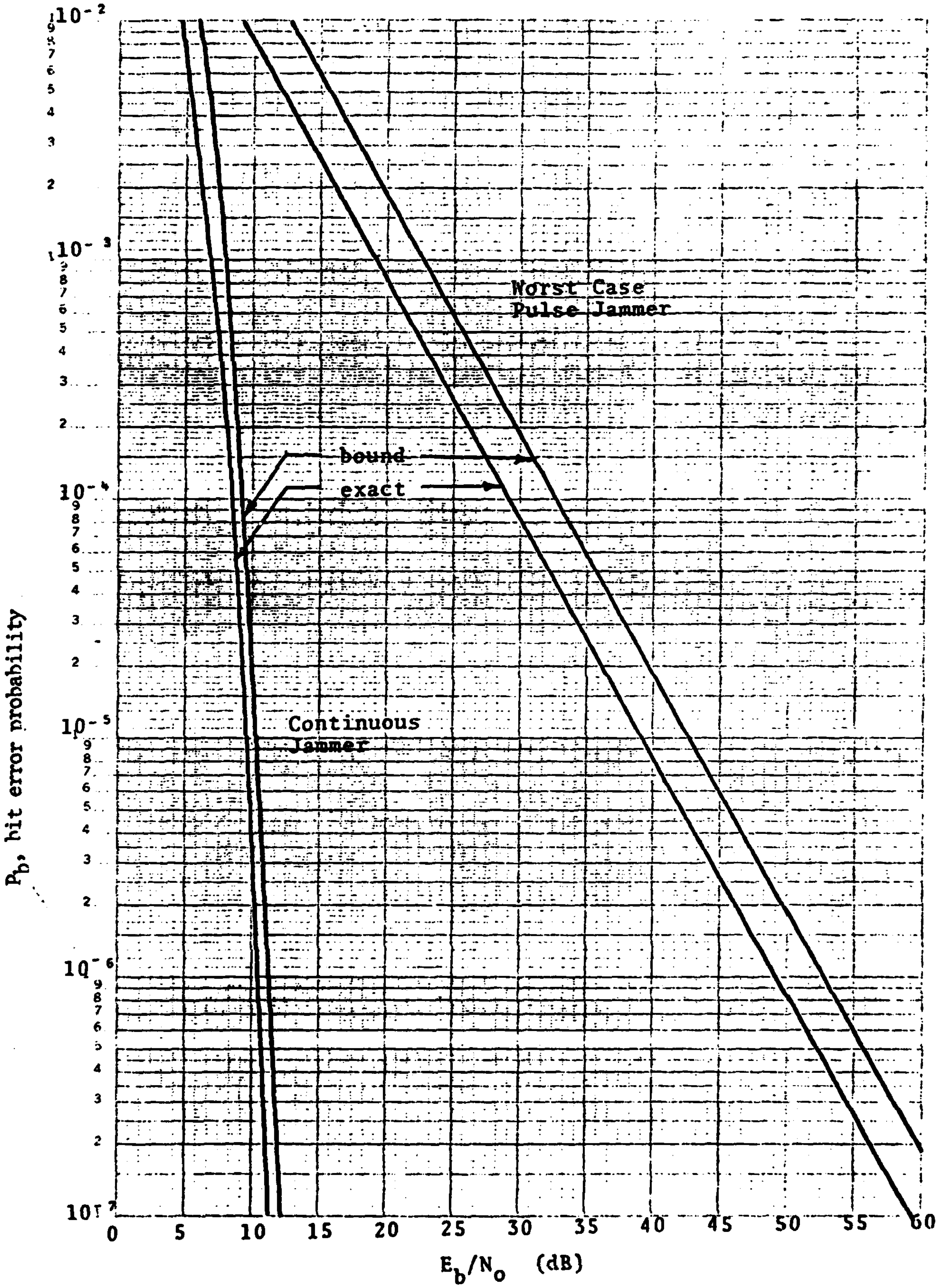
Suppose we assume convolutionally coding and ideal interleaving of the coded bits so that each coded bit is jammed with probability ρ independent of other coded bits. We also assume that the receiver knows perfectly when a jamming pulse occurs during a coded bit transmission. This system serves as an ideal performance limit. Less optimum receiver structures will be examined later.

Consider a path that diverges from the correct path in the trellis diagram that has distance k coded bits during the unmerged span. The Viterbi algorithm may incorrectly select this path if all k coded bits are jammed and the correlation of the received sequence with the incorrect path is larger than the correct path during the unmerged span. This occurs with probability

$$P_k \le \rho^k Q\left(\sqrt{2r\rho\left(\frac{E_b}{N_o}\right)k}\right)$$

In general it is very difficult to find the worst case pulse jamming parameter here. By using the upper bound on Q(x) we have the bit error union bound

Figure 5 Pulse Jamming DS/BPSK



$$P_b(\rho) \leq \tfrac{1}{2}\sum_{k=d_{min}}^{\infty} N_k \left[\rho e^{-\rho r\left(\frac{E_b}{N_o}\right)}\right]^k$$

By differentiation we maximize the bound with the choice

$$\rho = \begin{cases} \dfrac{1}{r(E_b/N_o)} & , \quad \dfrac{E_b}{N_o} > \dfrac{1}{r} \\[2ex] 1 & , \quad \dfrac{E_b}{N_o} \leq \dfrac{1}{r} \end{cases}$$

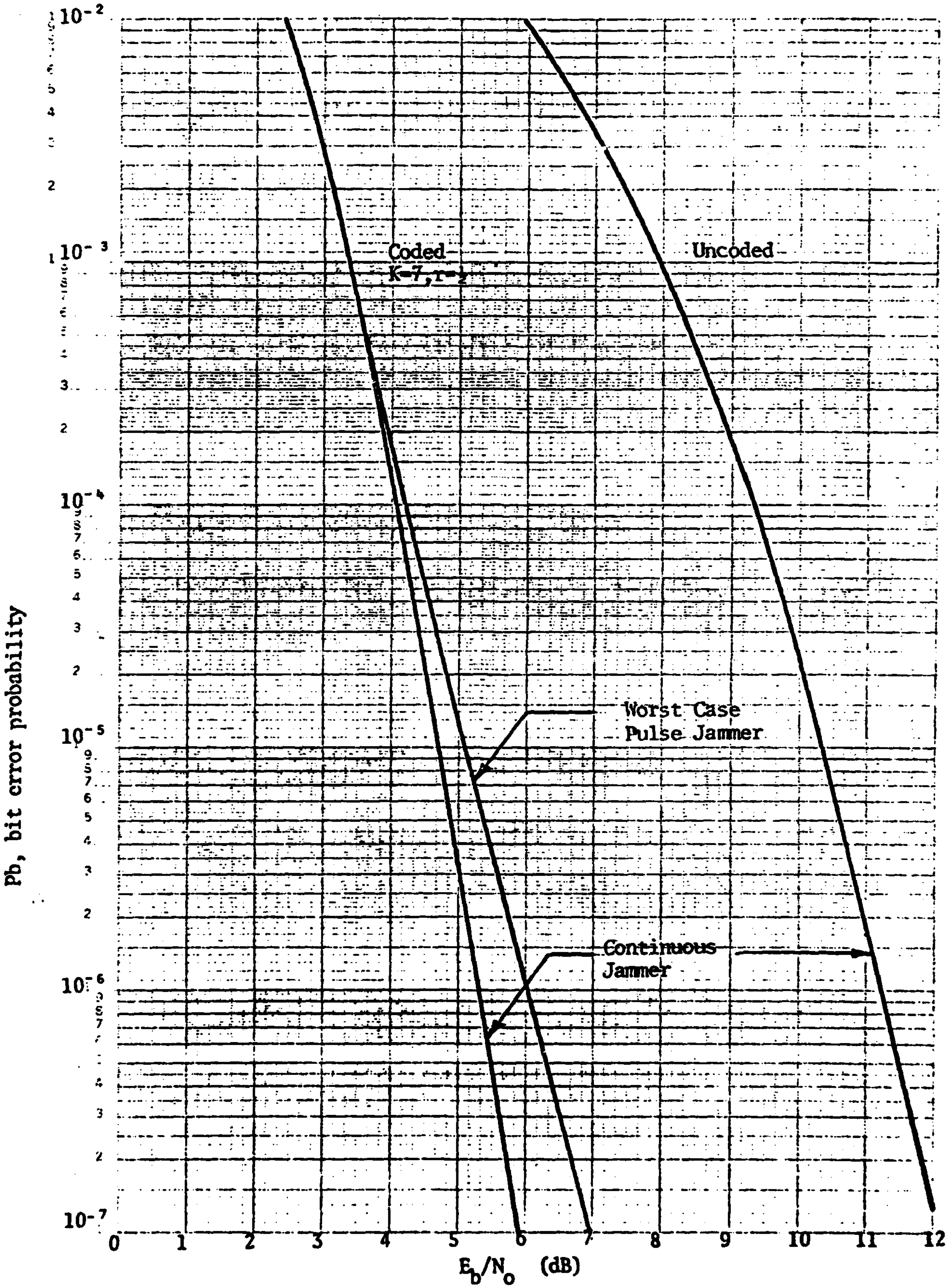
Thus for $E_b/N_o > 1/r$ we have

$$P_b \leq \tfrac{1}{2}\sum_{k=d_{min}}^{\infty} N_k \left[\frac{e^{-1}}{r(E_b/N_o)}\right]^k$$

Figure 6 shows this worst case pulse jammer for the K=7, $r=\tfrac{1}{2}$ convolutional code. Also shown is the no pulse ($\rho=1$) cases with the bound on $Q(x)$. Note the impact of pulse jamming is effectively neutralized with coding and interleaving.

It is clear from this analysis that lower rate codes (r small) tend to force the worst case toward continuous jamming. Since we use direct sequence spreading there is no bandwidth penalty for using low rate codes in this application. Also it should be noted that without interleaving coding would do no good.

Figure 6 Coding/Interleaving DS/BPSK



IV. Frequency Hopped System

Frequency hopping is typically used with noncoherent MFSK modulation where we take

$$M = 2^K$$

This modulation technique is also a form of orthogonal block coding where K data bits determine one of 2^K orthogonal codewords or waveforms to be transmitted over the channel. Spectrum spreading is achieved by shifting the carrier frequency of the MFSK pulse by a pseudo random amount determined by a PN generator.

Since CW tones look like the MFSK signals, this type of jamming waveform can place the most energy into a noncoherent MFSK detector. This, however, is not significantally worst than the worst case partial band noise jammer. For ease of analysis we shall assume the jammer uses Gaussian noise waveforms.

A. Broadband Jamming

For broadband jamming the performance of noncoherent MFSK signals is that for the additive white Gaussian noise channel with single-sided noise spectral density

$$N_o = \frac{J}{W}$$

Here we have the symbol error probability

$$P_s = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{k+1} e^{-\frac{k}{k+1}\left(\frac{E_s}{N_o}\right)}$$

and the bit error probability

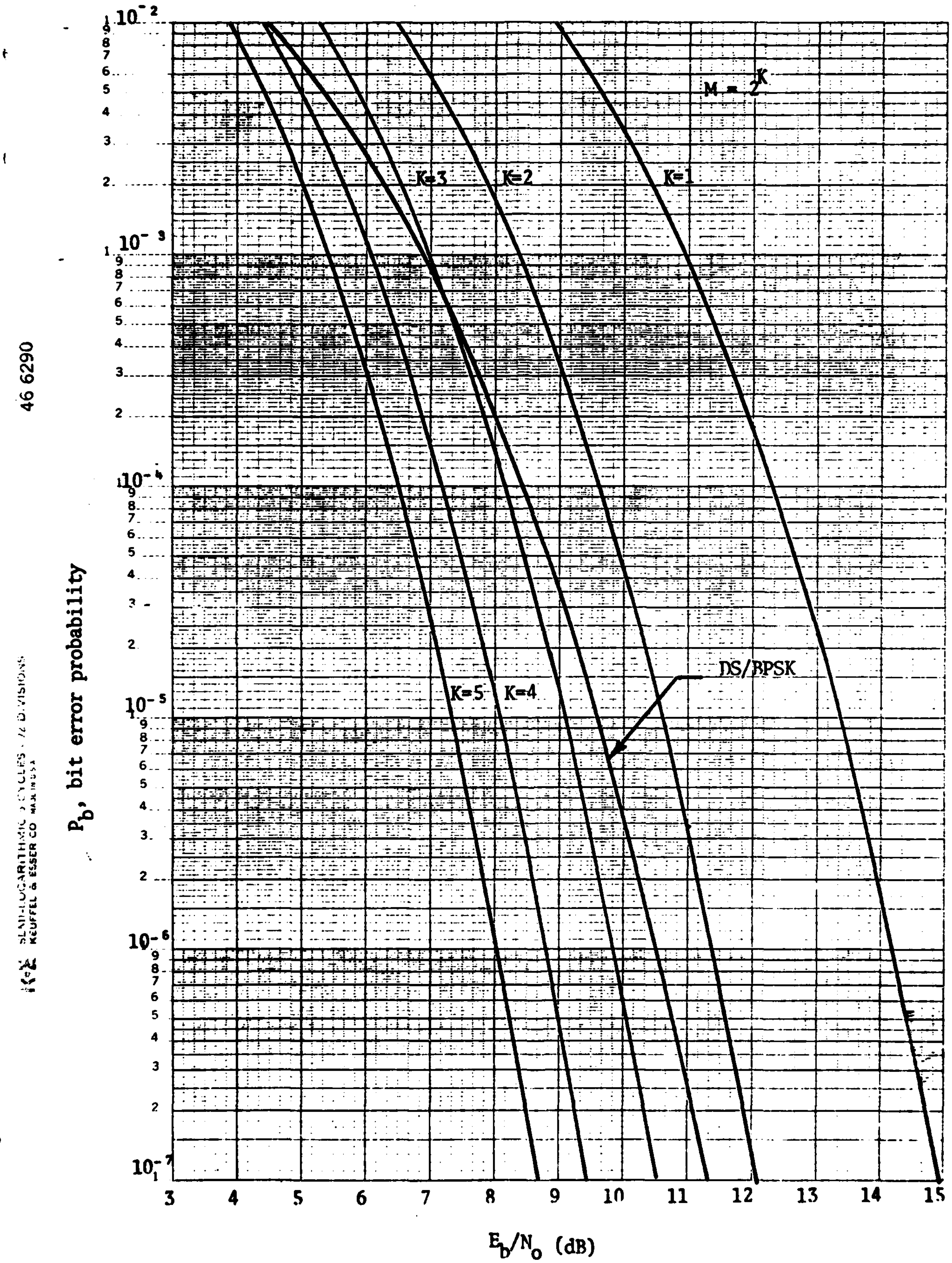
$$P_b = \frac{\frac{1}{2} M}{M-1} P_s$$

$$= \frac{2^{K-1}}{2^K-1} \sum_{\ell=1}^{2^K-1} \binom{2^K-1}{\ell} (-1)^{\ell+1} \frac{1}{\ell+1} e^{-\frac{\ell K}{\ell+1}\left(\frac{E_b}{N_o}\right)}$$

This is shown in Figure 7 for M = 2,4,8,16 and 32. Also shown here is the coherent DS/BPSK uncoded bit error probability with continuous jamming.

We again find it convenient to use simpler to evaluate upper bounds especially for the analysis of more complex systems. Here we note the pairwise error probability

Figure 7 Broadband FH/MFSK



$$P(s_m \to s_{\hat{m}}) = \tfrac{1}{2} e^{-\frac{E_s}{2N_o}}$$

where $P(s_m \to s_{\hat{m}})$ is the probability that the detector output of symbol $s_{\hat{m}}$ is greater than that of the transmitted symbol s_m. Here

$$E_s = KE_b$$

The union bound gives the symbol error bound

$$P_s \leq (M-1)\tfrac{1}{2} e^{-K\left(\frac{E_b}{2N_o}\right)}$$

and bit error bound

$$P_b \leq \tfrac{1}{4} \cdot M\, e^{-K\left(\frac{E_b}{2N_o}\right)}$$

$$= \tfrac{1}{4} e^{-K\left[\left(\frac{E_b}{2N_o}\right) - \ln 2\right]}$$

Figure 8 shows the actual bit error probabilities (dotted lines) and the corresponding union bounds (solid lines) for various values of $M = 2^K$. From this it seems we are justified in using the simple union upper bounds.

B. Partial Band Jamming/No Diversity

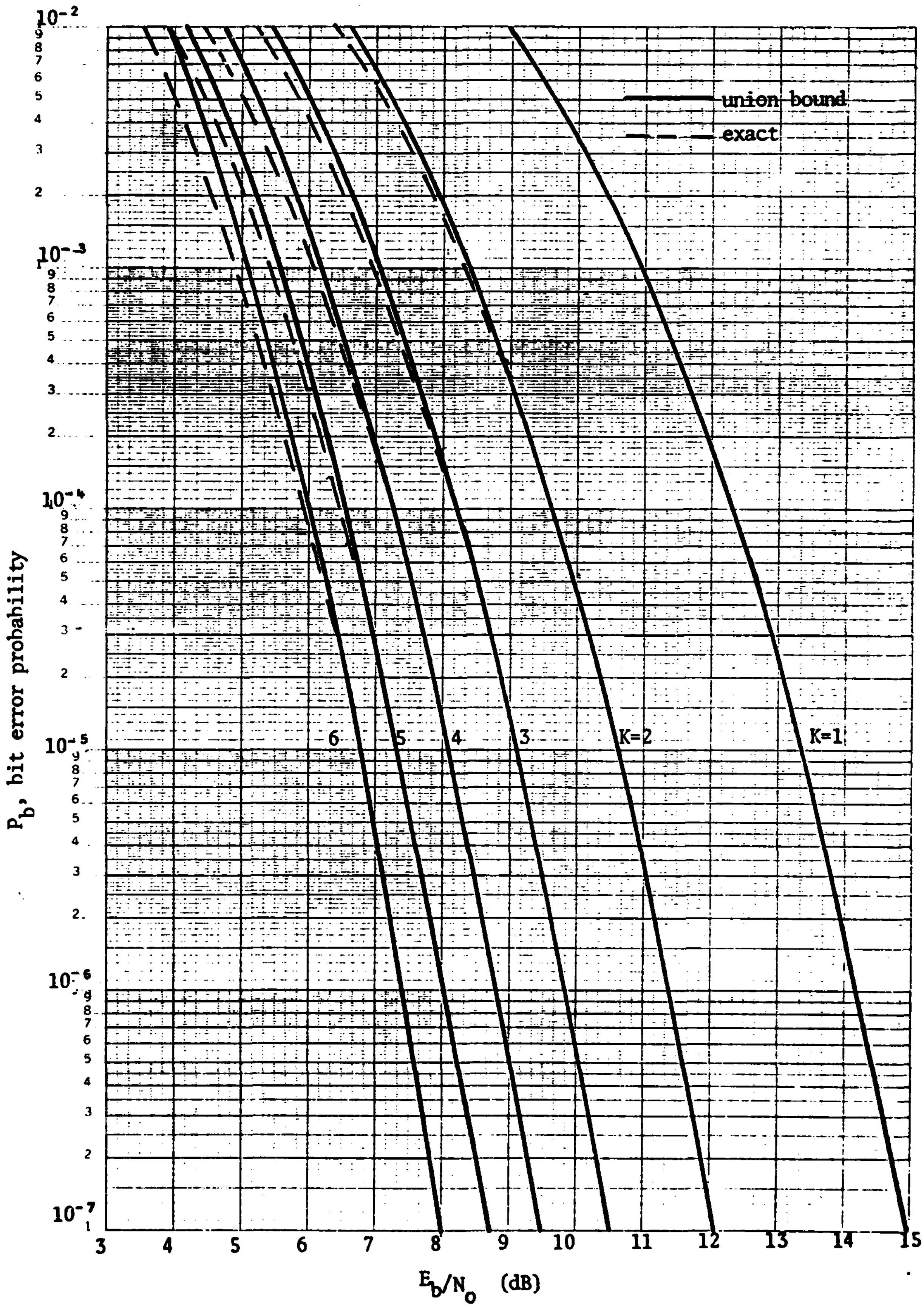
The impact of partial band jamming on FH systems is analogous to the impact pulse jamming has on direct sequence systems. The result is much like a Rayleigh fading channel.

Assume a partial band noise jammer with parameter $0 \leq \rho \leq 1$. The probability of error is given by

$$P_b(\rho) = \rho \frac{2^{K-1}}{2^K-1} \sum_{\ell=1}^{2^K-1} \binom{2^K-1}{\ell} (-1)^{\ell+1} \frac{1}{\ell+1} e^{-\rho \frac{\ell K}{\ell+1}\left(\frac{E_b}{N_o}\right)}$$

This is generally difficult to work with so we examine the union bound

Figure 8. Union Bound, Broadband FH/MFSK



$$P_b(\rho) \leq \tfrac{1}{4} M \rho e^{-\rho K \left(\frac{E_b}{2N_o}\right)}$$

This bound is minimized by

$$\rho = \begin{cases} \dfrac{2}{K(E_b/N_o)} & ; \quad \dfrac{E_b}{N_o} > \dfrac{2}{K} \\ 1 & ; \quad \dfrac{E_b}{N_o} \leq \dfrac{2}{K} \end{cases}$$

This yields the bit error bound

$$P_b \leq \frac{2^{K-1}}{eK(E_b/N_o)}$$

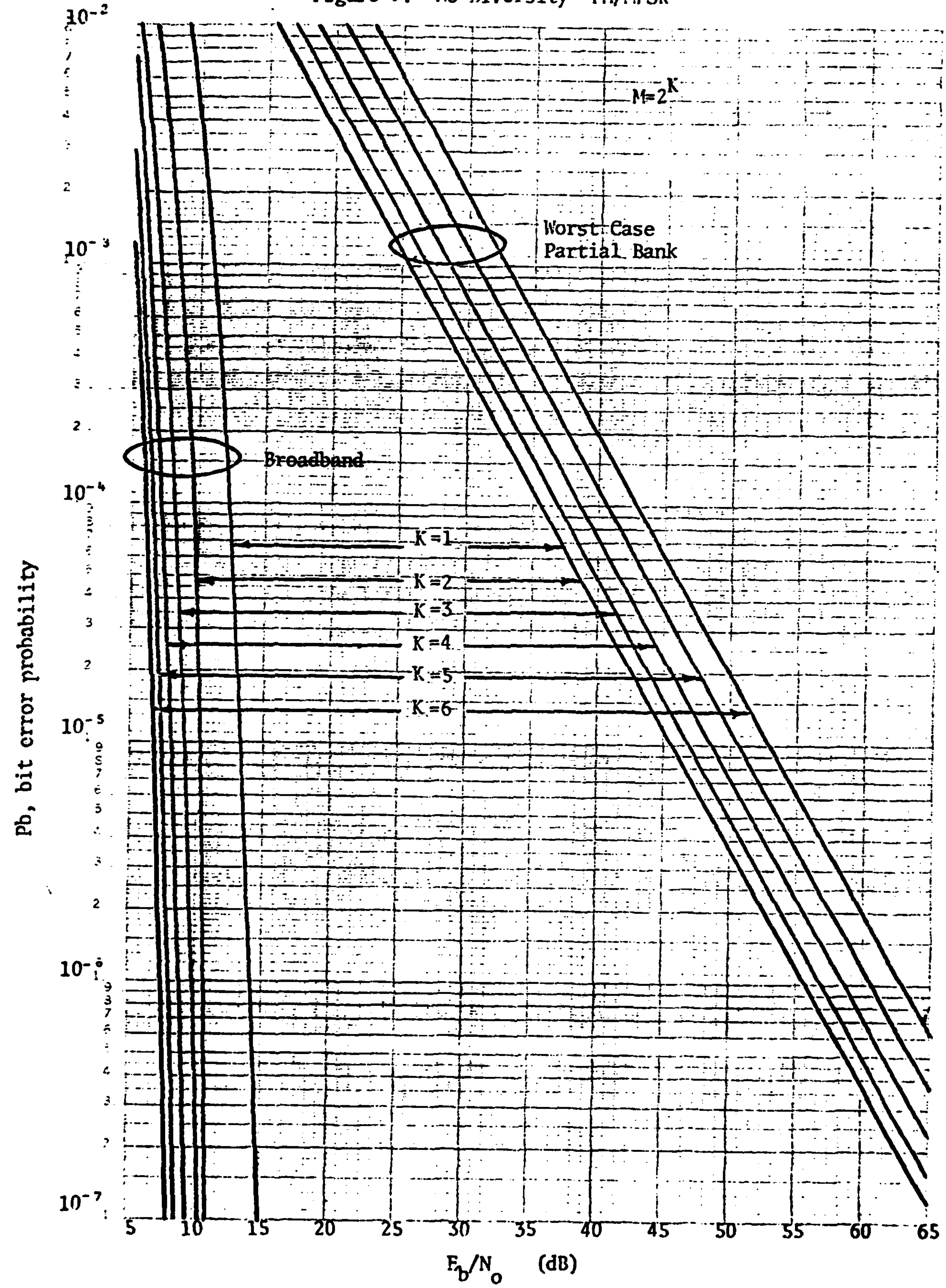
when $E_b/N_o > 2/K$.

Here we see that as K increases the worst case bit error bound increases. These are shown in figure 9 where we compare the union bounds for broadband noise jamming and worst case partial band noise jamming. As with pulse jamming against uncoded DS systems, we see that worst case partial band noise jamming has a tremendous impact on the performance of FH systems. At $P_b = 10^{-6}$ we have at a 40 dB degredation due to worst case partial band jamming compared to broadband jamming.

C. Partial Band Jamming/Diversity

As noted above MFSK modulation is also a form of orthogonal coding. To use this coding effectively we need to "break up" the impact of jamming into independent components much like the way we can combat Rayleigh fading. This can be done using "diversity" which is dual to interleaving used in com-

Figure 9. No Diversity FH/MFSK



batting pulse jamming of DS systems.

We take the original MFSK pulse of duration T_s seconds and divide it into m MFSK chips of duration $T_c = T_s/m$ seconds. Each MFSK chip is then independently hopped where independently hopping each MFSK chip means that we lose phase information among the m MFSK chips that make up the original MFSK signal. Here we assume the receiver noncoherently combines the m chip energy detectors for each of the M de-hopped signal frequencies. This is a sub-optimum but convenient receiver structure. We also assume the ideal receiver that knows exactly when a particular MFSK chip is jammed or not. That is, the receiver knows when the hopped frequency falls into the portion of the band that is being jammed.

In this situation we need to use a Chernoff bound with parameter $0 \leq \lambda \leq 1$ where [1]

$$P(s_m \to s_{\hat{m}}) \leq \frac{1}{2}\left[\frac{\rho}{1-\lambda^2}\, e^{-\rho\left(\frac{\lambda}{1+\lambda}\right)\frac{E_c}{N_o}}\right]^m$$

where

$$E_c = \frac{E_s}{m} = \frac{KE_b}{m}$$

This Chernoff bound can be compared with the exact result for $\rho = 1$, $m = 1$, and $K = 1$ where

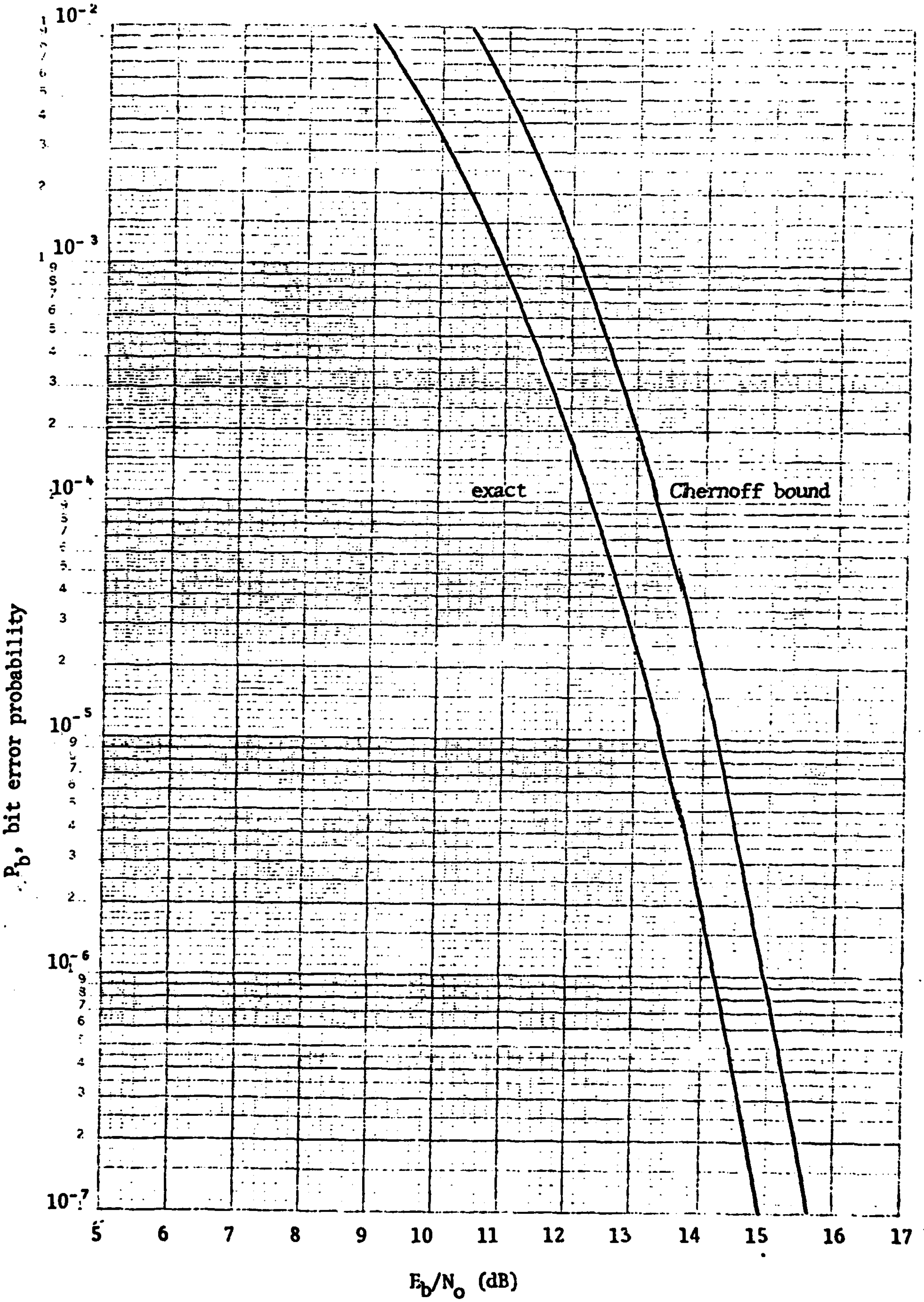
$$P(s_m = s_{\hat{m}}) = \frac{1}{2}\, e^{-\frac{E_b}{2N_o}}$$

$$< \frac{1}{2}\left(\frac{1}{1-\lambda^2}\right) e^{-\left(\frac{\lambda}{1+\lambda}\right)\left(\frac{E_b}{N_o}\right)}$$

for $0 \leq \lambda \leq 1$. Minimizing the Chernoff bound with respect of $\lambda \in [0,1]$ we compare the Chernoff bound with the exact bit error probability for this case in Figure 10. Again we see that the bound is fairly tight.

Returning to the general expression if we maximize the bound given above with respect to $0 \leq \rho \leq 1$ and minimize it with respect to $0 \leq \lambda \leq 1$ we obtain $\lambda = \frac{1}{2}$,

Figure 10 Broadband Jammer FH/BFSK



and

$$\rho = \begin{cases} \dfrac{3m}{K(E_b/N_o)} & , \dfrac{E_b}{N_o} > \dfrac{3m}{K} \\ 1 & , \dfrac{E_b}{N_o} \le \dfrac{3m}{K} \end{cases}$$

$$P(s_m \to \hat{s}_m) \le \frac{1}{2} \left[\frac{4}{e(E_c/N_o)}\right]^m$$

$$= \frac{1}{2} \left[\frac{4m}{eK(E_b/N_o)}\right]^m$$

The worst case bit error bound then becomes

$$P_b \le 2^{K-2} \left[\frac{4m}{eK(E_b/N_o)}\right]^m$$

Although the diversity parameter m is an integer we can to a good approximation minimize this bound assuming m is real valued. Choosing the minimizing value

$$m^* = \frac{K}{4}\left(\frac{E_b}{N_o}\right)$$

we obtain the bound

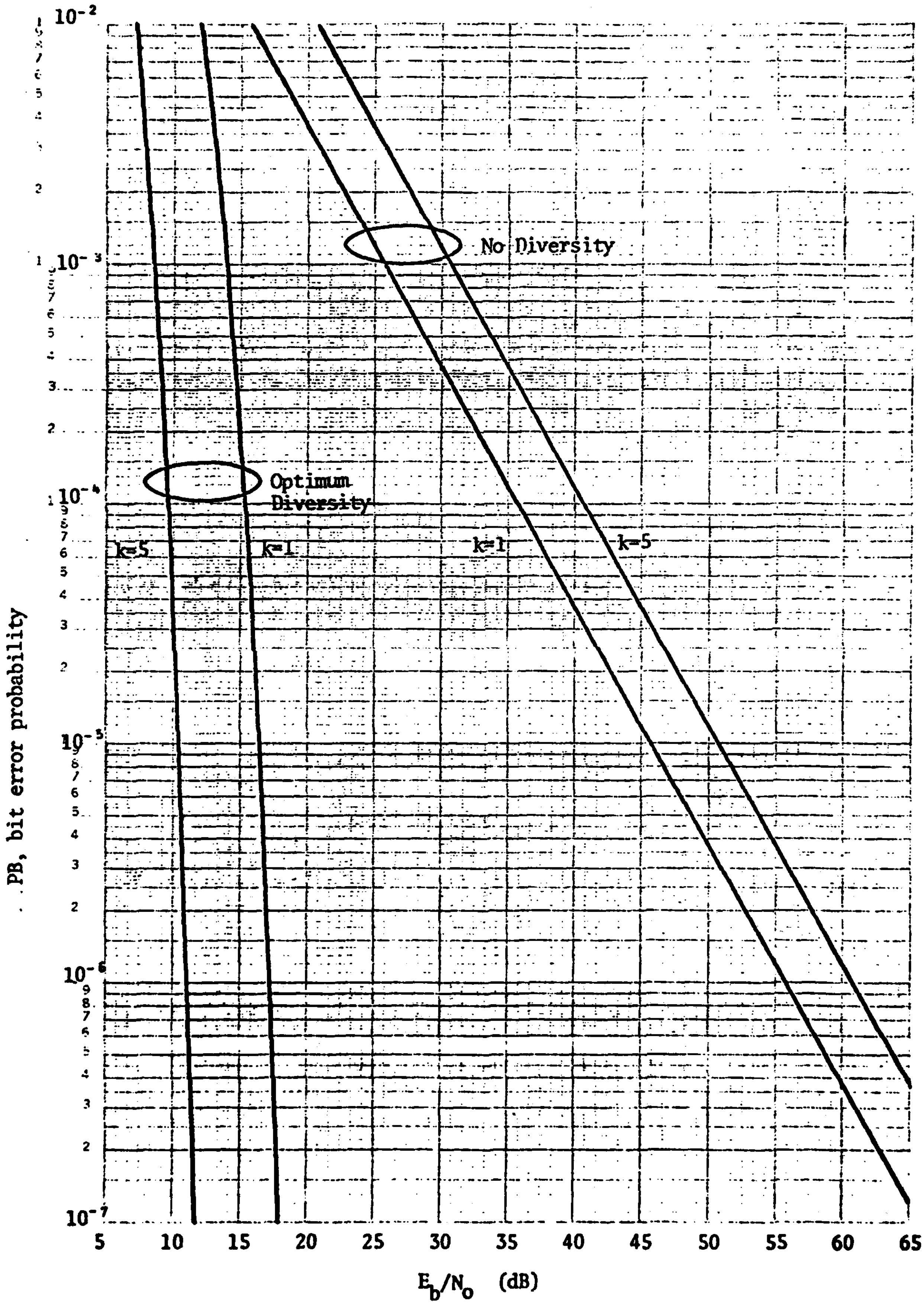
$$P_b \le \frac{1}{4} e^{-K\left[\left(\frac{E_b}{4N_o}\right) - \ln 2\right]}$$

This also yields

$$\rho = \frac{3}{4}$$

Figure 11 shows this optimum diversity performance compared with no diversity against worst case partial band jamming for M=2 and M=5. Note the optimum diversity case against worst case partial bandjamming is only 3dB worst than the broadband noise jammer case.

Figure 11 Optimum Diversity FH/MFSK



Although MFSK signals can be viewed as orthogonal coding we can also further code the MFSK symbols using more conventional codes. For conventional binary convolutional codes with BFSK we have the bit error bound

$$P_b \leq \frac{1}{2} \sum_{k=d_{min}}^{\infty} N_k \left[\frac{\rho}{1-\lambda^2} e^{-\rho \left(\frac{\lambda}{1+\lambda}\right) \left(\frac{E_c}{N_o}\right)} \right]^{km}$$

where

$$E_c = \frac{rE_b}{m}$$

r = code rate in bits/channel bit

Hence maximizing with respect to $0 \leq \rho \leq 1$ and minimizing with respect λ and m yields

$$\lambda = \frac{1}{2}$$

$$\rho = \frac{3}{4}$$

$$m^* = \frac{r}{4} \left(\frac{E_b}{N_o}\right)$$

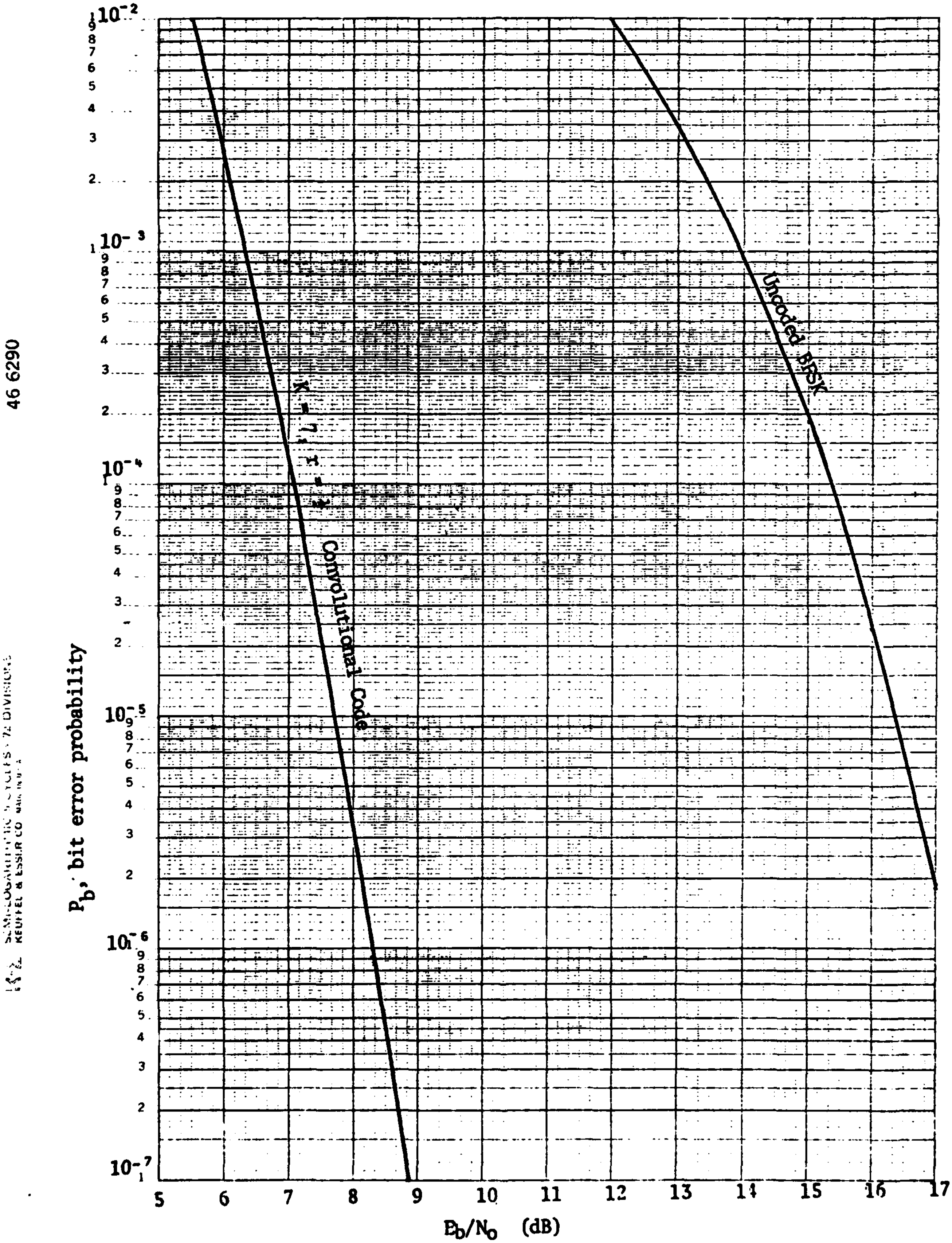
and the bound

$$P_b \leq \frac{1}{2} \sum_{k=d_{min}}^{\infty} N_k e^{-\frac{r}{4} \left(\frac{E_b}{N_o}\right) k}$$

This bound is 6 dB worst than the bound for the same code with coherent BPSK/DS. Figure 12 compares the uncoded BFSK with the $K = 7$, $r = \frac{1}{2}$ standard binary convolutional code.

For M-ary alphabets we can use orthogonal convolutional codes of constraint length K where $M = 2^K$ and also dual -K codes.

Figure 12. Optimum Diversity FH/BFSK



Choosing the maximizing

$$\rho = \frac{3}{4}$$

and minimizing

$$\lambda = \frac{1}{2}$$

and $$\hat{m}^* = \frac{1}{4(E_b/N_o)}$$

where $\hat{m}^*$ is the optimum diversity per data bit we have

Dual-3:

$$P_b \le \frac{2D^4}{[1-2D-5D^2]^2}\,, \qquad D = e^{-\frac{3}{8}\left(\frac{E_b}{N_o}\right)}$$

Dual-5:

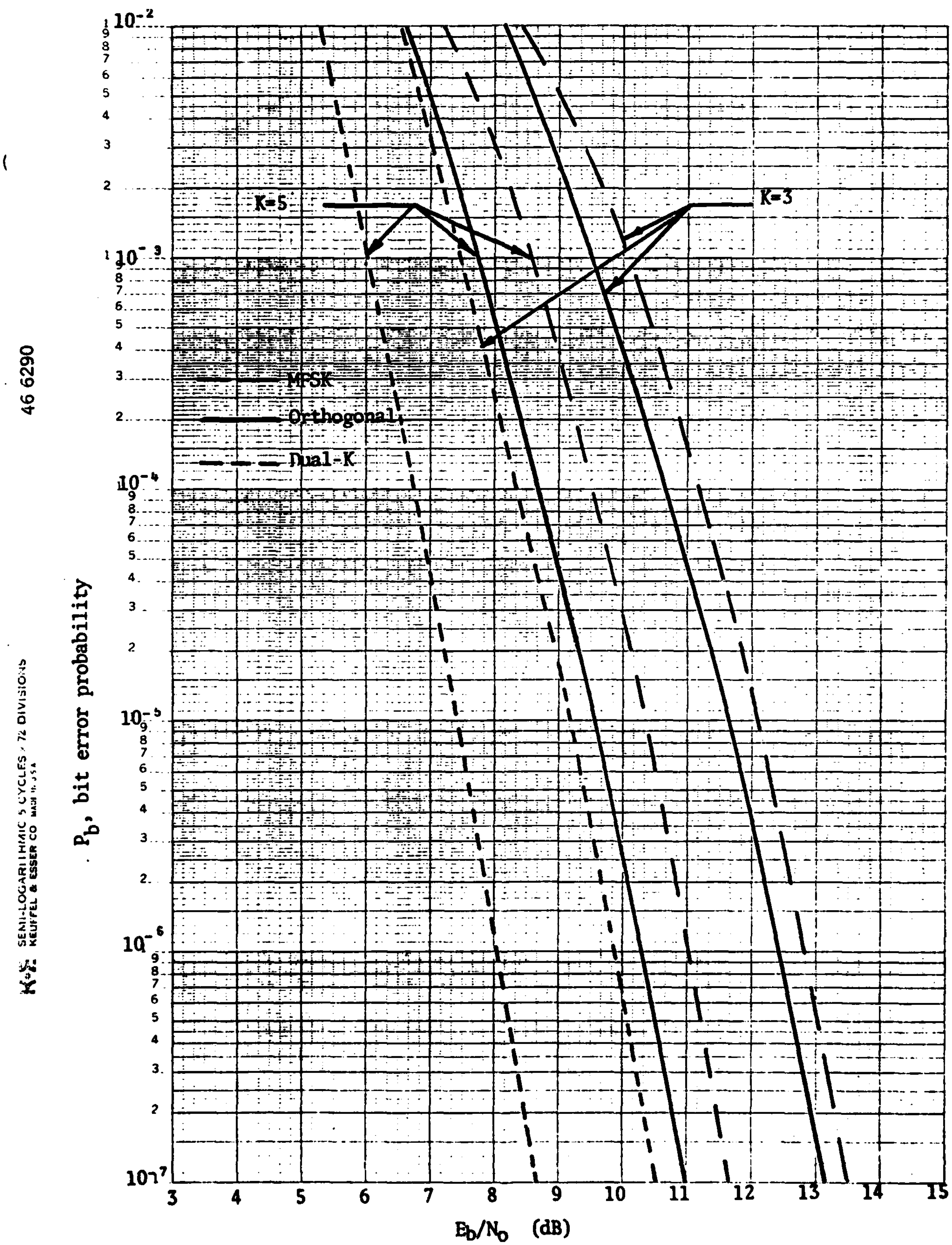
$$P_b \le \frac{8D^4}{[1-2D-29D^2]^2} \qquad D = e^{-\frac{5}{8}\left(\frac{E_b}{N_o}\right)}$$

Orthogonal:

$$P_b \le \frac{\frac{1}{2}D^K}{[1-2D]^2} \qquad D = e^{-\frac{1}{4}\left(\frac{E_b}{N_o}\right)}$$

These bit error bounds are shown in Figure 13 along with conventional MFSK.

Figure 13. Optimum Diversity FH/MFSK



V. Conclusion

Without coding and interleaving (or diversity) jamming signals can do considerably more harm than a white Gaussian noise channel with the same effective

$$\left.\frac{E_b}{N_o}\right|_{\text{effective}} = \frac{PG}{J/S}$$

At $P_b = 10^{-6}$, for example, there is a difference of approximately 40 dB between the ideal white Gaussian noise channel and the worst case jammer against uncoded DS and FH systems. With coding and interleaver (or diversity), however, this large difference is effectively eliminated.

The above results assumed ideal receivers that could detect when pulse or partial band jamming is in effect on a particular transmitted symbol. In this grant work we are currently investigating various receiver types including those that have channel measurement data. At HF frequencies we feel this is an important factor to consider. Various general types of receiver characteristics include

- Channel Output
  - Hard Quantized
  - Soft Quantized
  - Threshold

- Jammer State
  - Known
  - Unknown

- Decision Rule
  - Maximum Likelihood
  - Maximum Metric

- Channel State
  - Unknown
  - Known Noise Distribution
  - Known Propagation Distribution
  - Skywave/Groundwave

The analysis of more complex systems such as those with complex channel characteristics and suboptimum receiver structures, becomes extremely difficult. To simplify this task we will use union Bhattacharyya bounds for maximum likelihood receivers and Chernoff bounds for suboptimum receivers. In this note we have demonstrated the use of these bounds and how they can be effectively used for the analysis of complex communication systems.

Although in theory the coherent DS/BPSK systems seem to perform better than the noncoherent FH/MFSK systems, practical implementation considerations strongly favor the noncoherent FH/MFSK approach. Most important of these practical advantages are:

- easier synchronization
- wider spread bandwidths
- does not require a contiguous band

Most important is the robustness of noncoherent FH/MFSK compared to coherent DS/BPSK systems. Low probability of intercept (LPI) requirements and vulnerability to repeat-back jamming may, however, lead to hybrid designs usually in the form of frequency hopping of (direct sequence, orthogonal, or Gold code) binary BPSK sequences with either coherent or noncoherent reception.

VI. References

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Note No. 3

FACTOR OF ONE HALF IN ERROR BOUNDS

to

NAVAL RESEARCH LABORATORY

(Contract Award No. N00014-80-K-0935)

for

HF COMMUNICATION NETWORK SIGNALS

USING CHANNEL EVALUATION DATA

Principal Investigator

Jim K. Omura
Professor
System Science Department
University of California
Los Angeles, California

January, 1981

FACTOR OF ONE HALF IN ERROR BOUNDS

This note shows that the commonly used error bounds, Bhattacharyya and Chernoff bounds, can be reduced by a factor of one half. For this reason most of the bit error probability bounds used in this grant research will include this factor which tightens the bounds.

I. INTRODUCTION

Jacobs [1] gave sufficient conditions for reducing the standard Chernoff bound by a factor of one half. We rederive this result here and give less restrictive but harder to verify sufficient conditions. We also present some related results of Hellman and Raviv [2] which shows that all Bhattacharyya bounds can be reduced by a factor of one half.

Let Z be a continuous random variable that can have one of two probability densities:

$$\begin{aligned} H_1 &: \quad f_1(z) \quad , \quad -\infty < z < \infty \\ H_2 &: \quad f_2(z) \quad , \quad -\infty < z < \infty \end{aligned} \tag{1}$$

where we have a-priori probabilities for these two hypotheses denoted

$$\pi_1 = \Pr\{H_1\} \text{ and } \pi_2 = \Pr\{H_2\}.$$

We assume an arbitrary deterministic decision rule characterized by the following decision function: Given an observed value z of the random variable Z let

$$\phi(z) = \begin{cases} 1 \quad , & \text{decide } H_1 \\ 0 \quad , & \text{decide } H_2 \end{cases} \tag{2}$$

In terms of this decision function we have conditional error probabilities

$$\begin{aligned} P_{E_1} &= \Pr\{\text{decide } H_2 | H_1\} \\ &= \int_{-\infty}^{\infty} [1-\phi(z)] f_1(z) dz \end{aligned} \tag{3}$$

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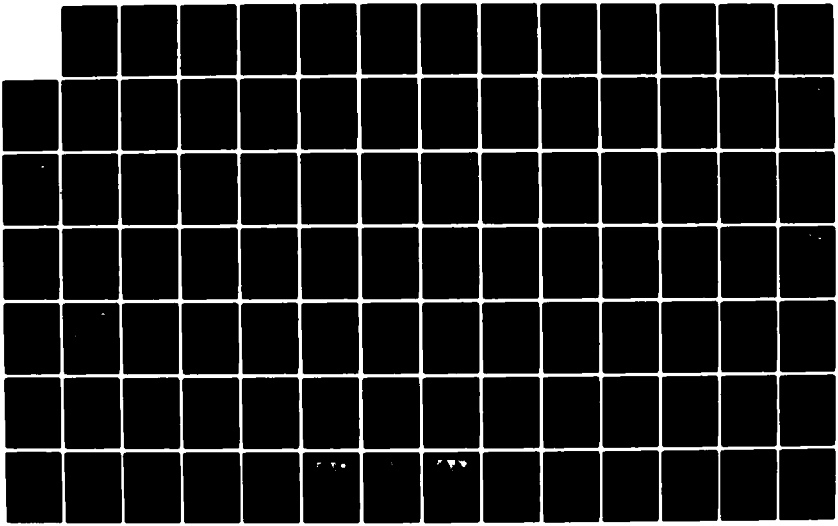

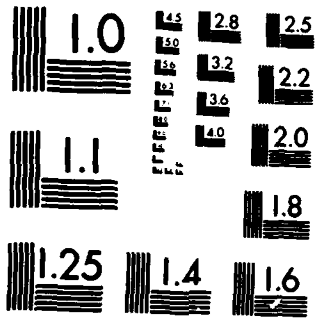

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and

$$P_{E_2} = \Pr\{\text{decide } H_1 | H_2\}$$
$$= \int_{-\infty}^{\infty} \phi(z) f_2(z) dz \tag{4}$$

The average error probability is

$$P_E = \pi_1 P_{E_1} + \pi_2 P_{E_2}$$
$$= \int_{-\infty}^{\infty} \{\pi_1 f_1(z) [1-\phi(z)] + \pi_2 f_2(z)\, \phi(z)\} dz \tag{5}$$

Often error probabilities are difficult to evaluate and so easily computed bounds are often used. In the following we examine Bhattacharyya and Chernoff bounds for various decision rules.

II. MAXIMUM A-POSTERIORI (MAP)

The decision rule that minimized P_E is the MAP rule,

$$\phi(z) = \begin{cases} 1 & , \quad \pi_1 f_1(z) \geq \pi_2 f_2(z) \\ 0 & , \quad \pi_1 f_1(z) < \pi_2 f_2(z) \end{cases} \tag{6}$$

which satisfies the inequalities

$$\phi(z) \leq \left[\frac{\pi_1 f_1(z)}{\pi_2 f_2(z)}\right]^{\alpha} \tag{7}$$

$$1-\phi(z) \leq \left[\frac{\pi_2 f_2(z)}{\pi_1 f_1(z)}\right]^{\beta} \tag{8}$$

for any $\alpha \geq 0$, $\beta \geq 0$. These inequalities are typically used to derive the bounds

$$P_{E_1} \leq \int_{-\infty}^{\infty} \left[\frac{\pi_2 f_2(z)}{\pi_1 f_1(z)}\right]^{\beta} f_1(z) dz$$

$$= \left(\frac{\pi_2}{\pi_1}\right)^{\beta} \int_{-\infty}^{\infty} f_1(z)^{1-\beta} f_2(z)^{\beta} dz \tag{9}$$

and

$$P_{E_2} \le \int_{-\infty}^{\infty} \left[\frac{\pi_1 f_1(z)}{\pi_2 f_2(z)}\right]^{\alpha} f_2(z)dz$$

$$= \left(\frac{\pi_1}{\pi_2}\right)^{\alpha} \int_{-\infty}^{\infty} f_1(z)^{\alpha} f_2(z)^{1-\alpha} dz \tag{10}$$

Next define the function

$$B(\alpha) = \int_{-\infty}^{\infty} f_1(z)^{\alpha} f_2(z)^{1-\alpha} dz \tag{11}$$

Then the average error probability is

$$P_E \le \pi_1^{1-\beta} \pi_2^{\beta} B(1-\beta) + \pi_1^{\alpha} \pi_2^{1-\alpha} B(\alpha) \tag{12}$$

for any $\alpha \ge 0$, $\beta \ge 0$. In general we would choose α and β to minimize these bounds. The special case where

$$\alpha = \beta = \frac{1}{2}$$

results in the Bhattacharyya bound

$$P_E \le 2\sqrt{\pi_1 \pi_2}\, B\left(\frac{1}{2}\right)$$

$$\le B\left(\frac{1}{2}\right)$$

$$= \int_{-\infty}^{\infty} \sqrt{f_1(z) f_2(z)} dz \tag{13}$$

since

$$\sqrt{\pi_1 \pi_2} \le \frac{1}{2} \tag{14}$$

In most cases of interest such as when

$$f_1(z) = f_2(-z) \text{ for all } z \tag{15}$$

we have $\alpha = \frac{1}{2}$ minimizing the function $B(\alpha)$. When $f_1(z)$ and $f_2(z)$ are conditional probabilities of a communication channel model, this is usually the case.

Let us now reexamine the general form for the average error probability using the MAP decision rule. Note that

$$P_E = \int_{-\infty}^{\infty} \left\{ \pi_1 f_1(z) [1-\phi(z)] + \pi_2 f_2(z)\phi(z) \right\} dz$$

$$= \int_{-\infty}^{\infty} \min \left\{ \pi_1 f_1(z),\ \pi_2 f_2(z) \right\} dz \tag{16}$$

Following Hellman and Raviv [2] we note that for any $a \geq 0$, $b \geq 0$ and $0 \leq \alpha \leq 1$ we have

$$\min\{a,b\} \leq a^{\alpha} b^{1-\alpha}. \tag{17}$$

This yields the upper bound on the average error probability

$$P_E \leq \int_{-\infty}^{\infty} \left[\pi_1 f_1(z) \right]^{\alpha} \left[\pi_2 f_2(z) \right]^{1-\alpha} dz$$

$$= \pi_1^{\alpha} \pi_2^{1-\alpha} B(\alpha) \tag{18}$$

If the minimizing choice of α is in the unit interval [0,1] then this bound is always a factor of one-hald smaller than the bound given in (12). In particular for the Bhattacharyya bound where $\alpha = \frac{1}{2}$, this bound due to Hellman and Riviv shows that there is always a factor of one-half,

$$P_E \leq \frac{1}{2} \int_{-\infty}^{\infty} \sqrt{f_1(z)\, f_2(z)}\, dz \tag{19}$$

Thus the commonly used Bhattacharyya bound particularly in transfer function bit error bounds for convolutional codes can be tightened by a factor of one-half.

III. MAXIMUM LIKELIHOOD (ML)

The ML decision rule,

$$\phi(z) = \begin{cases} 1 \;, & f_1(z) \geq f_2(z) \\ 0 \;, & f_1(z) < f_2(z) \end{cases} \tag{20}$$

tends to keep both conditional probabilities closer in value but only minimizes P_E when $\pi_1 = \pi_2 = 1/2$, the equal a-priori probability case. In general we have inequalities

$$\phi(z) \leq \left[\frac{f_1(z)}{f_2(z)}\right]^{\alpha} \qquad (21)$$

and

$$[1-\phi(z)] \leq \left[\frac{f_2(z)}{f_1(z)}\right]^{\beta} \qquad (22)$$

resulting in conditional error bounds

$$P_{E_1} \leq B(1-\beta) \qquad (23)$$

and

$$P_{E_2} \leq B(\alpha) \qquad (24)$$

The average error probability is simply

$$P_E \leq \pi_1 B(1-\beta) + \pi_2 B(\alpha) \qquad (25)$$

while the choice $\alpha = \beta = \frac{1}{2}$ which often minimizes this bound yields the usual Bhattacharyya bound

$$P_E \leq B\left(\frac{1}{2}\right) \qquad (26)$$

Again using the inequality (17) we can show a tighter bound as follows:

$$\begin{aligned}
P_E &= \int_{-\infty}^{\infty} \left\{\pi_1 f_1(z)\,[1-\phi(z)] + \pi_2 f_2(z)\phi(z)\right\} dz \\
&\leq \max\left\{\pi_1,\pi_2\right\} \int_{-\infty}^{\infty} \left\{f_1(z)\,[1-\phi(z)] + f_2(z)\,\phi(z)\right\} dz \\
&= \max\left\{\pi_1,\pi_2\right\} \int_{-\infty}^{\infty} \min\left\{f_1(z), f_2(z)\right\} dz \\
&\leq \max\left\{\pi_1,\pi_2\right\} \int_{-\infty}^{\infty} f_1(z)^{\alpha} f_2(z)^{1-\alpha} dz \\
&= \max\left\{\pi_1,\pi_2\right\} B(\alpha) \qquad (27)
\end{aligned}$$

for $0 \leq \alpha \leq 1$. For the case

$$\pi_1 = \pi_2 = \frac{1}{2}$$

and $\alpha = \frac{1}{2}$ we again have a factor of one-half. Most cases of interest have equal a-priori probabilities.

IV. MAXIMUM METRIC AND CHERNOFF BOUNDS

We now assume that Z is some sort of metric used to make the decision when

$Z = z$.

$$\begin{aligned} z &\geq 0 \quad \text{choose } H_1 \\ z &< 0 \quad \text{choose } H_2. \end{aligned} \tag{28}$$

The decision function is then

$$\phi(z) = \begin{cases} 1 & , \quad z > 0 \\ 0 & , \quad z < 0 \end{cases} \tag{29}$$

and conditional errors are

$$\begin{aligned} P_{E_1} &= \int_{-\infty}^{\infty} [1-\phi(z)]f_1(z)dz \\ &= \int_{-\infty}^{0} f_1(z)dz \end{aligned} \tag{30}$$

and

$$\begin{aligned} P_{E_2} &= \int_{-\infty}^{\infty} \phi(z)f_2(z)dz \\ &= \int_{0}^{\infty} f_2(z)dz \end{aligned} \tag{31}$$

For $\alpha \geq 0$ and $\beta \geq 0$ we have the standard Chernoff bounds

$$P_{E_1} \leq \int_{-\infty}^{\infty} e^{-\alpha z} f_1(z)dz \tag{32}$$

and

$$P_{E_2} \leq \int_{-\infty}^{\infty} e^{\beta z} f_2(z)dz \tag{33}$$

Thus, the averaged error probability is

$$P_E \leq \pi_1 C_1(\alpha) + \pi_2 C_2(\beta) \tag{34}$$

where

$$C_1(\alpha) = \int_{-\infty}^{\infty} e^{-\alpha z} f_1(z)dz \tag{35}$$

and

$$C_2(\beta) = \int_{-\infty}^{\infty} e^{\beta z} f_2(z)dz \tag{36}$$

Note that in general if P_{E_1} and P_{E_2} are less than .5 then the Chernoff bounds are minimized by non-negative parameters α and β. Hence, the Chernoff bounds apply for all real values of α and β.

Jacobs [1] considered the conditions

$$f_1(-z) \geq f_1(z) \qquad \text{all } z \leq 0 \tag{37a}$$

and

$$f_2(-z) \geq f_2(z) \qquad \text{all } z \geq 0 \tag{37b}$$

Then using the inequality

$$\begin{aligned} \frac{e^{\omega}+e^{-\omega}}{2} &= \cosh \omega \\ &\geq 1 \qquad \text{all } \omega \end{aligned} \tag{38}$$

and change of variables of integration he showed the following inequalities,

$$\begin{aligned} C_1(\alpha) &= \int_{-\infty}^{\infty} e^{-\alpha z} f_1(z)\,dz \\ &= \int_{-\infty}^{0} e^{-\alpha z} f_1(z)\,dz + \int_{0}^{\infty} e^{-\alpha x} f_1(x)\,dx \\ &= \int_{-\infty}^{0} e^{-\alpha z} f_1(z)\,dz + \int_{-\infty}^{0} e^{\alpha z} f_1(-z)\,dz \\ &\geq \int_{-\infty}^{0} e^{-\alpha z} f_1(z)\,dz + \int_{-\infty}^{0} e^{\alpha z} f_1(z)\,dz \\ &= 2\int_{-\infty}^{0} \cosh \alpha z \cdot f_1(z)\,dz \\ &\geq 2\int_{-\infty}^{0} f_1(z)\,dz \end{aligned} \tag{39}$$

or

$$P_{E_1} \leq \frac{1}{2} C_1(\alpha) \tag{40}$$

and similarly

$$P_{E_2} \leq \frac{1}{2} C_2(\beta). \tag{41}$$

Thus, the often satisfied conditions given by Jacobs in (37) results in a factor of one-half in the usual Chernoff bounds.

Less restrictive but more difficult to prove conditions are that

$$\int_0^{\infty} e^{-\alpha^* z} f_1(z)\,dz \geq \int_{-\infty}^{0} e^{\alpha^* z} f_1(z)\,dz \tag{42a}$$

and

$$\int_{-\infty}^{0} e^{\beta^* z} f_2(z)\,dz \geq \int_0^{\infty} e^{-\beta^* z} f_2(z)\,dz \tag{42b}$$

where α^* minimizes $C_1(\alpha)$ and β^* minimizes $C_2(\beta^*)$. Note that for the special case of $\alpha^* = 0$ we have

$$\begin{aligned} \int_0^{\infty} f_1(z)\,dz &\geq \int_{-\infty}^{0} f_1(z)\,dz \\ &= P_{E_1} \end{aligned} \tag{43}$$

which is always satisfied when

$$P_{E_1} < \frac{1}{2} .$$

Indeed, conditions (42) are also true for some non-negative range of α values. We assume it is true for the minimizing choices of the Chernoff bound parameters. Note that conditions (42) are less restrictive than those of (37) since (37) implies (42). Now consider the inequalities,

$$\begin{aligned} C_1(\alpha) &\geq C_1(\alpha^*) \\ &= \int_{-\infty}^{0} e^{-\alpha^* z} f_1(z)\,dz \\ &= \int_{-\infty}^{0} e^{-\alpha^* z} f_1(z)\,dz + \int_0^{\infty} e^{-\alpha^* z} f_1(z)\,dz \\ &\geq \int_{-\infty}^{0} e^{-\alpha^* z} f_1(z)\,dz + \int_{-\infty}^{0} e^{\alpha^* z} f_1(z)\,dz \\ &= 2\int_{-\infty}^{0} \cosh \alpha^* z \cdot f_1(z)\,dz \end{aligned}$$

$$\geq 2\int_{-\infty}^{0} f_1(z)dz$$

$$= 2\, P_{E_1} \tag{44}$$

or

$$P_{E_1} \leq \frac{1}{2}\, C_1(\alpha). \tag{45}$$

and similarly

$$P_{E_2} \leq \frac{1}{2}\, C_2(\beta) \tag{46}$$

Next for the special case where

$$\pi_1 = \pi_2 = \frac{1}{2}$$

and

$$\alpha^* = \beta^*$$

sufficient conditions can be given by

$$\int_0^{\infty} e^{-\alpha^* z} f_1(z)dz \geq \int_0^{\infty} e^{-\alpha^* z} f_2(z)dz \tag{47a}$$

and

$$\int_{-\infty}^{0} e^{\alpha^* z} f_2(z)dz \geq \int_{-\infty}^{0} e^{\alpha^* z} f_1(z)dz \tag{47b}$$

Note that these conditions are always satisfied if our decision rule is a maximum likelihood decision rule where

$$f_2(z) \leq f_1(z) \qquad \text{for all } z \geq 0 \tag{48a}$$

and

$$f_2(z) > f_1(z) \qquad \text{for all } z < 0. \tag{48b}$$

Assuming conditions (47) we have

$$C_1(\alpha) + C_2(\beta)$$

$$\geq C_1(\alpha^*) + C_2(\alpha^*)$$

$$= \int_{-\infty}^{\infty} e^{-\alpha^* z} f_1(z)dz$$

$$+ \int_{-\infty}^{\infty} e^{\alpha^* z} f_2(z)dz$$

$$
\begin{aligned}
&= \int_{-\infty}^{0} e^{-\alpha^* z} f_1(z)dz + \int_{0}^{\infty} e^{-\alpha^* z} f_1(z)dz \\
&+ \int_{-\infty}^{0} e^{\alpha^* z} f_2(z)dz + \int_{0}^{\infty} e^{\alpha^* z} f_2(z)dz \\
&\geq \int_{-\infty}^{0} e^{-\alpha^* z} f_1(z)dz + \int_{0}^{\infty} e^{-\alpha^* z} f_2(z)dz \\
&+ \int_{-\infty}^{0} e^{\alpha^* z} f_1(z)dz + \int_{0}^{\infty} e^{\alpha^* z} f_2(z)dz \\
&= 2\int_{-\infty}^{0} \cosh \alpha^* z f_1(z)dz \\
&+ 2\int_{0}^{\infty} \cosh \alpha^* z f_2(z)dz \\
&\geq 2\, P_{E_1} + 2\, P_{E_2} \qquad (49)
\end{aligned}
$$

or

$$
\begin{aligned}
P_E &= \frac{1}{2} P_{E_1} + \frac{1}{2} P_{E_2} \\
&\leq \frac{1}{4} C_1(\alpha) + \frac{1}{4} C_2(\beta). \qquad (50)
\end{aligned}
$$

which is again a factor of one-half less than the original Chernoff bound on the average error probability (34) for $\pi_1 = \pi_2 = \frac{1}{2}$.

For the special case where Z happens to be a maximum likelihood metric,

$$
Z = \ell n \left[\frac{f_1(Z)}{f_2(Z)}\right], \qquad (51)
$$

then the conditions (47) hold and

$$
\begin{aligned}
C_1(\alpha) &= \int_{-\infty}^{\infty} e^{-\alpha z} f_1(z)dz \\
&= \int_{-\infty}^{\infty} \left[\frac{f_2(z)}{f_1(z)}\right]^{\alpha} f_1(z)dz \\
&= \int_{-\infty}^{\infty} f_1(z)^{1-\alpha} f_2(z)^{\alpha} dz \\
&= B(1-\alpha) \qquad (52)
\end{aligned}
$$

and

$$C_2(\beta) = \int_{-\infty}^{\infty} e^{\beta z} f_z(z)dz$$

$$= \int_{-\infty}^{\infty} \left[\frac{f_1(z)}{f_2(z)}\right]^{\beta} f_2(z)dz$$

$$= \int_{-\infty}^{\infty} f_1(z)^{\beta} f_2(z)^{1-\beta} dz$$

$$= B(\beta) \tag{53}$$

where $B(\cdot)$ is given by (11). Recall that $B\left(\frac{1}{2}\right)$ is the Bhattacharyya bound.

V. APPLICATIONS

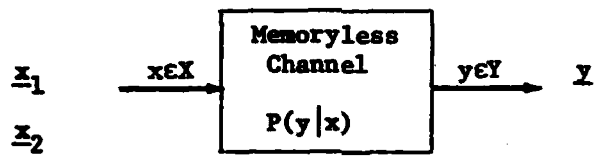
In most applications of interest we consider two sequences of length N,

$$\underline{x}_1,\ \underline{x}_2 \varepsilon X^N \tag{54}$$

that can be transmitted over a memoryless channel with input alphabet X and output alphabet Y and condition probability

$$P(y|x) \quad , \quad x\varepsilon X \quad , \quad y\varepsilon Y. \tag{55}$$

This is shown in the following figure.



$$P_N(\underline{y}|\underline{x}) = \prod_{n=1}^{N} P(y_n|x_n)$$

The receiver obtains a sequence

$$\underline{y}\varepsilon Y^N \tag{56}$$

from the channel and must decide between hypothesis

$$\begin{aligned} H_1&:\ \underline{x}_1 \text{ is sent} \\ H_2&:\ \underline{x}_2 \text{ is sent} \end{aligned} \tag{57}$$

which have a-priori probabilities

$$\pi_1 = \Pr\{H_1\} \text{ and } \pi_2 = \Pr\{H_2\}.$$

The receiver will typically use a metric

$$m(y,x) \quad x \epsilon X, \quad y \epsilon Y \tag{58}$$

and the decision rule where if and only if

$$\sum_{n=1}^{N} m(y_n,x_{1n}) \geq \sum_{n=1}^{N} m(y_n,x_{2n}) \tag{59}$$

then choose H_1. By defining

$$Z = \sum_{n=1}^{N} [m(y_n,x_{1n}) - m(y_n,x_{2n})] \tag{60}$$

we have the basic problem considered in previous sections.

For M sequences of length N denoted $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_M$ we have the decision rule: Given $\underline{y} \epsilon Y^N$ choose the sequence $\underline{x}_{\hat{m}}$ that has the largest total metric

$$\sum_{n=1}^{N} m(y_n,x_{\hat{m}n}) \tag{61}$$

for $\underline{x}_{\hat{m}} = (x_{\hat{m}1},x_{\hat{m}2},\ldots, x_{\hat{m}N}) \epsilon X^N$. The union bound for each conditional error probability is,

$$\begin{aligned} P_{E_m} &= \Pr\{\text{error} \,|\, \underline{x}_m\} \\ &\leq \sum_{\hat{m} \neq m} \Pr\{\text{decide } \underline{x}_{\hat{m}} \,|\, \underline{x}_m\} \end{aligned} \tag{62}$$

$$m = 1,2,\ldots, M.$$

Here we have

$$\Pr\{\text{deciding } \underline{x}_{\hat{m}} \,|\, \underline{x}_m\} \leq P(\underline{x}_m \rightarrow \underline{x}_{\hat{m}}) \tag{63}$$

where $P(\underline{x}_m \rightarrow \underline{x}_{\hat{m}})$ is the probability of deciding $\underline{x}_{\hat{m}}$ when $\underline{x}_m$ is sent assuming $\underline{x}_m$ and $\underline{x}_{\hat{m}}$ are the only two possible sequences. That is,

$$P(\underline{x}_m \rightarrow \underline{x}_{\hat{m}}) = \Pr\left\{ \sum_{n=1}^{N} [m(y_n,x_{\hat{m}n}) - m(y_n,x_{mn})] \geq 0 \,\middle|\, \underline{x}_m \right\} \tag{64}$$

which is the two hypothesis error probability. Here we apply our two hypothesis results discussed earlier.

VI. REFERENCES

[1] I. M. Jacobs, "Probability-of-Error Bounds for Binary Transmission on the Slowly Fading Rician Channel," IEEE Trans. Information Theory, vol. IT-12, pp. 431-441, October 1966.

[2] M. E. Hellman and J. Raviv, "Probability of Error, Equivocation, and the Chernoff Bound," IEEE Trans. Information Theory, vol. IT-16, pp. 368-372, July 1970.

Note No. 4

A GENERAL ANALYSIS OF ANTI-JAM COMMUNICATION SYSTEMS

to

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HF COMMUNICATION NETWORK SIGNALS

USING CHANNEL EVALUATION DATA

Principal Investigator

Jim K. Omura
Professor
System Science Department
University of California
Los Angeles, California

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A GENERAL ANALYSIS OF ANTI-JAM COMMUNICATION SYSTEMS

by

Jim K. Omura
Department of System Science
University of California
Los Angeles, California

I. INTRODUCTION

The performance of conventional uncoded communication systems for the idealized additive Gaussian noise channel is well understood [1,2]. For these cases closed form expressions are known for the bit error probabilities. When coding is used, however, exact bit error probability expressions are difficult to obtain and upper bounds are typically used to evaluate performance [3]. This is also often true in evaluating uncoded bit error probabilities when we have any of the following more complex signals and channels:

- new bandwidth efficient modulations
- intersymbol interference signals
- multipath signals
- interference due to other signals
- channel nonlinearities
- suboptimum receivers
- fading channels

In general once we consider more complex signals and channels, bounds on the bit error probabilities are usually required to practically evaluate bit error probabilities. Obtaining exact bit error probabilities for these more complex communication systems, especially those using coding, is often unrealistic.

In this note we shall consider the general anti-jam communication system illustrated in Figure 1. We will derive a general error bound which will serve

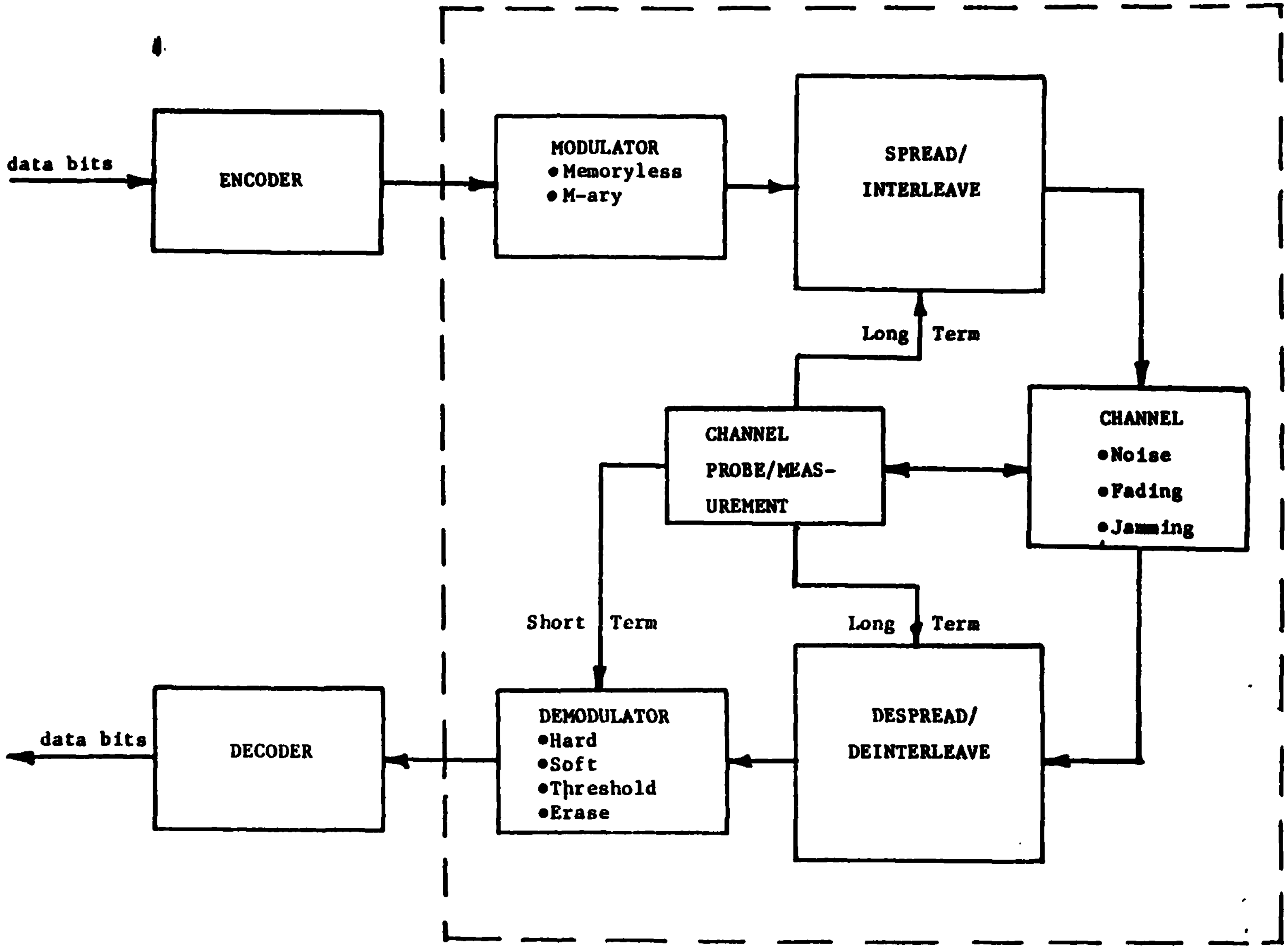


FIGURE 1. AJ SYSTEM

as the basis for evaluating the performance of all such complex communication systems. The key feature of this approach is the decoupling of the coding aspects of the system from the remaining part of the communication system. Specifically we compute the cutoff rate parameter [3,4,5,6]

$$R_0 = f\left(\frac{E_c}{N_0}\right) \quad \text{bits/channel use} \tag{1}$$

which represents the practically achievable reliable data rate per channel use. This cutoff rate will be a function of the equivalent channel signal energy to noise ratio

$$\frac{E_c}{N_0} = r\,\frac{E_b}{N_0} \tag{2}$$

which is shown here to be directly related to the energy-per-bit to noise ratio usually given by-(assuming jamming limits performance)

$$\frac{E_b}{N_0} = \frac{PG}{J/S}\ . \tag{3}$$

Here r is the data rate in bits per channel signal and we have the usual jamming parameters

$$\begin{aligned} PG &= \frac{W}{R} \quad \text{, processing gain} \\ S &= \text{signal power} \\ J &= \text{jammer power} \\ W &= \text{spread bandwidth} \\ R &= \text{data rate in bits per second.} \end{aligned} \tag{4}$$

For conventional coded direct sequence (DS) coherent BPSK signals E_c is the energy per coded bit while for m diversity frequency hopped (FH) noncoherent MFSK, E_c is the energy of each diversity "chip" signal.

For any specific code we then derive a bound on the coded bit error probability of the form*

$$P_b \le \frac{1}{2}\,B(R_0) \tag{5}$$

*The factor of one-half appears in most of the Chernoff bounds. This is discussed in another note.

which is only a function of the cutoff rate R_0. Since the function $B(R_0)$ is unique for each code and the cutoff rate parameter, R_0, is independent of the code used we are able to decouple the coding from the rest of the communication system. Thus to evaluate various anti-jam communication systems we can first compare them using the cutoff rate parameter. Codes can then be evaluated separately.

The use of diversity and conventional MFSK modulation can be viewed as a form of coding. Here too bit error bounds will be expressed in terms of the cutoff rate R_0. For some special cases where we know exact bit error probabilities we compare them with these general bounds and show that the bound is about 1 dB off in E_b/N_0 for bit error probabilities of 10^{-3} or less.

II. EQUIVALENT MEMORYLESS CHANNEL

The general AJ communication system is modeled as shown in Figure 1. Here we consider the subsystem shown inside the dotted lines as an equivalent memoryless channel available for sending coded data. We assume ideal interleaving which creates the memoryless property. Various types of receiver characteristics include:

- Channel Output
  - Hard Quantized
  - Soft Quantized
  - Threshold
- Jammer State Information
  - Unknown
  - Known
- Decision Rule
  - Maximum Likelihood
  - Maximum Metric

- Channel State Information
  - Noise Level
  - Signal Power

In some cases where channel characteristics may change slowly channel probes and measurements may be used to enchance communication. On a short-term basis a receiver may detect when the particular transmitted symbol is jammed or not. These are all included in the general communication system of Figure 1. Later we shall compare various special cases.

III. CUTOFF RATE AND BIT ERROR BOUNDS WITH NO JAMMER STATE INFORMATION

We assume the receiver does not know when a jamming signal is present in the channel output. For each example AJ system we have an equivalent memoryless channel as shown in Figure 2 with input symbols from alphabet X and output symbols from alphabet Y. We also assume that the receiver uses a metric m(y,x) to make decisions. The maximum likelihood decision rule corresponds to a particular choice of m(y,x).

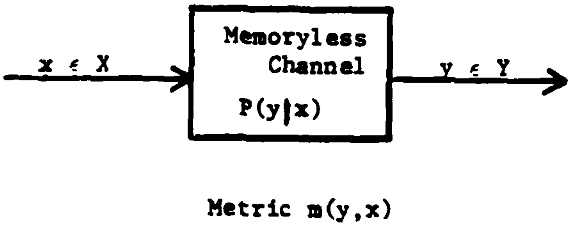


Figure 2 EQUIVALENT MEMORYLESS CHANNEL

Consider two sequences $\underline{x}, \hat{\underline{x}} \in X^N$ and the pairwise error probability of the receiver choosing $\hat{\underline{x}}$ when $\underline{x}$ is transmitted assuming these are the only two possible transmitted sequences. This probability is denoted $P(\underline{x} \to \hat{\underline{x}})$. Using the Chernoff bound with parameter $\lambda \geq 0$ we obtain

$$\begin{aligned} P(\underline{x} \to \hat{\underline{x}}) &= \Pr\left\{ \sum_{n=1}^{N} m(y_n, x_n) \leq \sum_{n=1}^{N} m(y_n, \hat{x}_n) \,\middle|\, \underline{x} \right\} \\ &= \Pr\left\{ \sum_{n=1}^{N} [\, m(y_n, \hat{x}_n) - m(y_n, x_n)] \geq 0 \,\middle|\, \underline{x} \right\} \\ &\leq E\left\{ e^{\lambda \sum_{n=1}^{N} [m(y_n, \hat{x}_n) - m(y_n, x_n)]} \,\middle|\, \underline{x} \right\} \qquad (6) \\ &= \prod_{n=1}^{N} E\left\{ e^{\lambda [m(y_n, \hat{x}_n) - m(y_n, x_n)]} \,\middle|\, x_n \right\} \end{aligned}$$

Defining

$$D(x, \hat{x}; \lambda) = E\left\{ e^{\lambda [m(y, \hat{x}) - m(y, x)]} \,\middle|\, x \right\} \tag{7}$$

we have the Chernoff bound,

$$P(\underline{x} \to \hat{\underline{x}}) \leq \prod_{n=1}^{N} D(x_n, \hat{x}_n; \lambda) \tag{8}$$

where

$$D(x, x; \lambda) = 1 \quad \text{all } x \in X. \tag{9}$$

Suppose all the components of $\underline{x}$ and $\hat{\underline{x}}$ are independently chosen according to the probability distribution

$$q(x), \; x \in X. \tag{10}$$

The cutoff rate is generally defined as,

$$R_0 = \max_{\lambda \geq 0} \; \max_{q} R_0(q; \lambda) \tag{11}$$

where $R_0(q, \lambda)$ is given by the relation

$$\begin{aligned} 2^{-R_0(q, \lambda)} &= E\left\{ D(x, \hat{x}; \lambda) \right\} \\ &= \sum_{x} \sum_{\hat{x}} q(x) q(\hat{x}) D(x, \hat{x}; \lambda) \end{aligned} \tag{12}$$

Throughout this note we consider symmetric channels where for alphabet size,

$$M = |X| \tag{13}$$

we have

$$q(x) = \frac{1}{M}\ ,\ x \in X \tag{14}$$

and

$$D(x,\hat{x};\lambda) = \begin{cases} D(\lambda) & ,\ \hat{x} \neq x \\ 1 & ,\ \hat{x} = x. \end{cases} \tag{15}$$

For this case

$$\sum_x \sum_{\hat{x}} q(x)\ q(\hat{x})\ D(x,\hat{x};\lambda) = \frac{1 + (M-1)\ D(\lambda)}{M} \tag{16}$$

and the cutoff rate is simply,

$$R_0 = \log_2 M - \log_2 [1+(M-1)D] \tag{17}$$

where

$$D = \min_{\lambda \geq 0} D(\lambda). \tag{18}$$

Note that here there is a simple one-to-one onto relationship between D and R_0.

For this symmetric case the pairwise error probability for two sequences $\underline{x},\hat{\underline{x}} \in X^N$ has the bound

$$P(\underline{x} \to \hat{\underline{x}}) \leq \prod_{n=1}^{N} D(x_n,\hat{x}_n;\lambda) = D(\lambda)^{w(\underline{x},\hat{\underline{x}})} \tag{19}$$

where $w(\underline{x},\hat{\underline{x}})$ is the Hamming distance between $\underline{x}$ and $\hat{\underline{x}}$. Minimizing the bound with respect to $\lambda \geq 0$ we have

$$P(\underline{x} \to \hat{\underline{x}}) \leq D^{w(\underline{x},\hat{\underline{x}})} \tag{20}$$

When using a specific code which consists of many sequences the decoded bit

error probability can be union bounded by pairwise error probabilities which result in bit error probabilities of the form

$$P_b \leq G(D) \tag{21}$$

where G( ) depends on the specific code. Since D can be expressed in terms of the cutoff rate R_0 we also have the equivalent bound

$$P_b \leq B(R_0) \tag{22}$$

where again $B(\cdot)$ depends on the specific code wheras the cutoff rate R_0 depends only on the memoryless channel.

If the metric happens to be a maximum liklihood metric which has the form

$$m(y,x) = a\ln p(y|x) + b \tag{23}$$

where $a > 0$, we have

$$\begin{aligned} D(x,\hat{x};\lambda) &= E\left\{ e^{\lambda[m(y,\hat{x}) - m(y,x)]} \Big| x \right\} \\ &= \sum_y e^{\lambda a[\ln p(y|\hat{x}) - \ln p(y|x)]} p(y|x) \\ &= \sum_y p(y|x) \left[\frac{p(y|\hat{x})}{p(y|x)}\right]^{\lambda a} \end{aligned} \tag{24}$$

This is usually minimized by $\lambda a = 1/2$. Thus

$$\min D(x,\hat{x};\lambda) = \sum_y \sqrt{p(y|x)p(y|\hat{x})} \tag{25}$$

which is the Bhattacharyya function [3]. The optimized Chernoff bound for the maximum liklihood metric results in the usual Bhattacharyya bound.

For all metrics of interest including the maximum likelihood metric, the general bound iven in (22) can be reduced by a factor of one half. In another note we show that under a difficult to check, but easily satisfied condition we have

$$P_b \leq \frac{1}{2} B(R_0). \tag{26}$$

Throughout this paper we assume this factor of one-half applies.

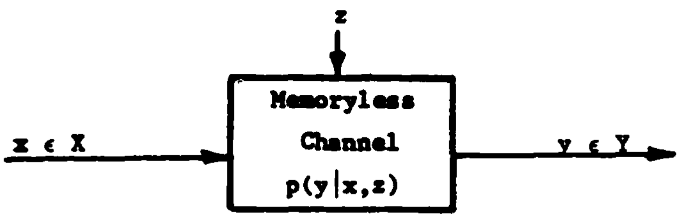
IV. CUTOFF RATE AND BIT ERROR BOUNDS WITH JAMMER STATE INFORMATION

We now assume that the jamming signal is present at the channel output with probability ρ. Thus for each use of the channel the jammer state is characterized by the random variable

$$
z = \begin{cases} 1, & \text{with probability } \rho \\ 0, & \text{with probability } 1-\rho. \end{cases} \tag{27}
$$

Assuming ideal interleaving and/or diversity we assume this random variable is independent of the other uses of the channel.

The equivalent memoryless channel with the receiver having jammer state information is shown in Figure 3. Here now the receiver uses metric $m(y,x \mid z)$.



Metric $m(y,x \mid z)$

Figure 3 EQUIVALENT MEMORYLESS CHANNEL

Consider two sequences $\underline{x}, \hat{\underline{x}} \in X^N$, jammer state sequence $\underline{z}$, and the pairwise error probability of the receiver choosing $\hat{\underline{x}}$ when $\underline{x}$ is transmitted assuming these are the only two possible transmitted sequences given $\underline{z}$. This probability is denoted $P(\underline{x} \to \hat{\underline{x}} \mid \underline{z})$. Using the Chernoff bound with parameter $\lambda \geq 0$ we obtain

$$
\begin{aligned}
P(\underline{x} \to \hat{\underline{x}} \mid \underline{z}) &= \Pr\left\{ \sum_{n=1}^{N} m(y_n, x_n \mid z_n) \leq \sum_{n=1}^{N} m(y_n, \hat{x}_n \mid z_n) \,\middle|\, \underline{x}, \underline{z} \right\} \\
&= \Pr\left\{ \sum_{n=1}^{N} [m(y_n, \hat{x}_n \mid z_n) - m(y_n, x_n \mid z_n)] \geq 0 \,\middle|\, \underline{x}, \underline{z} \right\} \\
&\leq E\left\{ e^{\lambda \sum_{n=1}^{N} [m(y_n, \hat{x}_n \mid z_n) - m(y_n, x_n \mid z_n)]} \,\middle|\, \underline{x}, \underline{z} \right\} \\
&= \prod_{n=1}^{N} E\left\{ e^{\lambda [m(y_n, \hat{x}_n \mid z_n) - m(y_n, x_n \mid z_n)]} \,\middle|\, x_n, z_n \right\}
\end{aligned} \tag{28}
$$

Now averaging over the sequence $\underline{z} = (z_1, z_2, \ldots, z_n)$ we have

$$P(\underline{x} \to \hat{\underline{x}}) \le \prod_{n=1}^{N} E\{ e^{\lambda \sum_{n=1}^{N} [m(y_n,\hat{x}_n \mid z_n) - m(y_n,x_n \mid z_n)]} \mid x_n \} \quad (29)$$

Defining

$$D(x,\hat{x};\lambda) = E\{e^{\lambda[m(y,\hat{x} \mid z) - m(y,x \mid z)]} \mid x\} \quad (30)$$

we have the Chernoff bound

$$P(\underline{x} \to \hat{\underline{x}}) \le \prod_{n=1}^{N} D(x_n,\hat{x}_n;\lambda) \quad (31)$$

where

$$D(x,x;\lambda) = 1 \quad \text{all } x \in X . \quad (32)$$

The rest of the analysis is identical to that of the previous section. In fact when we have the special case,

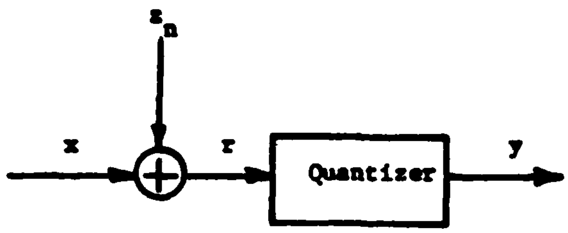
$$m(y,x \mid z) = m(y,x) \quad (33)$$

we obtain all the results of the previous section using the channel probability

$$P(y \mid x) = (1-\rho)p(y \mid x,z = 0) + \rho P(y \mid x,z = 1). \quad (34)$$

V. DS/BPSK EXAMPLES

The use of direct sequence (DS) spreading reduces the jamming signal to an equivalent white Gaussian noise term after despreading at the receiver. If the jammer uses pulse jamming with fraction of jamming time ρ then the equivalent discrete time channel is modeled as shown in Figure 4. Here we assume ideal interleaving so that the channel is memoryless. The quantizer determines the choice of channel outputs being hard or soft decisions. We now examine several special cases.



$$X = \begin{cases} E_c \ , & \text{with probability } 1/2 \\ E_c \ , & \text{with probability } 1/2 \end{cases}$$

$$n \sim N\left(0, \frac{N_0}{2\rho}\right)$$

$$Z = \begin{cases} 1 \ , & \text{with probability } \rho \\ 0 \ , & \text{with probability } 1-\rho \end{cases}$$

Figure 4. EQUIVALENT DS/BPSK CHANNEL
WITH PULSE JAMMING.

A. Soft Decision and Known Jammer State

Ideal performance is achieved when the receiver has complete channel output information, knows the jammer state for each use of the channel, and uses the maximum likelihood metric

$$m(y,x \mid z) = \ell n\ p(y \mid x,z). \tag{35}$$

This yeilds the Bhattacharyya bound

$$\begin{aligned} D(x,\hat{x};\ 1/2) &= E\left\{ \int_{-\infty}^{\infty} \sqrt{p(y \mid x,z)\ p\ (y \mid \ddot{x},z)}\ \ dy \right\} \\ &= \begin{cases} 1 & , \ \hat{x} = x \\ \rho e^{-\rho \frac{E_c}{N_0}} & , \ \hat{x} \neq x \end{cases} \end{aligned} \tag{36}$$

or

$$D = \rho e^{-\rho \frac{E_c}{N_o}} \tag{37}$$

For the uncoded case where $N = 1$ and $E_c = E_b$ we have the bit error bound

$$P_b \leq \frac{1}{2} D$$
$$= \frac{1}{2} \rho e^{-\rho \frac{E_b}{N_o}} \tag{38}$$

wheras the exact bit error probability is

$$P_b = \rho Q\left(\sqrt{\rho \frac{2E_b}{N_o}}\right) \tag{39}$$

For continuous jamming where $\rho = 1$ we have the exact bit error probability and the bound shown in Figure 5. For the worst case value of ρ that maximizes the bit error probability we obtain the worst case pulse jamming curve for the exact bit error probability also shown in Figure 5. When ρ is chosen to maximize the parameter D we have the corresponding worst case pulse jammer bit error bound shown in Figure 5 .

Using a binary code of rate r bits per channel use we have the bit error probability bound

$$P_b \leq \frac{1}{2} G(D) \tag{40}$$

where

$$D = \rho e^{-\rho \frac{E_c}{N_o}} \tag{41}$$

and

$$\frac{E_c}{N_o} = r \frac{E_b}{N_o} \tag{42}$$

again choosing ρ that maximizes D we obtain the bit error bound shown in Figure 6 for the $K = 7$, $r = 1/2$ convolutional code which has parameters

$$G(D) = \sum_{k=d_{min}}^{\infty} N_k D^k ,$$
$$d_{min} = 10 \tag{43}$$

Figure 5 Pulse Jamming DS/BPSK

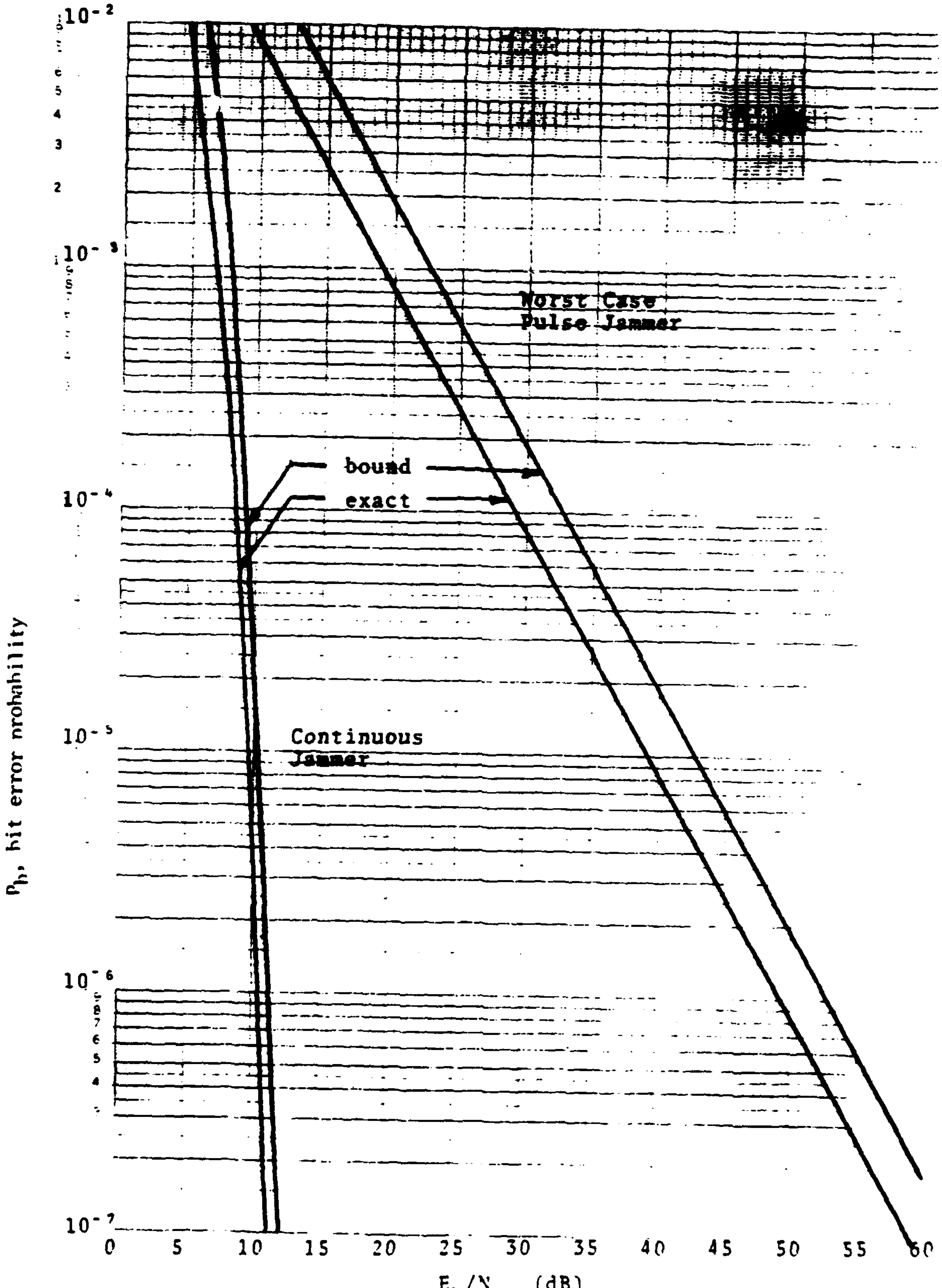
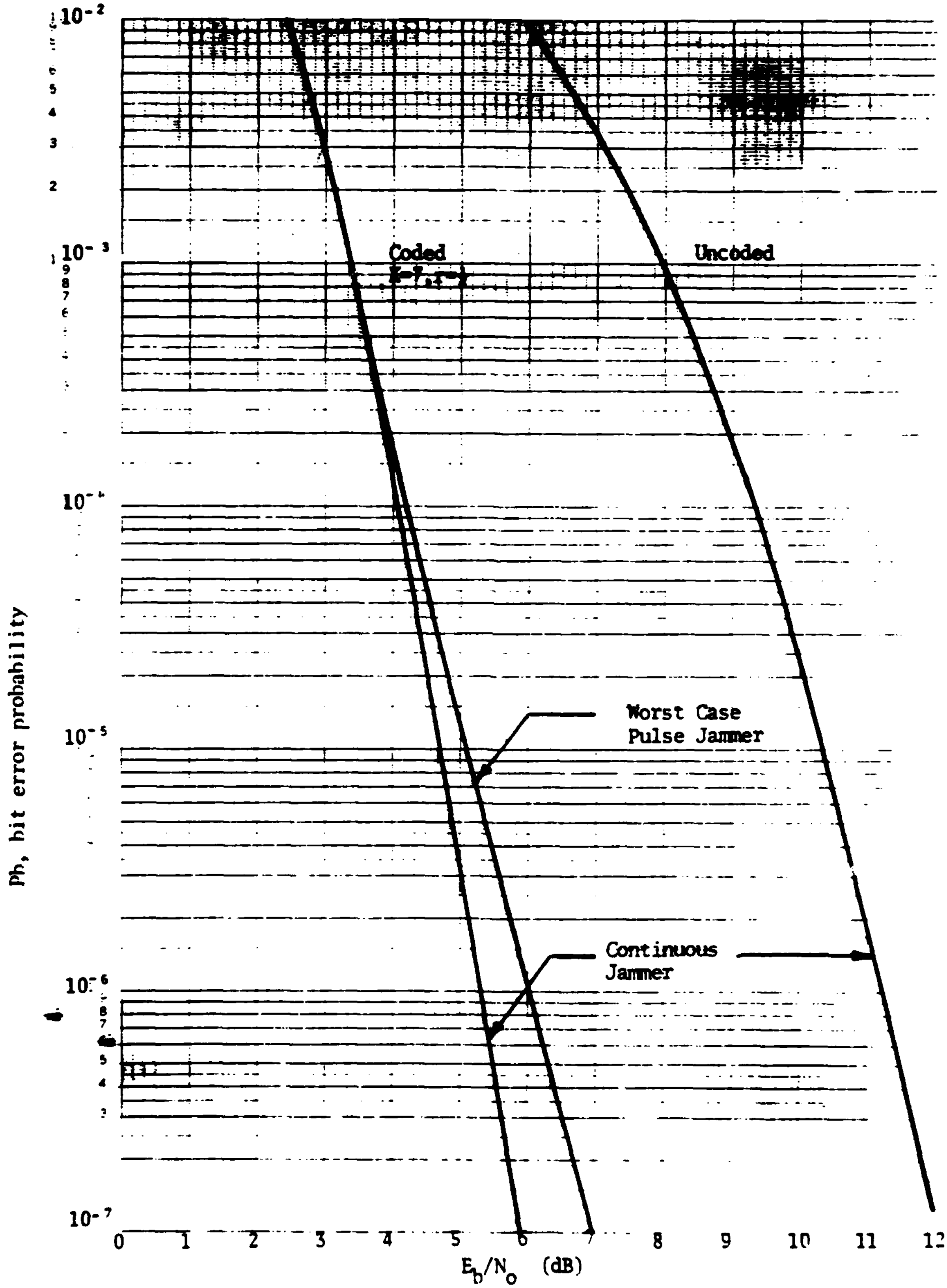


Figure 6 Coding/Interleaving DS/BPSK



and

$$N_{10} = 36 \qquad N_{14} = 1{,}404$$
$$N_{11} = 0 \qquad N_{15} = 0$$
$$N_{12} = 20 \qquad N_{16} = 11{,}633 \tag{44}$$
$$N_{13} = 0 \qquad N_{17} = 0$$

Note that since the cutoff rate R_0 and D are related by

$$R_0 = 1 - \log_2(1+D) \quad \text{bits/channel use} \tag{45}$$

or

$$D = 2^{(1-R_0)} - 1 \tag{46}$$

we have for this code

$$P_b \leq \frac{1}{2} \; B(R_0) \tag{47}$$

where

$$B(R_0) = \sum_{k=d_{min}}^{\infty} N_k (2^{(1-R_0)}-1)^k \tag{48}$$

B. Hard Decision and Known Jammer Satate

The hard decision quantizer of Figure 4 results in a binary symmetric channel (BSC) with crossover probability

$$\epsilon\,(z) = \begin{cases} p & , z = 1 \\ 0 & , z = 0 \end{cases} \tag{49}$$

where

$$p = Q\left(\sqrt{\rho \frac{2E_c}{N_o}} \right) \tag{50}$$

Suppose the receiver knows the jammer state and uses the maximum likelihood metric. Then we have the Bhattacharyya bound

$$D(x,x;\ 1/2) = E\{\sum_{y=0}^{1} \sqrt{p(y|x,z)\ p\ (y|\hat{x},z)}\}$$

$$= \begin{cases} 1 & ;\ \hat{x} = x \\ \rho\ \sqrt{4p(1-p)} & ;\ \hat{x} \neq x \end{cases} \tag{51}$$

or

$$D = \rho\ \sqrt{4Q(\sqrt{\rho\frac{2E_c}{N_o}})\ [1-Q(\sqrt{\rho\frac{2E_c}{N_o}})]} \tag{52}$$

Observing that in Figure 5 for E_c/N_o of interest we have

$$Q(\sqrt{\rho\frac{2E_c}{N_o}}) \simeq \frac{1}{2}\ e^{-\rho\frac{E_c}{N_o}} \tag{53}$$

and

$$D \simeq \sqrt{2}\ \rho e^{-\rho\frac{E_c}{2N_o}} \tag{54}$$

which is approximately $10 \log_{10} 2 = 3$ dB in E_c/N_o from the soft decision channel. This is expected based on the conventional additive white Gaussian noise channel.

C. Soft Decision and Unknown Jammer State

We assume the receiver does not know the jammer state and uses the metric

$$m(y,x) = yx$$

which would be a maximum likelihood metric if we had a simple continuous jammer where $\rho = 1$. Then

$$\begin{aligned} D(x,\hat{x};\lambda) &= E\ \{e^{\lambda[y(\hat{x} - x)]/N_o}|x\} \\ &= E\ \{e^{\lambda(x + z_n)(\hat{x} - x)/N_o}\} \\ &= e^{\lambda x(\hat{x} - x)/N_o}\ E\{e^{\lambda z_n(x - x)/N_o}\} \\ &= e^{\lambda x(\hat{x} - x)/N_o}\ \{1-\rho+\rho e^{\lambda^2(\hat{x} - x)^2/\rho}\} \end{aligned}$$

or

$$D(\lambda) = e^{-2\lambda E_c/N_o}\ \{1-\rho+\rho e^{4\lambda^2 E_c/(\rho N_o)}\} \tag{55}$$

Note that if

$$\rho \leq 4\lambda^2 \left(\frac{E_c}{N_o}\right) \tag{56}$$

the derivative of $D(\lambda)$ with respect to ρ is negative suggesting ρ should be chosen as small as possible to maximize this function. In general regardless of the choice of λ by decreasing ρ $D(\lambda)$ will exceed one and we have either a poor bound and/or this soft decision receiver without jammer state knowledge yeilds poor performance. To check this actual performance consider two sequences $\underline{x}, \hat{\underline{x}} \in X^N$

where

$$w = w(\underline{x}, \hat{\underline{x}}). \tag{57}$$

Then

$$\begin{aligned}
P(\underline{x} \rightarrow \hat{\underline{x}}) &= Pr\{(\underline{y}, \hat{\underline{x}} - \underline{x}) \geq 0 \mid \underline{x}\} \\
&= Pr\{\sum_{n=1}^{N} y_n(\hat{x}_n - x_n) \geq 0 \mid \underline{x}\} \\
&= Pr\{\sum_{i=1}^{w} (-\sqrt{E_c} + z_i n_i) \geq 0\} \\
&= Pr\{\sum_{i=1}^{w} z_i n_i \geq w\sqrt{E_c}\} \\
&= \sum_{\ell=1}^{w} Pr\{\sum_{i=1}^{w} z_i n_i \geq w\sqrt{E_c} \mid \sum_{i=1}^{w} z_i = \ell\} Pr\{\sum_{i=1}^{w} z_i = \ell\} \\
&= \sum_{\ell=1}^{w} \binom{w}{\ell} \rho^{\ell}(1-\rho)^{w-\ell} Q\left(\sqrt{2w^2\left(\frac{E_c}{\ell N_o}\right)}\right) \\
&\leq D(\lambda)^w
\end{aligned} \tag{58}$$

Instead of choosing ρ to maximize $P(\underline{x} \rightarrow \hat{\underline{x}})$ choose

$$\rho = \frac{1}{2w^2\left(\frac{E_c}{N_o}\right)} \tag{59}$$

and note that

$$\begin{aligned}
Q\left(\sqrt{2\rho w^2\left(\frac{E_c}{\ell N_o}\right)}\right) &= Q\left(\sqrt{\frac{1}{\ell}}\right) \\
&> Q(1)
\end{aligned} \tag{60}$$

Then

$$P(\underline{x} \to \underline{\hat{x}}) \geq \sum_{\ell=1}^{w} \binom{w}{\ell} \rho^{\ell}(1-\rho)^{w-\ell} Q(1)$$

$$= [1- (1-\rho)^{w}]\ Q(1)$$

$$= [1- (1- \frac{1}{2w^2(\frac{E_c}{N_o})})^{w}\ ]\ Q(1) \tag{61}$$

This shows that for the worst case choice of ρ $P(\underline{x} \to \underline{\hat{x}})$ cannot decrease exponentially fast with w and hence the soft decision channel with no jammer state information yeilds poor performance.

D. Hard Decision and Unknown Jammer State

For the hard decision channel the memoryless channel becomes a BSC with crossover probability

$$\epsilon = \rho Q(\sqrt{\rho \frac{2E_c}{N_o}}) \tag{62}$$

A receiver that has no jammer state knowledge uses the metric

$$m(y,x) = -w(y,x) \tag{63}$$

and has

$$\min D(x,\hat{x};\lambda) = \begin{cases} 1 & , \hat{x} = x \\ \sqrt{4\epsilon(1-\epsilon)}, & \hat{x} \neq x \end{cases} \tag{64}$$

or

$$D = \sqrt{4\rho Q(\sqrt{\rho\frac{2E_c}{N_o}})[1 - \rho Q(\sqrt{\rho\frac{2E_c}{N_o}})]} \tag{65}$$

which when compared with the hard decision channel with known jammer state differs by roughly $\sqrt{\rho}$. Against the worst case pulse jammer the hard decision receiver with no jammer state knowledge does much better than the soft decision channel with no jammer state knowledge. This is because here

$$D < 1 \tag{66}$$

and

$$P(\underline{x} \to \underline{\hat{x}}) \leq \frac{1}{2} D^{w(\underline{x},\underline{\hat{x}})} \tag{67}$$

decreases at least exponentially with $w(\underline{x}, \hat{\underline{x}})$ wheras this is not true for the soft decision channel with no jammer state knowledge.

E. R_0 Evaluation

The cutoff rate for each coded bit in the worst case pulse jamming environment is given by

$$R_0 = 1 - \log_2(1+D) \quad \text{bits/channel use} \tag{68}$$

where D is a function of the equivalent coded bit energy - to - noise ratio E_c/N_o where

$$E_c = S\, T_c \tag{69}$$

where T_c is the coded bit duration or BPSK modulation pulse duration and

$$N_o = \frac{J}{W}. \tag{70}$$

Here D is given by

(1) soft decision and known jammer state

$$D_1 = \max_{0 \le \rho \le 1} \quad \rho e^{-\rho\left(\frac{E_c}{N_o}\right)} \tag{71}$$

(2) hard decision and know jammer state

$$D_2 = \max_{0 \le \rho \le 1} \quad 2\rho \sqrt{Q\left(\sqrt{2\rho\left(\frac{E_c}{N_o}\right)}\right) \left[1 - Q\left(\sqrt{2\rho\left(\frac{E_c}{N_o}\right)}\right)\right]} \tag{72}$$

(3) soft decision and unknown jammer state

$$D_3 = \min\{1, D^*\} \tag{73}$$

where

$$D^* = \max_{0 \le \rho \le 1} \; \min_{\lambda \ge 0} \; e^{-2\lambda\left(\frac{E_c}{N_o}\right)} \left\{1 - + e^{\frac{4\lambda^2}{\rho}\left(\frac{E_c}{N_o}\right)}\right\} \tag{74}$$

(4) hard decision and unknown jammer state

$$D_4 = \max_{0 \le \rho \le 1} \quad \sqrt{4\rho Q\left(\sqrt{2\rho\left(\frac{E_c}{N_o}\right)}\right) \left[1 - \rho Q\left(\sqrt{2\rho\left(\frac{E_c}{N_o}\right)}\right)\right]} \tag{75}$$

These four examples are shown in Figure 7. Figure 8 shows the maximizing values of ρ. For each of these cases for a specific code characterized by a function $B(\cdot)$ we have the bit error bound

$$P_b \le \frac{1}{2} B(R_0). \tag{76}$$

The $K = 7$, $r = 1/2$ convolutional code example was given by (43) and (44). Here R_0 and D depend on E_c/N_0 where $E_c = E_b/2$.

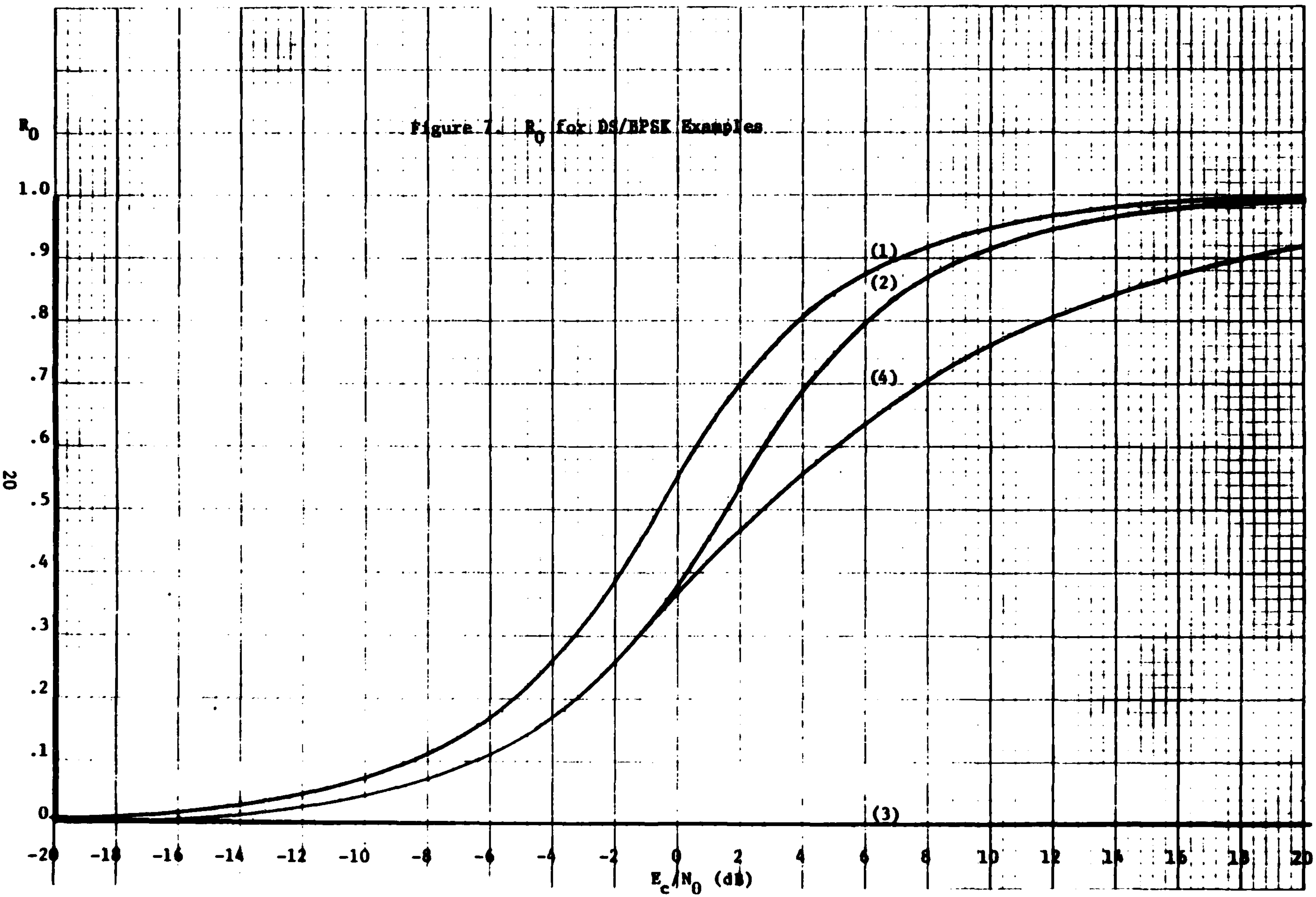


Figure 7. R_0 for DS/BPSK Examples

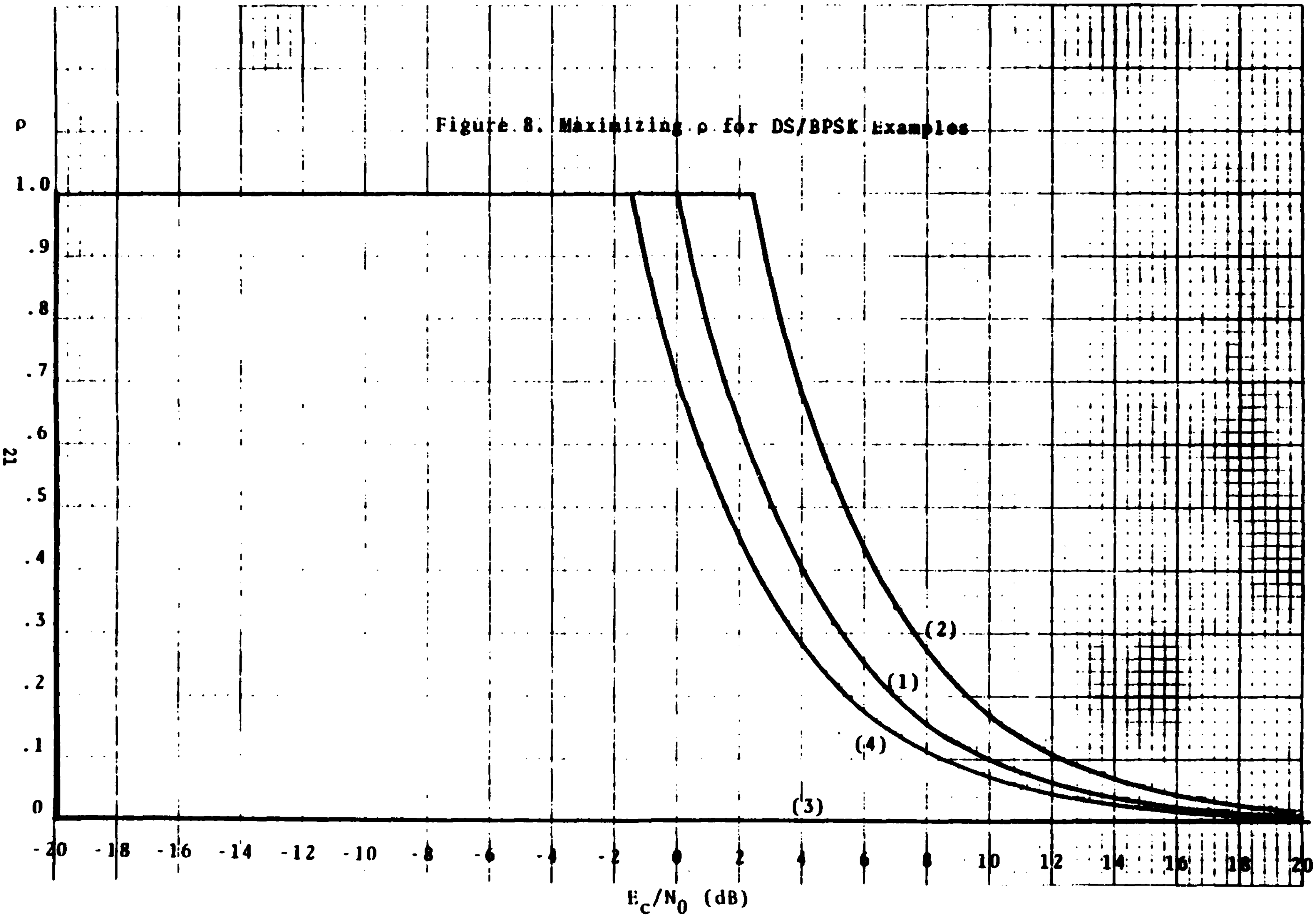


Figure 8. Maximizing ρ for DS/BPSK Examples

VI. FH/MFSK EXAMPLES

We next consider several examples of noncoherent MFSK signals that are frequency hopped to combat jamming. We assume a partial band noise jammer that jams bandwidth W_J with noise spectral density J/W_J. Defining N_o as J/W we have

$$\frac{J}{W_J} = \frac{J}{W} \cdot \frac{W}{W_J}$$
$$= N_o/\rho \tag{77}$$

where

$$\rho = \frac{W_J}{W} \tag{78}$$

is the fraction of the total band W this is jammed. During each hop of duration T_c we assume one MFSK "chip" is transmitted resulting, after dehopping, in an equivalent additive noise channel with noise $Zn(t)$ where

$$Z = \begin{cases} 1, & \text{with probability } \rho \\ 0, & \text{with probability } 1-\rho \end{cases} \tag{79}$$

and n(t) is a white Gaussian noise process with power spectral density N_0/ρ.

A. Soft Decision and Known Jammer State

Ideal performance is achieved when the receiver has complete channel output information, knowledge of the jammer state for each use of the channel, and uses the maximum likelihood metric. The maximum likelihood metric, however, involves a zeroth order Bessel function, $I_o(\cdot)$, which is not convenient as the simple energy detector outputs. We consider here the input alphabet $X=\{1,2,\ldots M\}$ the channel output symbols $y \in R^M$ where

$$y = (y^{(1)}, y^{(2)}, \ldots y^{(M)}) \tag{80}$$

where

$$y^{(\ell)} = \ell^{th} \text{ chip energy detector output}$$
$$\ell = 1,2,\ldots,M. \tag{81}$$

The metric we consider has the form

$$m(y,x|z) = \lambda(z)\, y^{(x)} \tag{82}$$

When a channel input sequence of symbols

$$\underline{x} = (x_1, x_2, \ldots, x_N) \tag{83}$$

is transmitted the total metric is simply

$$\sum_{n=1}^{N} m(y_n, x_n | z_n) = \sum_{n=1}^{N} \lambda(z_n)\, y_n^{(x_n)} \tag{84}$$

which is a weighted noncoherent sum of the chip energy detector outputs corresponding to input sequence $\underline{x}$. Here $\lambda(z_n)$ is some function which incorporates the Chernoff bound parameter which we examine next.

For this case we have for $\hat{x} \neq x$

$$\begin{aligned} D(x,\hat{x};\lambda) &= E\{e^{\frac{\rho\lambda(z)}{N_o}[y^{(\hat{x})} - y^{(x)}]}\Big| x\} \\ &= E\{E\{e^{\frac{\rho\lambda(z)}{N_o} y^{(\hat{x})}}\Big| x,z\} \\ &\quad \cdot E\{e^{\frac{-\rho\lambda(z)}{N_o} y^{(x)}}\Big| x,z\} | x\} \end{aligned} \tag{85}$$

where

$$E\{e^{\frac{\rho\lambda(z)}{N_o} y^{(\hat{x})}}\Big| x,z\} = \begin{cases} 1 & , \quad z=1 \\ \dfrac{1}{1-\lambda_1} & , \quad z=1 \end{cases} \tag{86}$$

$$E\{e^{\frac{-\rho\lambda(z)}{N_o} y^{(x)}}\Big| x,z\} = \begin{cases} e^{-\rho\lambda_o\left(\frac{E_c}{N_o}\right)} & , \quad z = 0 \\ \dfrac{1}{1+\lambda_1}\, e^{-\frac{\lambda_1}{1+\lambda_1}\rho\left(\frac{E_c}{N_o}\right)} & , \quad z = 1 \end{cases} \tag{87}$$

and

$$\lambda(z) = \begin{cases} \lambda_o & , \quad z = 0 \\ \lambda_1 & , \quad z = 1 \end{cases} \tag{88}$$

Choosing $\lambda_o = \infty$ we have for $\lambda = \lambda_1$,

$$D(x,\hat{x};\lambda) = \rho\frac{1}{1-\lambda^2} e^{-\frac{\lambda}{1+\lambda}\rho(\frac{E_c}{N_o})} \tag{89}$$

for

$$0 \leq \rho \leq 1$$

$$0 \leq \lambda \leq 1$$

This is a symmetric case where

$$D(x,\hat{x};\lambda) = \begin{cases} 1 & , \quad \hat{x} = x \\ \frac{\rho}{1-\lambda^2} e^{-\frac{\lambda}{1+\lambda}\rho(\frac{E_c}{N_o})} & , \quad \hat{x} \neq x \end{cases} \tag{90}$$

so that

$$D(\lambda) = \frac{\rho}{1-\lambda^2} e^{-\frac{\lambda}{1+\lambda}\rho(\frac{E_c}{N_o})} \tag{91}$$

For the BFSK case (M = 2) where we have no coding and $E_c = E_b$, the true bit error probability and the bound based on $D(\lambda)$ is as follows for the $\rho = 1$ broadband jamming case:

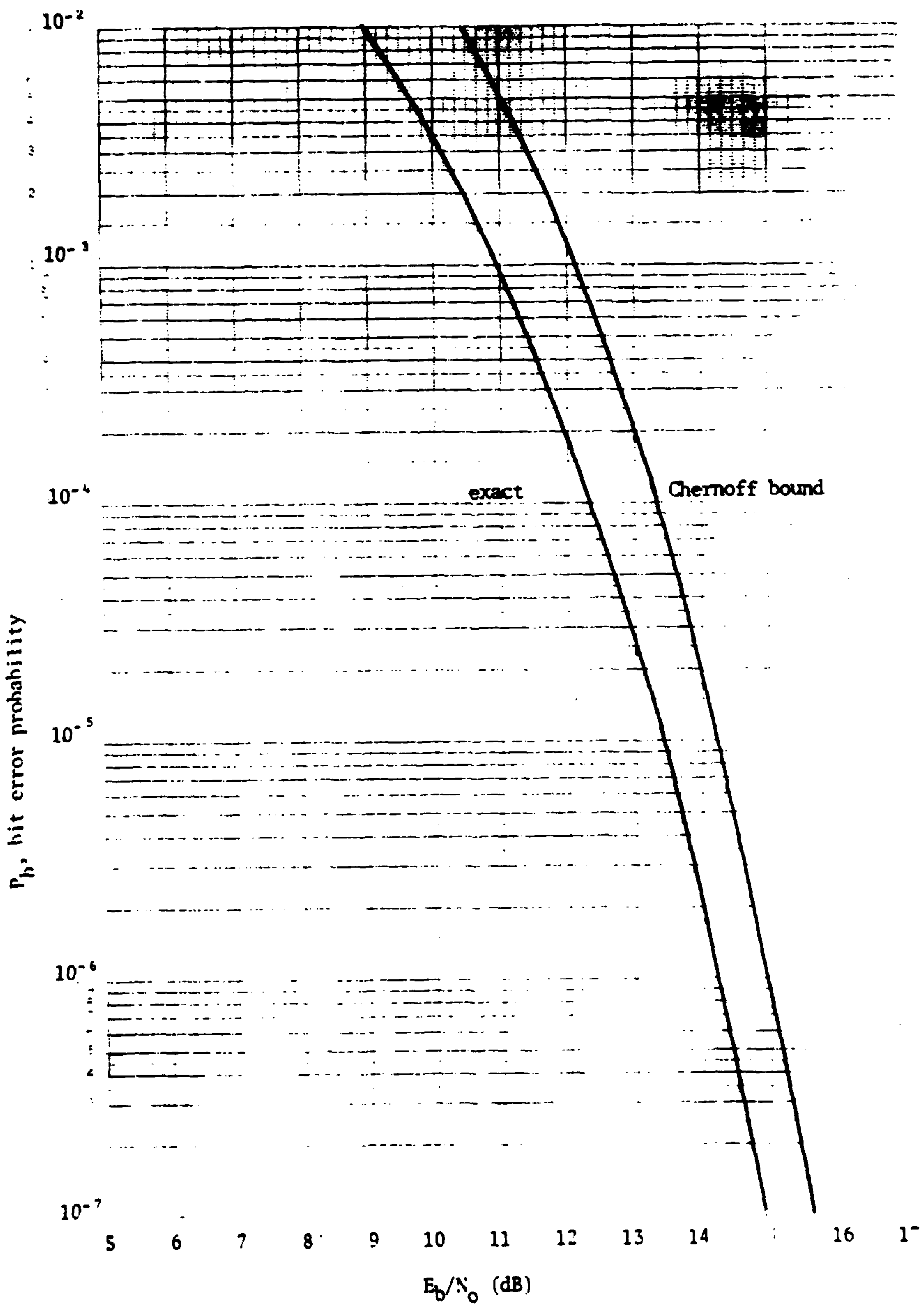
$$\begin{aligned} P_b &= \frac{1}{2} e^{-\frac{E_b}{2N_o}} \\ &\leq \frac{1}{2} D(\lambda) \\ &= \frac{1}{2} \left(\frac{1}{1-\lambda^2}\right) e^{-\frac{\lambda}{1+\lambda}(\frac{E_b}{N_o})} \end{aligned} \tag{92}$$

This exact bit error probability and the bound minimized with $0 \leq \lambda \leq 1$ are shown in Figure 9. There is roughly a one dB difference.

In general for the worst case ρ and minimixing λ given by

$$\rho = \begin{cases} 1 & ; \quad \frac{E_c}{N_o} < 3 \\ \frac{3}{(\frac{E_c}{N_o})} & ; \quad \frac{E_c}{N_o} \geq 3 \end{cases} \tag{93}$$

Figure 9 Broadband Jammer FH/BFSK



and

$$\lambda = 1/2 \quad \text{if} \quad \frac{E_c}{N_o} \geq 3 \tag{94}$$

we have

$$D = \begin{cases} \min\limits_{0\leq\lambda\leq 1} \dfrac{1}{1-\lambda^2}\, e^{-\frac{\lambda}{1+\lambda}\left(\frac{E_c}{N_o}\right)} & ; \dfrac{E_c}{N_o} < 3 \\ \dfrac{4e^{-1}}{\left(\frac{E_c}{N_o}\right)} & ; \dfrac{E_c}{N_o} \geq 3 \end{cases} \tag{95}$$

B. Hard Decision and Known Jammer State

The hard decision channel has output alphabet $Y = X = \{1,2,3,\ldots,M\}$

where

$$p(y|x,z) = \begin{cases} 1 & , y=x, z=0 \\ 0 & , y\neq x, z=0 \\ 1-\epsilon & , y=x, z=1 \\ \dfrac{\epsilon}{M-1} & , y\neq x, z=1 \end{cases} \tag{96}$$

where

$$\epsilon = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{k+1}\, e^{-\frac{k}{k+1}\rho\left(\frac{E_c}{N_o}\right)} \tag{97}$$

The maximum likelihood metric has the form

$$m(y,x|z) = -\lambda(z)\, w(y,x) \tag{98}$$

where

$$\lambda(z) = \begin{cases} \lambda_o & , z = 0 \\ \lambda_1 & , z = 1 \end{cases} \tag{99}$$

and we take the limit

$$\lambda_o = \infty \tag{100}$$

This results in the Bhattacharyya bound for $\hat{x} \neq x$,

$$D = E\left\{ \sum_y \sqrt{p(y|x,z)\, p(y|\hat{x},z)} \right\}$$

$$= \rho \sum_y \sqrt{p(y|x,1)\, p(y|\hat{x},1)}$$

$$= \rho\left[2\sqrt{\frac{\varepsilon(1-\varepsilon)}{M-1}} + (M-2)\frac{\varepsilon}{M-1}\right] \tag{101}$$

where ε depends on ρ and E_c/N_o as shown in (97).

C. Soft Decision and Unknown Jammer State

For the soft decision channel where the receiver has no knowledge of the jammer state the metric is simply

$$m(y,x) = y^{(x)}, \tag{102}$$

the x^{th} energy detector output $y^{(x)}$. When $\hat{x} \neq x$ we have

$$D(\lambda) = E\left\{ e^{\frac{\rho}{N_o}\lambda[y^{(\hat{x})} - y^{(x)}]} \Big| x \right\}$$

$$= (1-\rho)\, e^{-\lambda\rho\left(\frac{E_c}{N_o}\right)} + \rho\frac{1}{1-\lambda^2}\, e^{-\frac{\lambda}{1+\lambda}\rho\left(\frac{E_c}{N_o}\right)} \tag{103}$$

where

$$0 \leq \lambda \leq 1\ ,\ 0 \leq \rho \leq 1.$$

Note that from the lower bound

$$D(\lambda) \geq (1-\rho)\, e^{-\lambda\rho\left(\frac{E_c}{N_o}\right)} \tag{104}$$

by choosing ρ arbitrarily small regardless of λ we have

$$\max_{0\leq\rho\leq 1} D(\lambda) \geq 1. \tag{105}$$

The bound using $D(\lambda)$ is thus useless and we would expect that in general against the worst case partial band jammer, this soft decision receiver with no jammer state knowledge results in very poor performance.

D. Hard Decision and Unknown Jammer State

The hard decision M-ary channel has input and output alphabets

$$X = Y = \{1,2,3,\ldots,M\} \tag{106}$$

and conditional probability

$$p(y|x) = \begin{cases} 1-\rho\varepsilon & , \quad y=x \\ \dfrac{\rho\varepsilon}{M-1} & , \quad y\neq x \end{cases} \tag{107}$$

where

$$\varepsilon = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{k+1} e^{-\frac{k}{k+1}\rho\left(\frac{E_c}{N_o}\right)} \tag{108}$$

Without any jammer state knowledge the receiver uses the metric

$$M(y,x) = -w(y,x) \tag{109}$$

This is a maximum likelihood metric based on the unknown jammer state conditional probability. Here we have

$$D(x,\hat{x}) = \sum_y \sqrt{p(y|x)\, p(y|\hat{x})}$$
$$= \begin{cases} 1 & , \quad \hat{x} = x \\ D & , \quad \hat{x} \neq x \end{cases} \tag{110}$$

Where

$$D = 2\sqrt{\frac{\rho\varepsilon(1-\rho\varepsilon)}{M-1}} + (m-2)\frac{\rho\varepsilon}{M-1} \tag{111}$$

which is similar to the hard decision example with known jammer state differing by roughly $\sqrt{\rho}$.

E. R_o Evaluation

The cutoff rate for each coded bit in the worst case pulse jamming environment is given by

$$R_o = \log_2 M - \log_2 [1+(M-1)D] \tag{112}$$

where D is a function of the equivalent MFSK chip energy-to-noise ratio E_c/N_o where

$$E_c = ST_c \tag{113}$$

where T_c is the MFSK chip duration and

$$N_o = \frac{J}{W} \tag{114}$$

Here D is given by

(1) soft decision and known jammer state

$$D_1 = \begin{cases} \min\limits_{0 \le \lambda \le 1} \dfrac{1}{1-\lambda^2} e^{-\frac{\lambda}{1+\lambda}\left(\frac{E_c}{N_o}\right)} & ; \quad \dfrac{E_c}{N_o} < 3 \\ \dfrac{4e^{-1}}{\left(\frac{E_c}{N_o}\right)} & ; \quad \dfrac{E_c}{N_o} \ge 3 \end{cases} \tag{115}$$

(2) hard decision and known jammer state

$$D_2 = \max_{0 \le \rho \le 1} \left\{ 2\rho\sqrt{\frac{\varepsilon(1-\varepsilon)}{M-1}} + (M-2)\ \frac{\rho\varepsilon}{M-1} \right\} \tag{116}$$

where

$$\varepsilon = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1}\ \frac{1}{k+1}\ e^{-\frac{k}{k+1}\rho\left(\frac{E_c}{N_o}\right)} \tag{117}$$

(3) soft decision and unknown jammer state

$$D_3 = 1 \tag{118}$$

(4) hard decision and unknown jammer state

$$D_4 = \max_{0 \le \rho \le 1} \left\{ 2\sqrt{\frac{\rho\varepsilon(1-\rho\varepsilon)}{M-1}} + (m-2)\ \frac{\rho\varepsilon}{M-1} \right\} \tag{119}$$

where ε is the same as in the hard decision case with known jammer state.

These four examples are shown in Figure 10 for R_0 and Figure 11 for the maximizing value of ρ. For the special case of $M=2$ we can use binary convolutional codes characterized by a function $B(\cdot)$ in

$$P_b \le \frac{1}{2}\ B(R_0). \tag{120}$$

The $K = 7$, $r = 1/2$ convolutional code example was given by (43) and (44). Figure 12 also shows this code bit error bound for $\rho=1$ and the soft decision receiver.

F. Diversity and Coding

In the previous analysis the parameters D and R_o related by

$$R_o = \log_2 M - \log_2[1+(M-1)D] \tag{121}$$

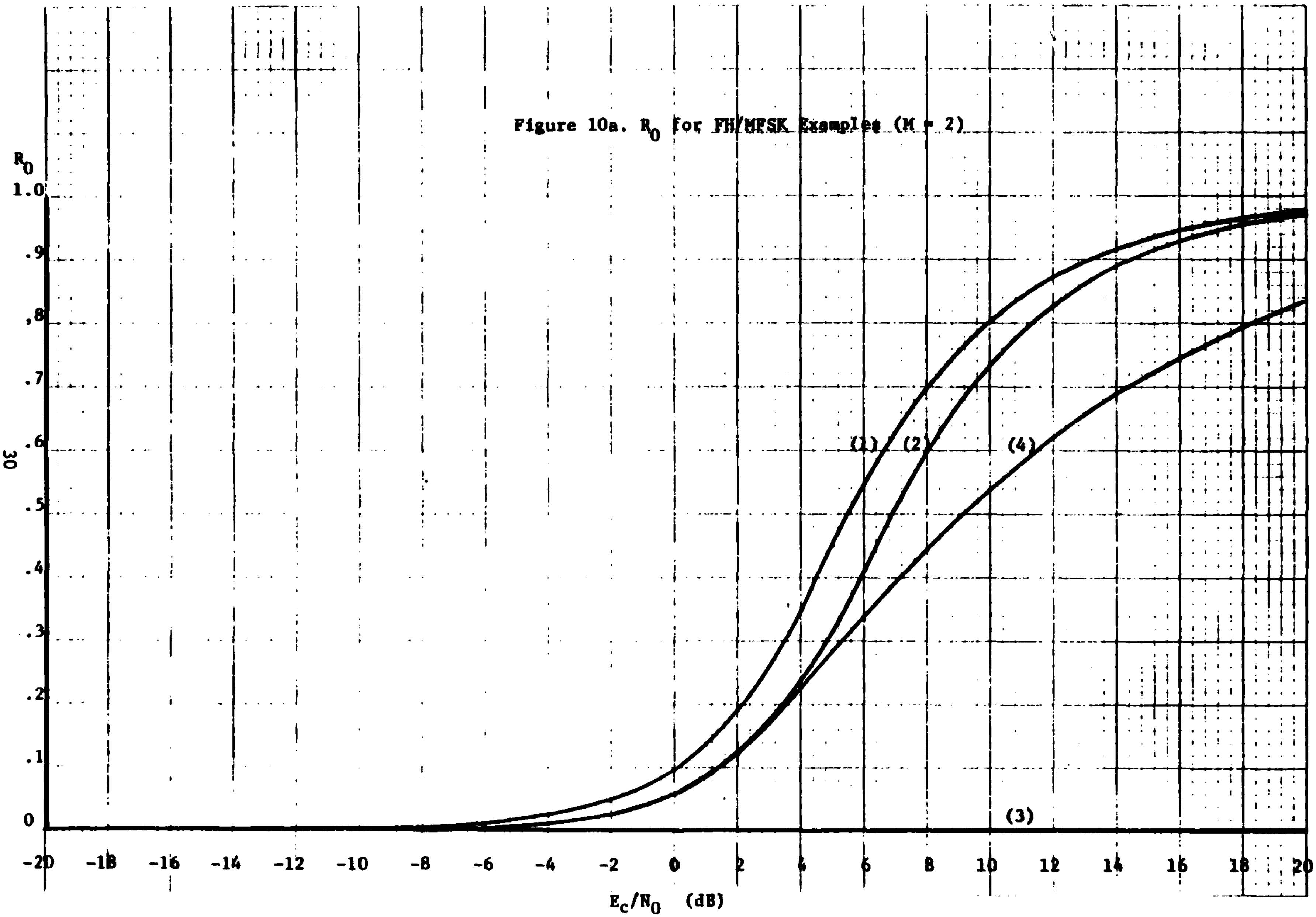
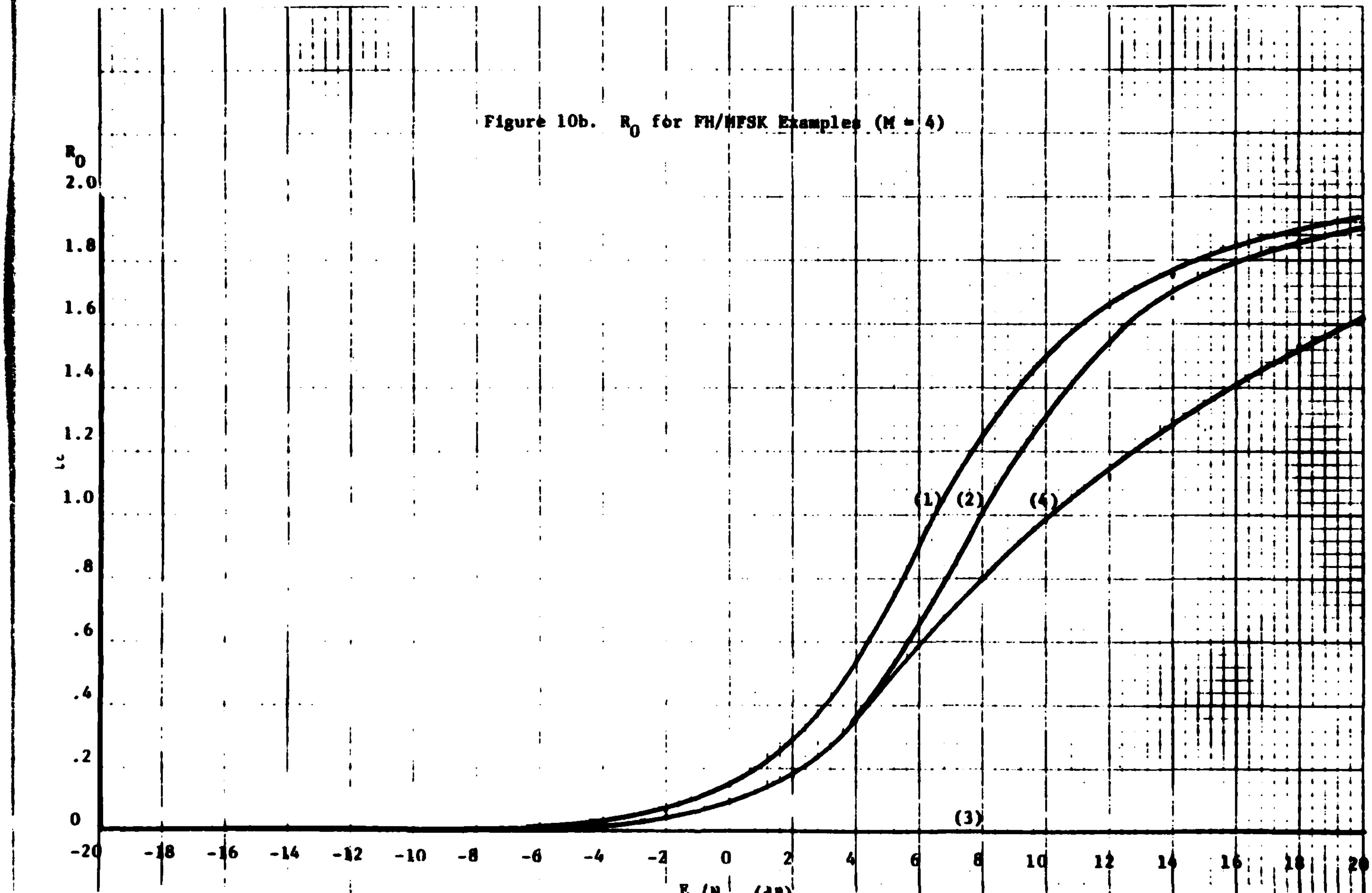


Figure 10a. R_0 for FH/MFSK Examples ($M = 2$)

Figure 10b. R_0 for FH/MFSK Examples (M = 4)



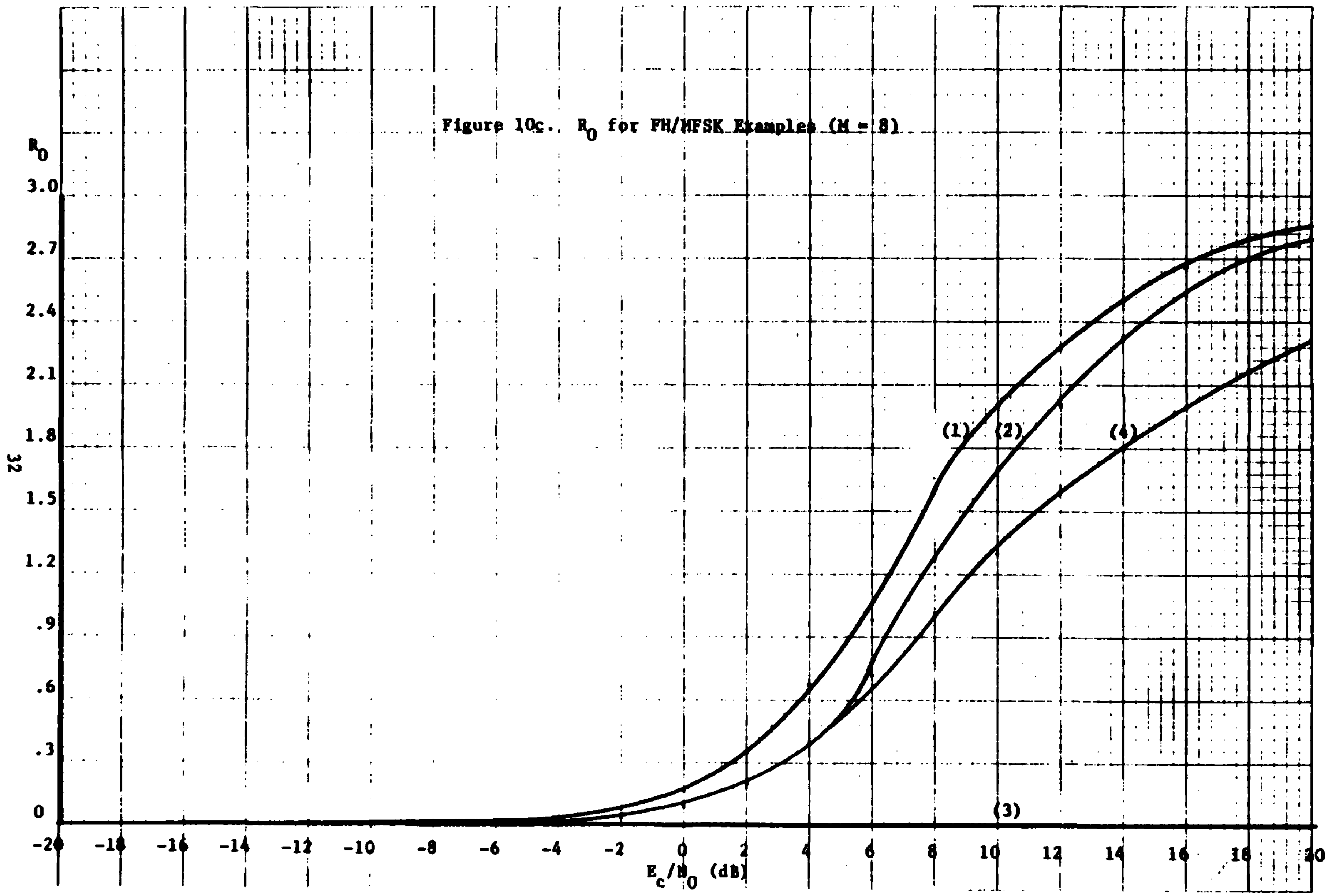
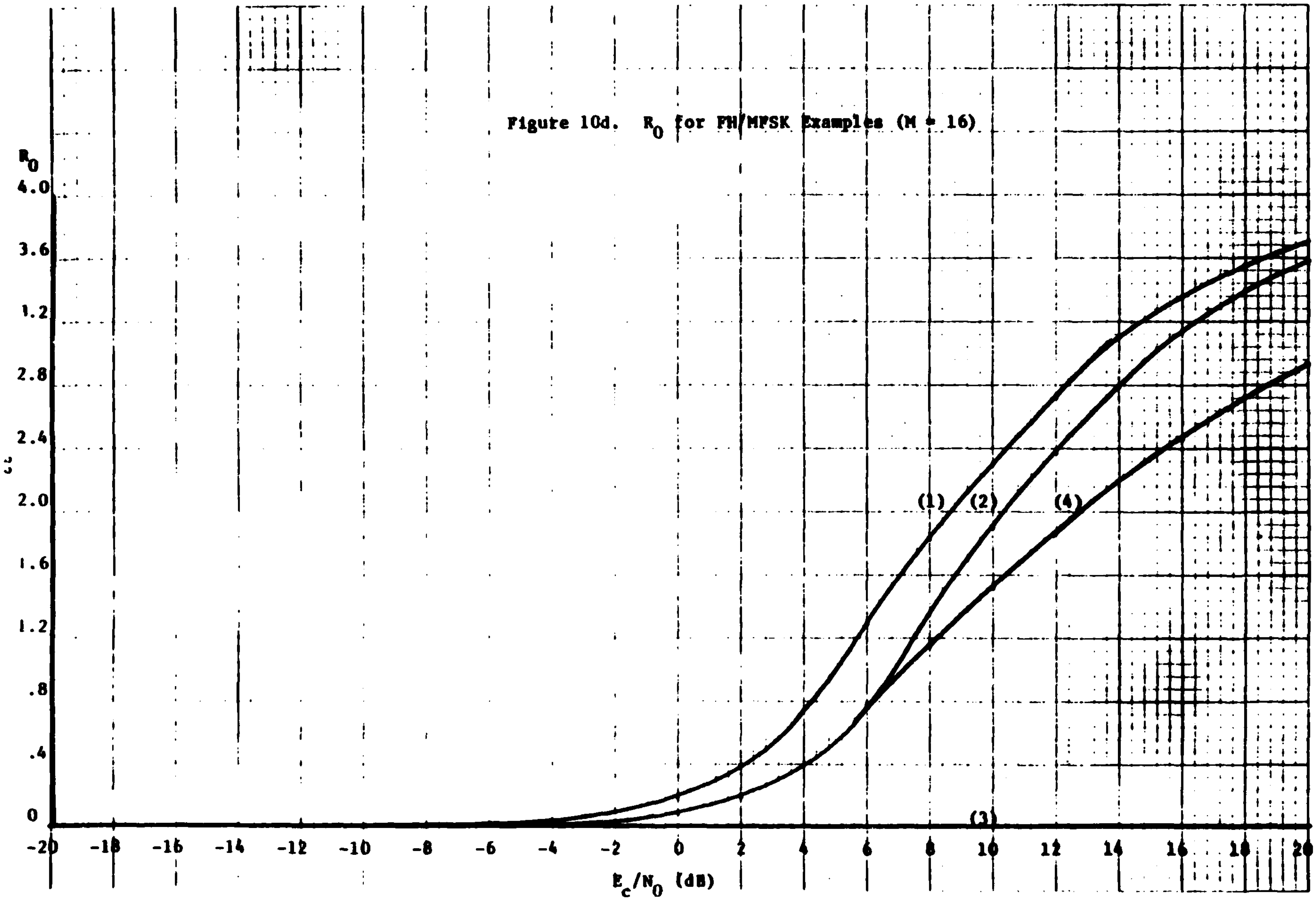


Figure 10c. R_0 for FH/MFSK Examples (M = 8)
R_0
3.0
2.7
2.4
2.1
1.8
1.5
1.2
.9
.6
.3
0
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
20
E_c/N_0 (dB)
(1)
(2)
(3)
(4)

Figure 10d. R_0 for FH/MFSK Examples ($M = 16$)



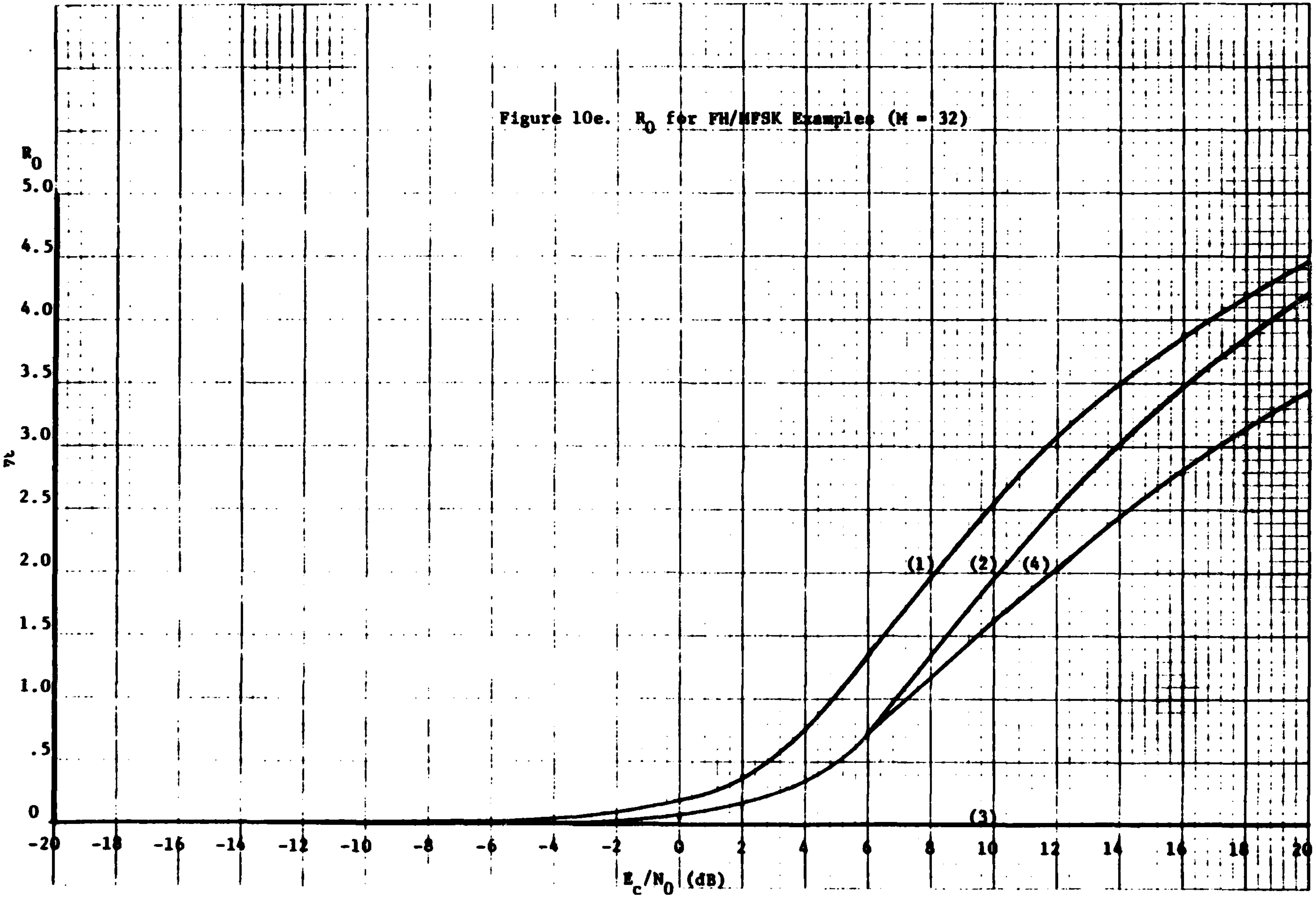


Figure 10e. R_0 for FH/MFSK Examples (M = 32)

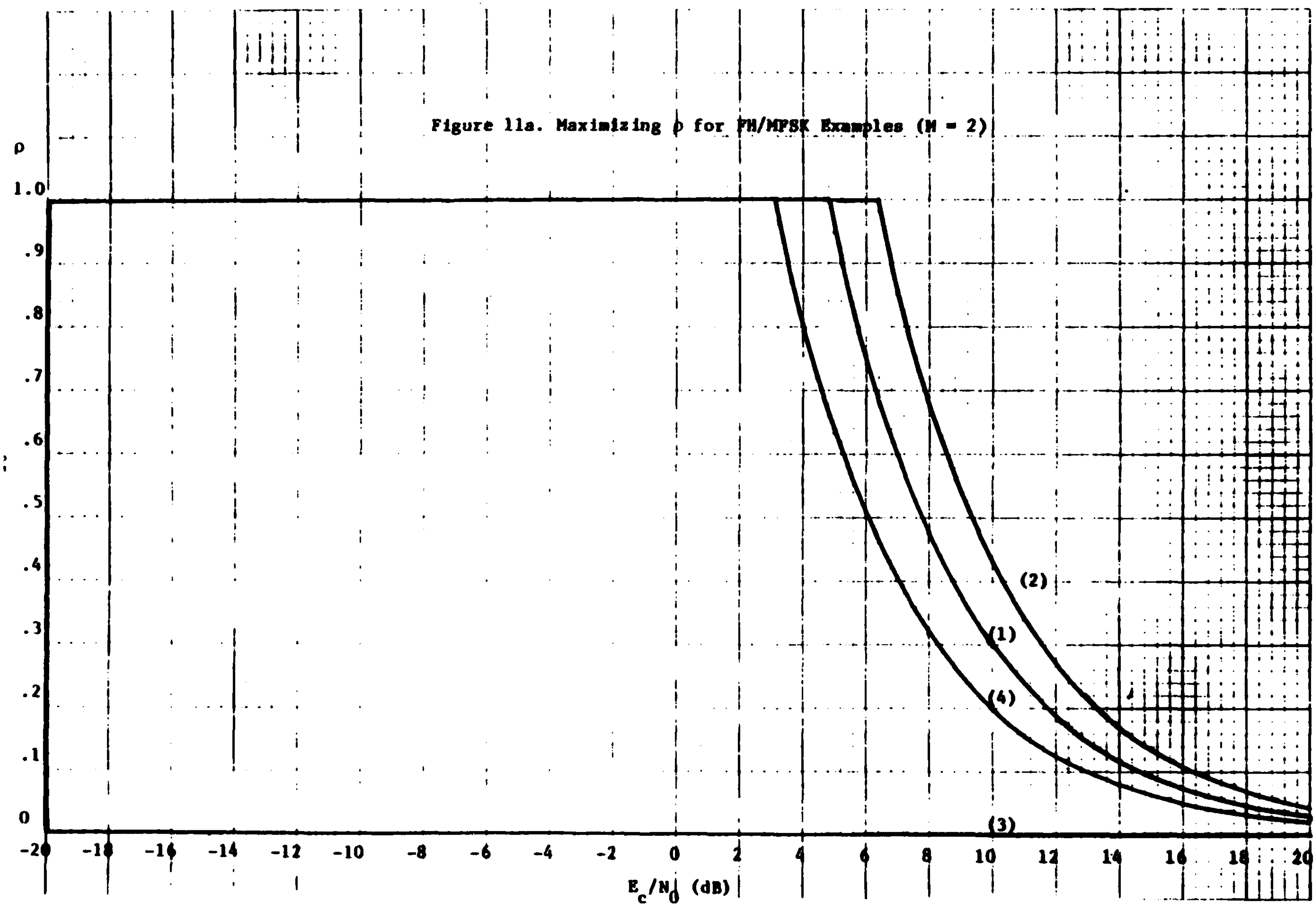


Figure 11a. Maximizing ρ for FH/MPSK Examples (M = 2)

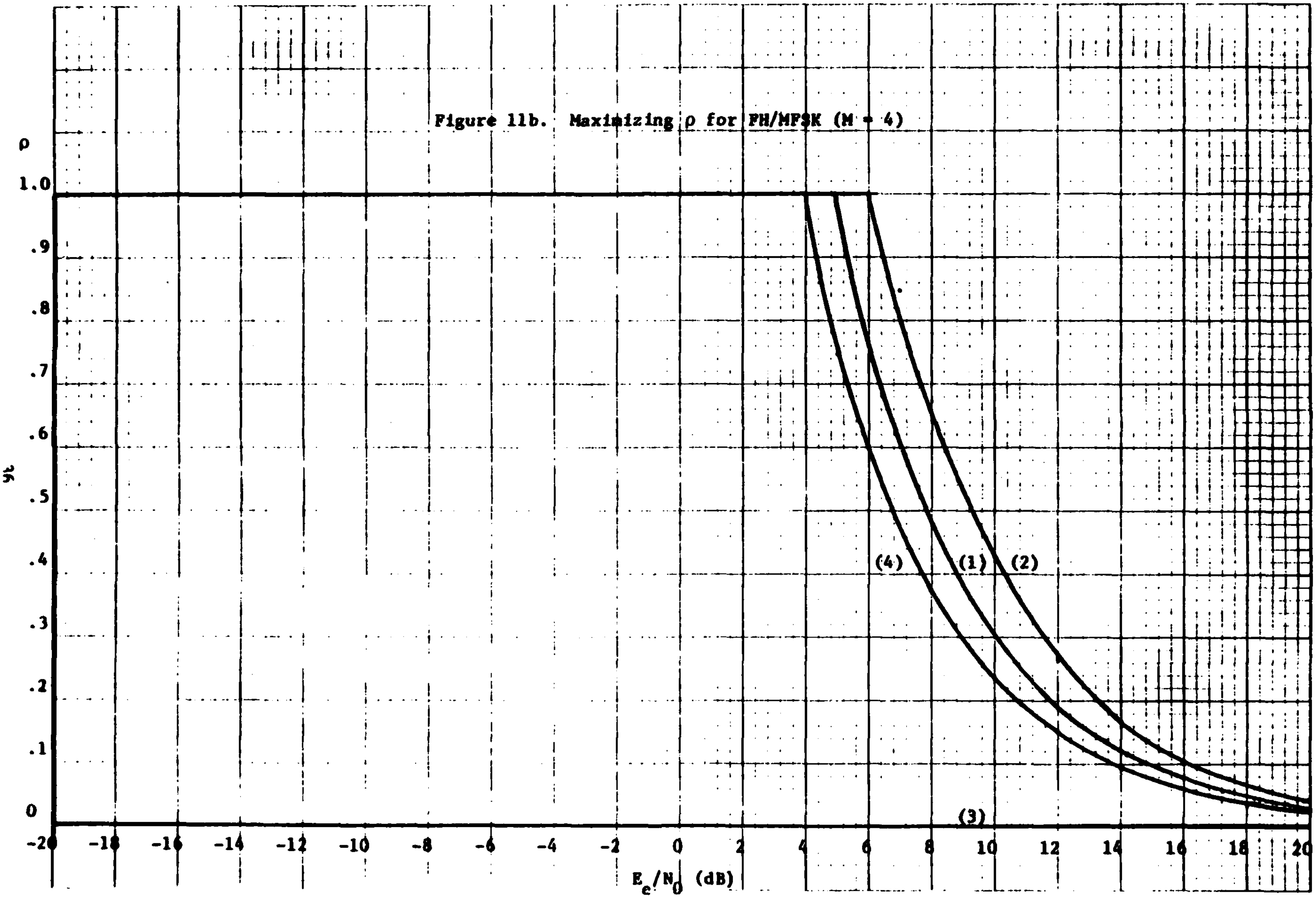


Figure 11b. Maximizing ρ for FH/MFSK ($M = 4$)

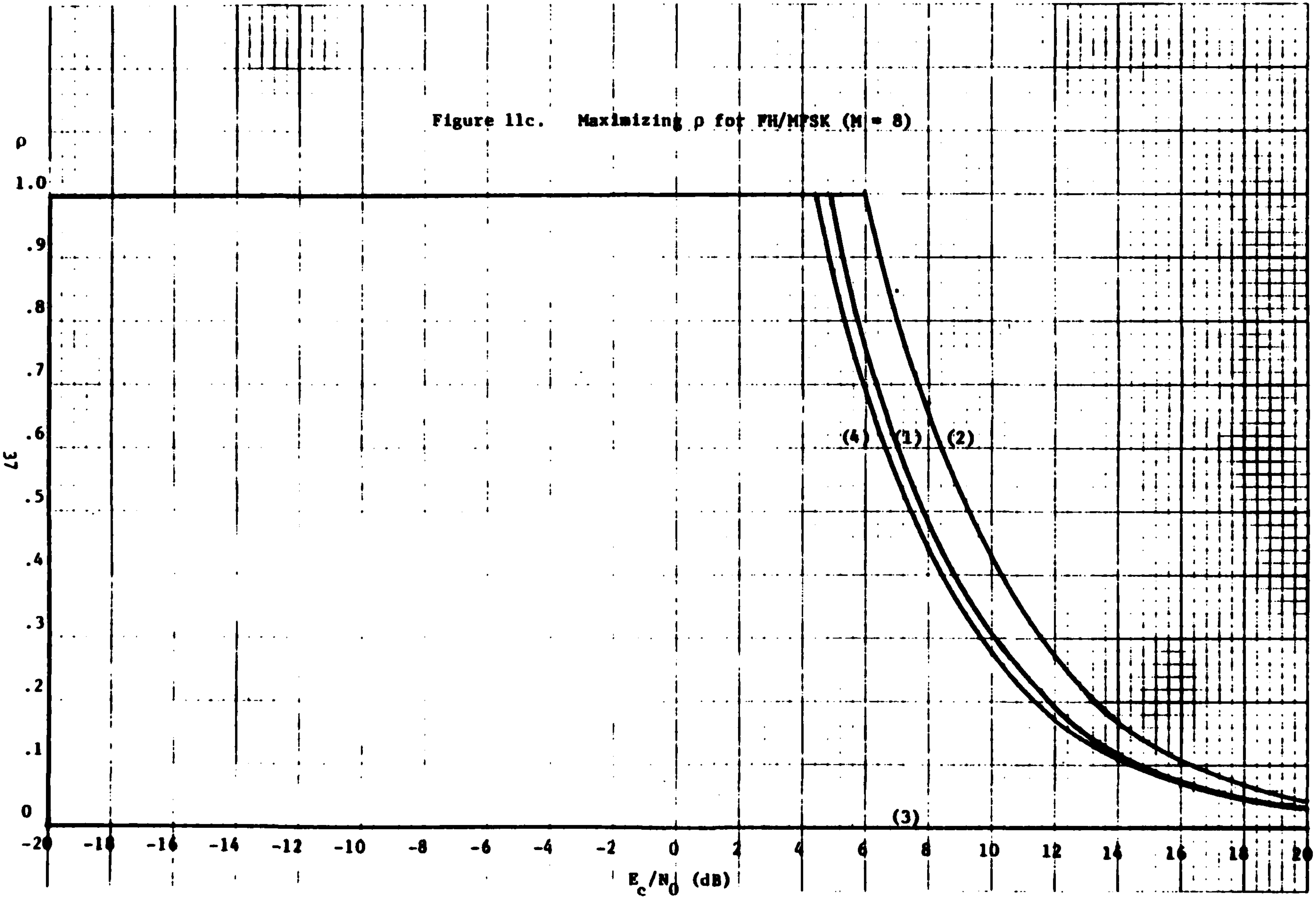


Figure 11c. Maximizing ρ for FH/MPSK (M = 8)

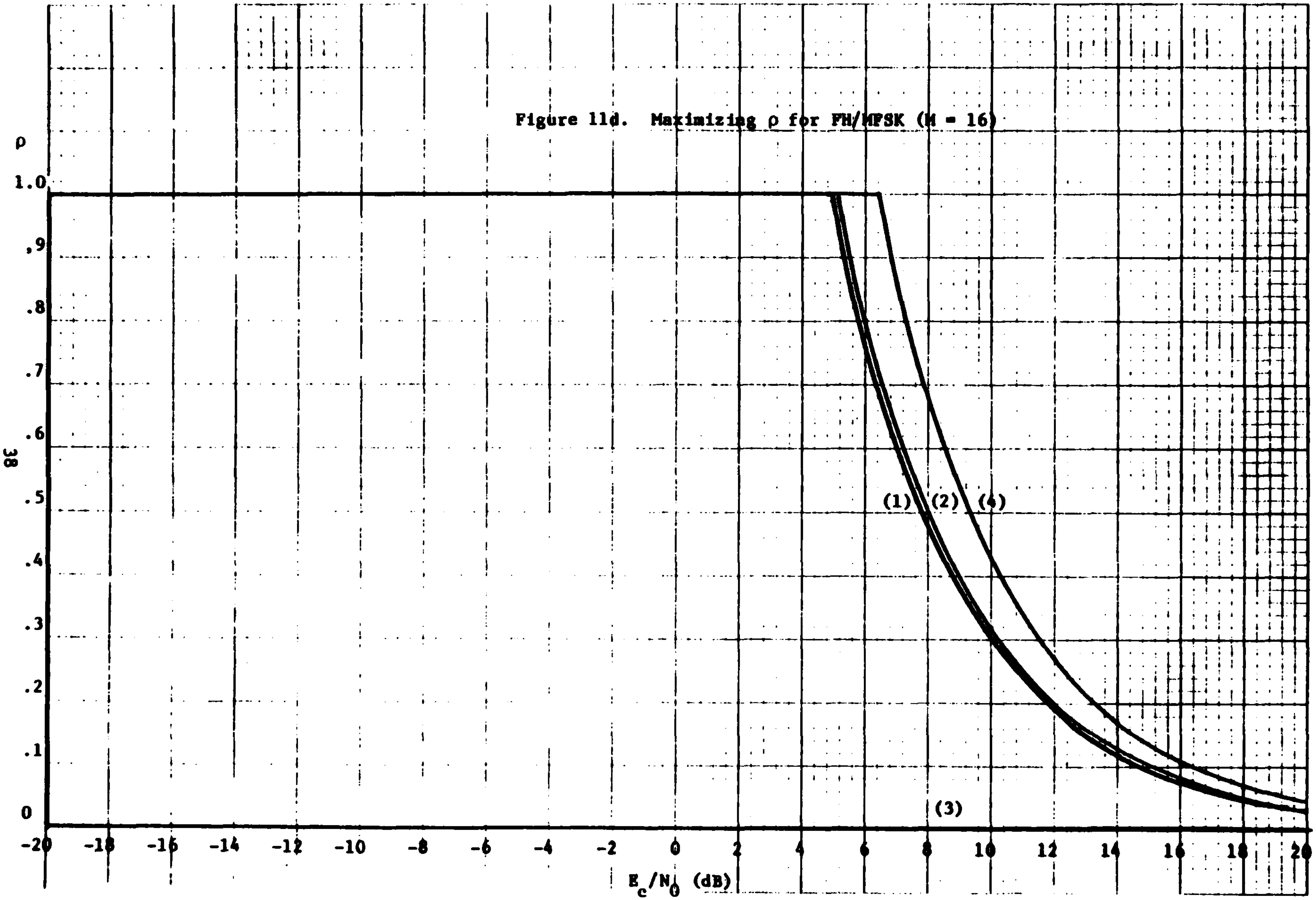
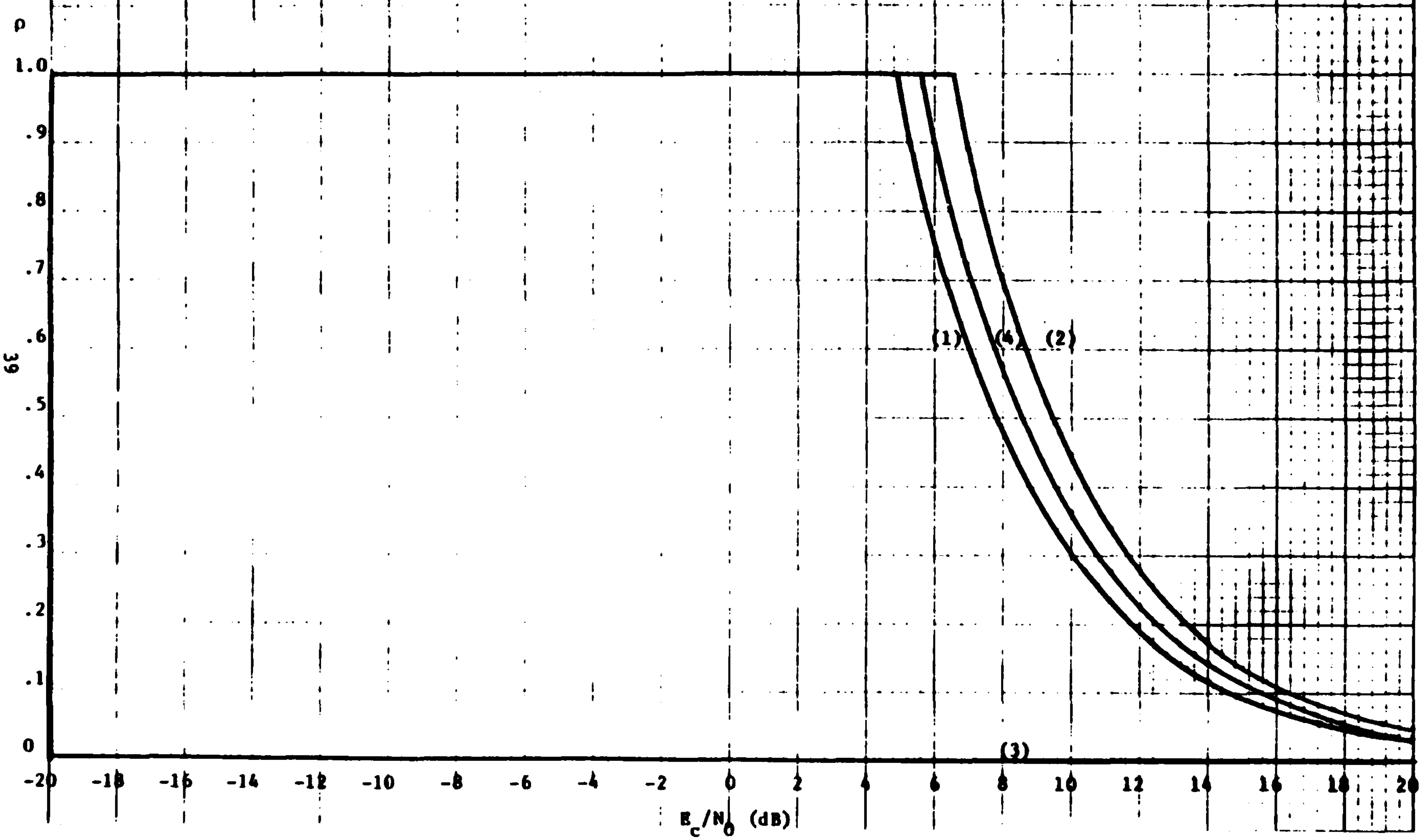


Figure 11d. Maximizing ρ for FH/MFSK (M = 16)

Figure 11e: Maximizing ρ for FH/MFSK ($M = 32$)



and

$$D = \frac{M2^{-R_o} - 1}{M - 1} \tag{122}$$

are given in terms of E_c/N_o, the energy per MFSK chip - to - noise ratio. To emphasize this we write

$$D = D(\frac{E_c}{N_o}) \tag{123}$$

and

$$R_o = R_o(\frac{E_c}{N_o}) \tag{124}$$

Conventional MFSK modulation has symbol error probability bound

$$P_s \leq \frac{1}{2} \quad (M-1)D(\frac{E_c}{N_o}) \tag{125}$$

and bit error bound

$$\begin{aligned} P_b &= \frac{\frac{1}{2} M}{M-1} \; P_s \\ &\leq \frac{1}{4} \; MD(\frac{E_c}{N_o}) \end{aligned} \tag{126}$$

Since here for $K = \log_2 M$ we have

$$E_c = KE_b \tag{127}$$

we can express this as

$$P_b \leq 2^{K-2} D(K\frac{E_b}{N_o}) \tag{128}$$

For the special case of boradband noise jamming ($\rho = 1$) with a soft decision receiver having jammer state knowledge ,

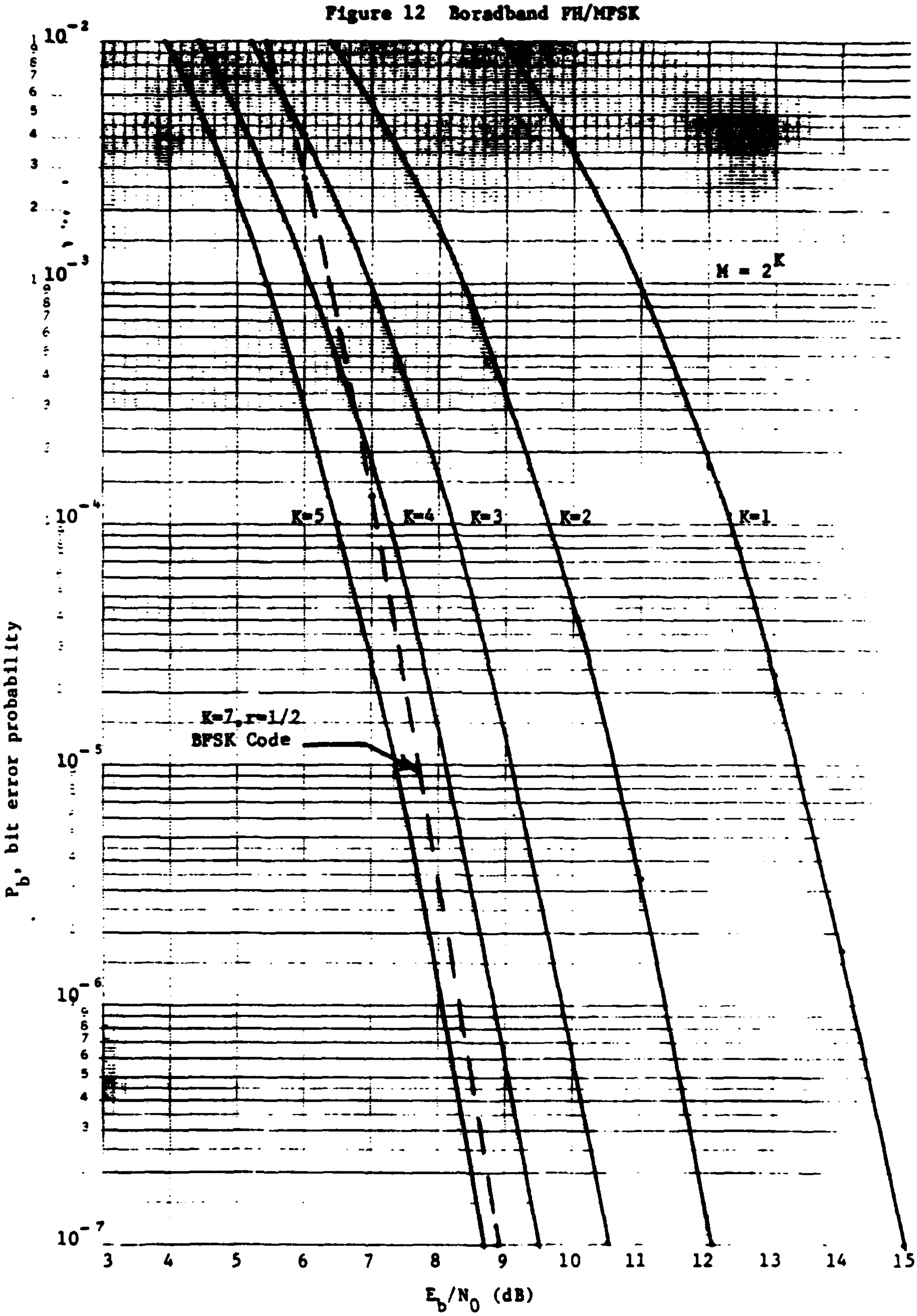
$$D = \min_{0 \leq \lambda \leq 1} \frac{1}{1-\lambda^2} \; e^{-\frac{\lambda}{1+\lambda}\left(\frac{E_c}{N_o}\right)} \tag{129}$$

whereas the exact bit error probability expression is

$$P_b = \frac{2^{k-1}}{2^k - 1} \sum_{\ell=1}^{2^k - 1} \binom{2^k - 1}{\ell} (-1)^{\ell+1} \frac{1}{\ell+1} e^{-\frac{\ell}{\ell+1} K(\frac{E_b}{N_o})} \tag{130}$$

The difference between these exact bit error probabilities shown in Figure 12 and the bounds given by (128) is exactly the same as the binary case (K=1) shown in Figure 9.

Figure 12 Boradband FH/MPSK



MFSK modulation can be regarded as a form of coding. Certainly with diversity it is also a form of coding. For m diversity MFSK modulation we have symbol error bound

$$P_s \leq \frac{1}{2} (M-1) D(\frac{E_c}{N_o})^m \tag{131}$$

and bit error bounding using $K = \log_2 M$ and

$$E_c = \frac{K}{m} E_b, \tag{132}$$

given by

$$P_b \leq 2^{K-2} D((\frac{K}{m})(\frac{E_b}{N_o}))^m \tag{133}$$

For soft decision with known jammer state and no diversity we have the results shown in Figure 13. This is compared with the optimum diversity in Figure 14.

Next we can consider more general codes that use M-ary alphabets. Conventional MFSK with m diversity is merely a block code with M sequences of blocklength m.

(1) Orthogonal Convolutional Codes

An orthogonal convolutional of constraint length K with m diversity generates m $M=2^K \sim$ary symbols per bit and has the bit error bound

$$P_b \leq \frac{D(\frac{1}{m}(\frac{E_b}{N_o}))^{mK}}{2[1-2D(\frac{1}{m}(\frac{E_b}{N_o}))^m]^2} \tag{134}$$

(2) Dual - K Rate $\frac{1}{m}$ codes

A dual - K code has two K bit registers which is a total constraint length of 2K bits or 2 $2^K \sim$ary data symbol. Here M consists of m_o distinct outputs at time n of the form

$$x_n^{(\ell)} = i_n \oplus g_\ell i_{n-1} \ ; \ \ell = 1,2,\ldots,m_o \tag{135}$$

where i_n and i_{n-1} are $2^K \sim$ary data symbols belonging to the Galois field $GF(2^K)$ consisting of $\{0,1,2,\ldots,2^K-1\}$ and g_ℓ are distinct nonzero elements in this field. Thus $m_o < 2^K$. Letting

$$\beta_\ell = \text{\# of repetitions of the } \ell^{\underline{th}} \text{ symbol} \tag{136}$$
$$\ell = 1, 2, \ldots, m_o$$

we have

Figure 13. No Diversity FH/MFSK

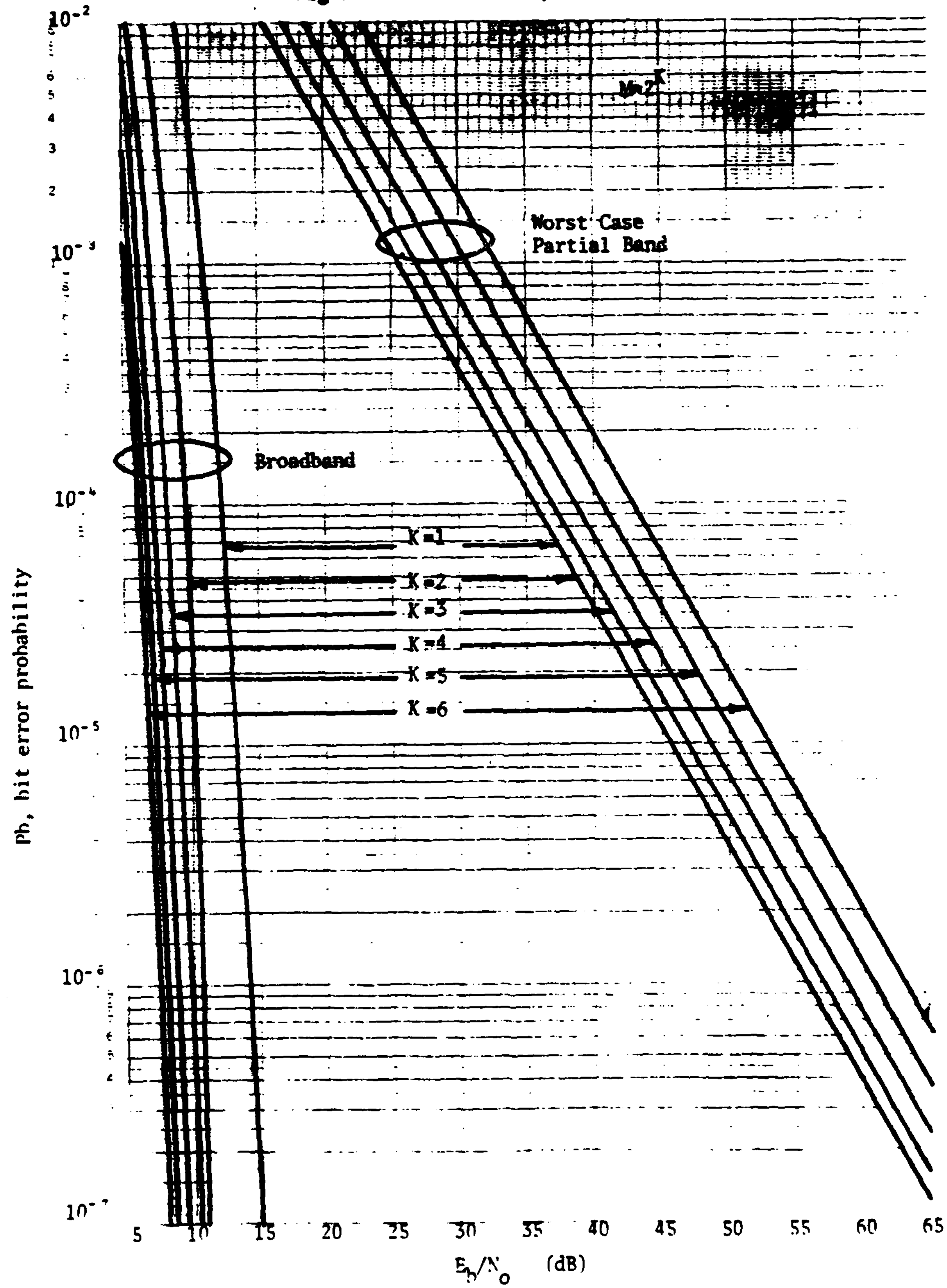
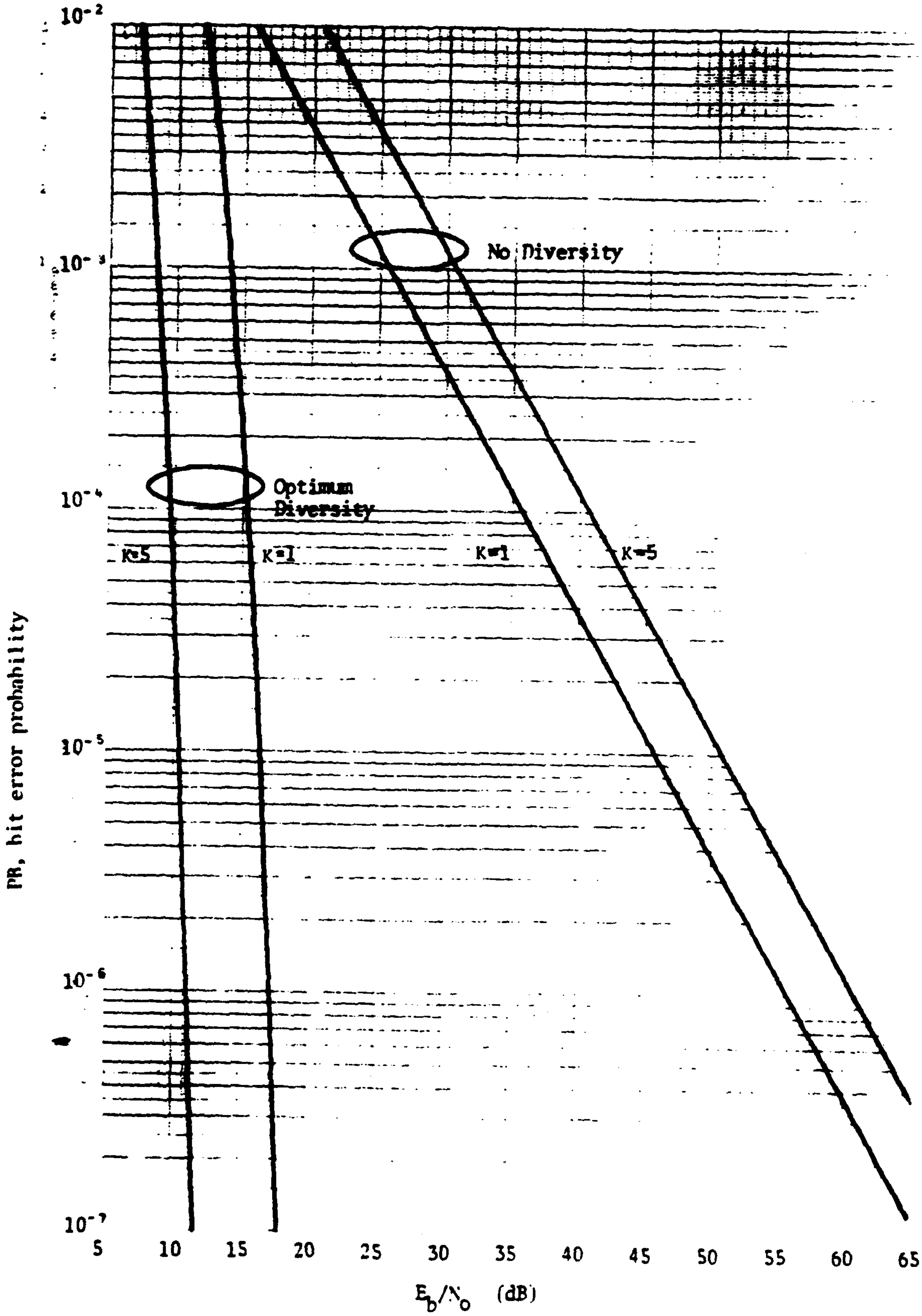


Figure 14 Optimum Diversity FH/MFSK



$$m = \beta_1 + \beta_2 + \cdots \beta_{m_o}. \tag{137}$$

The bit error bound is given by

$$P_b \leq \frac{2^{K-2}\, D^{2m}}{[1-D^{m-\beta_1}-D^{m-\beta_2}- \ldots -D^{m-\beta_{m_o}}-(2^K-1-m_o)D^m]^2} \tag{138}$$

where

$$D = D\left(\left(\frac{K}{m}\right)\left(\frac{E_b}{N_o}\right)\right) \tag{139}$$

since

$$E_c = \left(\frac{K}{m}\right) E_b \tag{140}$$

VII CONCLUSIONS

For a general class of M-ary channels given by Figures 1,2,and 3 we derive a parameter

$$D = \min_{\lambda \geq 0} E\{e^{\lambda[m(y,\hat{x};z)-m(y,x;z)]};x\} \tag{141}$$

for the worst case jammer. This includes arbitrary decision metric $m(y,x;z)$ which may or may not depend on jammer state information characterized by the random variable z. The cutoff rate is given by

$$R_o = \log_2 M - \log_2[1+(M-1)D] \tag{142}$$

in bits per channel M-ary symbol and thus

$$D = \frac{M2^{-R_o}-1}{M-1} \tag{143}$$

Both R_o and D are expressed in terms of E_c/N_o, the equivalent channel symbol energy - to - noise ratio. If a code of rate r bits per channel M-ary symbol is used then

$$E_c = rE_b \tag{144}$$

giving a bit error bound of the form

$$\begin{aligned} P_b &\leq \frac{1}{2}\ G\ (D) \\ &= \frac{1}{2}\ B\ (R_o) \end{aligned} \tag{145}$$

The use of D and R_o allows us to separate the channel characteristics given by D or R_o and the code characteristics given by the function G(D) or $B(R_o)$. Hence for a given E_c/N_o we can choose the type of system that yeilds a good value of R_o and then consider coding for this channel separately.

VIII REFERENCES

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Note No. 5

FADING DISPERSIVE CHANNELS

to

NAVAL RESEARCH LABORATORY

(Contract Award No. N00014-80-K-0935)

for

HF COMMUNICATION NETWORK SIGNALS

USING CHANNEL EVALUATION DATA

Principal Investigator

Jim K. Omura
Professor
System Science Department
University of California
Los Angeles, California

April, 1981

FADING DISPERSIVE CHANNELS

This is a tutorial note on fading dispersive channels, partly based on Kennedy [1]. Our goal is to derive as simply as possible some of the basic parameters associated with such channels and examine their relationships to basic signal parameters. We then evaluate the use of diversity and interleaving for two digital communication signals.

I. Introduction

Assume a narrow band signal

$$x(t) = A(t) \cos [w_o t + \psi(t)]$$

is transmitted through a fading dispersive channel as sketched in Figure 1. Basic parameters of this signal include signal duration,

T seconds,

and signal bandwidth

W Hz

We can approximate such a signal with the Fourier series expansion

$$x(t) = \sum_k \{a_k \cos w_k t + b_k \sin w_k t\} .$$

Since a fading dispersive channel is linear we can now consider the impact of the channel on each component.

A. $i^{\underline{th}}$ Scatterer on Cos wt.

Consider the signal at the receiver due to a single scatterer when the transmitted component is cos wt. This is shown in Figure 2. The $i^{\underline{th}}$ scatterer has parameters r_i, w_i, ρ_i and θ_i defined as follows in terms of the scatterer's impact on cos wt:

---

[1] R. S. Kennedy, Fading Dispersive Communication Channels, Wiley-Interscience, 1969.

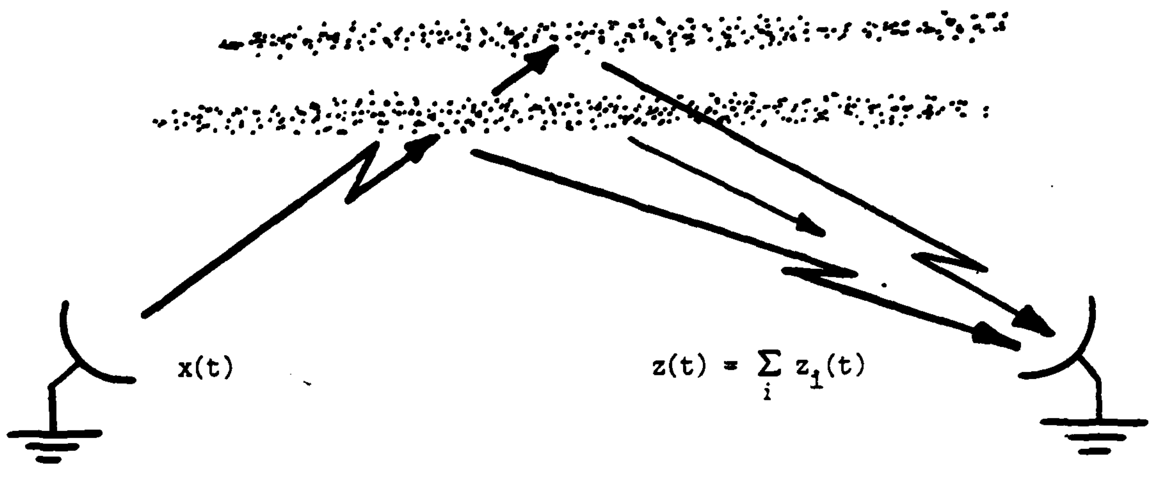


Figure 1. Fading Dispersive Channel

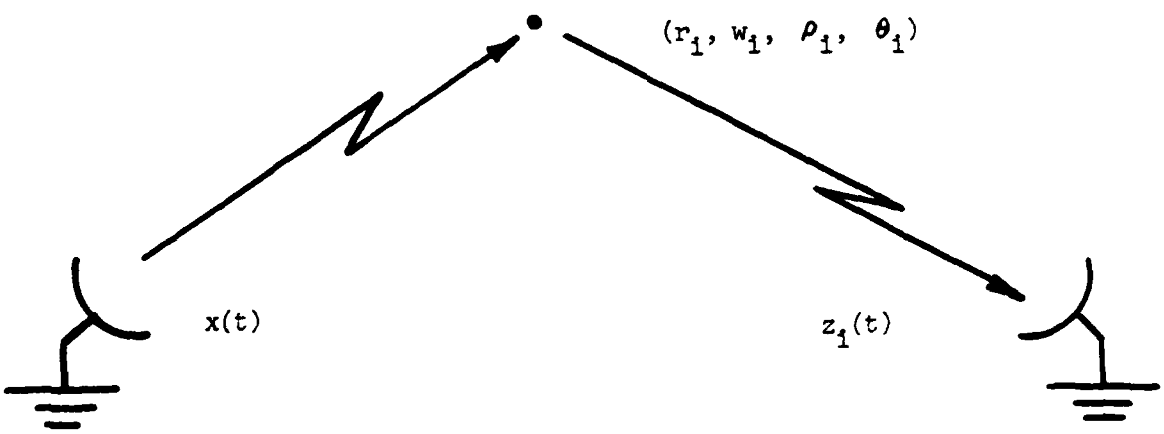


Figure 2. Signal of i^{th} Scatterer

$$
\begin{array}{l}
\cos wt \\
\quad \Big\downarrow \ \text{time delay by } r_i \\
\cos[w(t-r_i)] \\
\quad \Big\downarrow \ \text{doppler frequency shift by } w_i \\
\cos[(w-w_i)(t-r_i)] \\
\quad \Big\downarrow \ \text{amplitude } \rho_i \\
\rho_i \cos[(w-w_i)(t-r_i)] \\
\quad \Big\downarrow \ \text{phase } \Theta_i \ (\text{include } w_i r_i) \\
\rho_i \cos[(w-w_i)t-wr_i+\Theta_i]
\end{array}
$$

Thus the response to cos wt due to the $i^{\underline{th}}$ scatterer is

$$z_i(t;w) = \rho_i \cos[(w-w_i)t-wr_i+\Theta_i].$$

Typically ρ_i and Θ_i are random variables that are independent of each other and characterized as

$$\Theta_i \sim \text{uniform over } [0,2\pi]$$

and

$$\rho_i \sim \text{Rayleigh}$$

Also, these random variables are assumed independent for each scatterer.

B. Response to Cos wt.

The total signal at the receiver due to all scatterers when the signal is cos wt is thus

$$z(t;w) = \sum_i \rho_i \cos\,[(w-w_i)t-wr_i+\Theta_i]$$

where

$$E\,\{z(t;w)\} = 0$$

since for any fixed t,w,w_i, and r_i,

$$E\,\{\cos[(w-w_i)t-wr_i+\Theta_i]\} = 0$$

The autocorrelation of $z(t;w)$ is

$$R(t_1,t_2;w) = E\{z(t_1;w)z(t_2;w)\}$$
$$= \sum_i E\{\rho_i{}^2\}\ E\{\cos[(w-w_i)t_1-wr_i+\Theta]$$
$$\cdot\cos[(w-w_i)t_2 - wr_i+\Theta_i]\}$$
$$= \tfrac{1}{2}\sum_i E\{\rho_i{}^2\}\ \cos[(w-w_i)(t_1-t_2)]$$

Next define the index set for scatterers,

$$I(r,f) = \{i:r_i=r,w_i=2\pi f\}$$

and the scatter function

$$\sigma(r,f) = \sum_{i\epsilon I(r,f)} E\{\rho_i{}^2\}$$

The scatter function evaluated at r and f denotes the average energy contribution at the receiver due to scatterers at delay r having doppler shift f. With this definition we have

$$R(t_1,t_2:w) = \tfrac{1}{2}\iint\sigma(r,f)\ \cos[(w-2\pi f)(t_1-t_2)]drdf$$

C. Total Response

For the general narrowband signal

$$x(t) = A(t)\ \cos[w_o t + \psi(t)]$$

where $A(t)$ and $\psi(t)$ are "slowly varying" compared to the carrier $\cos w_o t$, the $i\underline{\text{th}}$ scatterer signal at the receiver is

$$\rho_i A(t-r_i)\ \cos[(w_o-w_i)t-w_o r_i + \psi(t-r_i) + \Theta_i]$$

resulting in the total signal due to all scatterers,

$$z(t) = \sum_i \rho_i A(t-r_i)\ \cos[(w_o-w_i)t-w_o r_i + \psi(t-r_i) + \Theta_i]$$

with autocorrelation

$$R(t_1,t_2) = E\{z(t_1)z(t_2)\}$$
$$= \sum_i \tfrac{1}{2}\ E\{\rho_i^2\}\ A(t_1-r_i)A(t_2-r_i)$$
$$\cdot\cos\ [(w_o-w_i)(t_1-t_2) + \psi(t_1-r_i)-\psi(t_2-r_i)]$$
$$= \tfrac{1}{2}\iint\sigma(r,f)A(t_1-r)A(t_2-r)$$
$$\cdot\cos\ [(w_o-2\pi f)(t_1-t_2) + \psi(t_1-r) - \psi(t_2-r)]drdf$$

II. Scatter Function $\sigma(r,f)$:

Two example sketches of the scatter function are shown in Figure 3. Figure 3(a) shows an F-layer scatter function at HF frequencies while Figure 3(b) shows a scatter function due to dipple needles used in Project West Ford.

We next consider two basic parameters of the scatter function $\sigma(r,f)$. Let

$$\sigma = \iint \sigma(r,f)drdf$$

be the total average energy due to all scatterers.

Delay Scatter Function

The delay scatter function is

$$\sigma(r) = \int \sigma(r,f)df.$$

This is the average energy due to scatterers at delay r. Associated with this function is the "multipath spread" parameter

$$L = \frac{\sigma^2}{\int \sigma^2(r)dr} \qquad \text{"Multipath Spread"}$$

Figure 4(b) shows a sketch of $\sigma(r)$ and the multipath spread parameter. Generally L is a measure of the range of delays due to scatterers.

Frequency Scatter Function

The frequency scatter function is

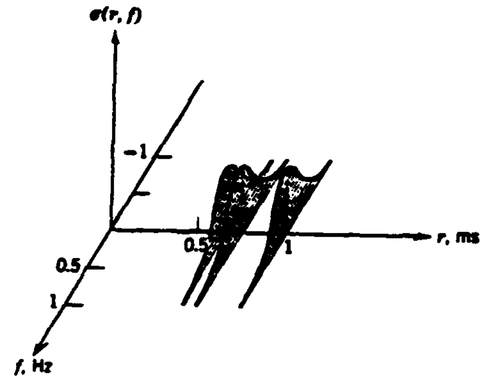
$$\sigma(f) = \int \sigma(r,f)dr.$$

This is the average energy due to scatterers at doppler shift f. Associated with this function is the "doppler spread" parameter

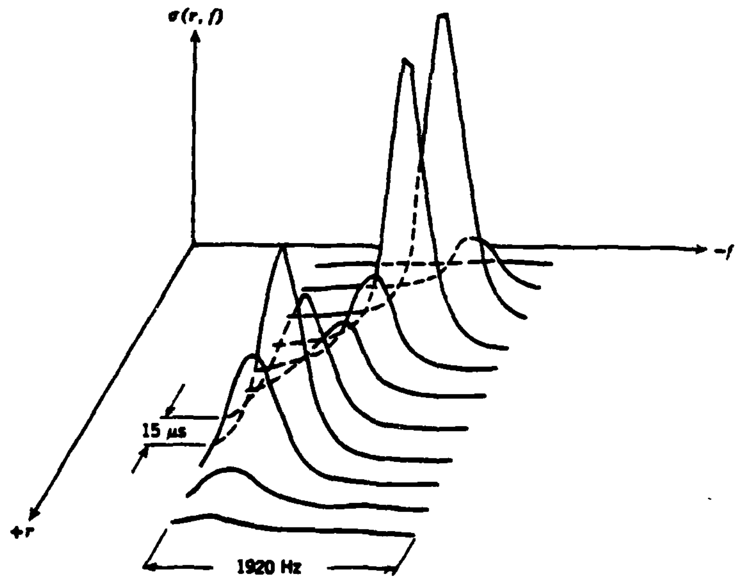
$$B = \frac{\sigma^2}{\int \sigma^2(f)df} \qquad \text{"Doppler Spread"}$$

Figure 4(a) shows a sketch of $\sigma(f)$ and the multipath spread parameter. Generally B is a measure of the range of doppler shifts due to scatterers.

For the uniform scatter function the parameters L and B are indeed

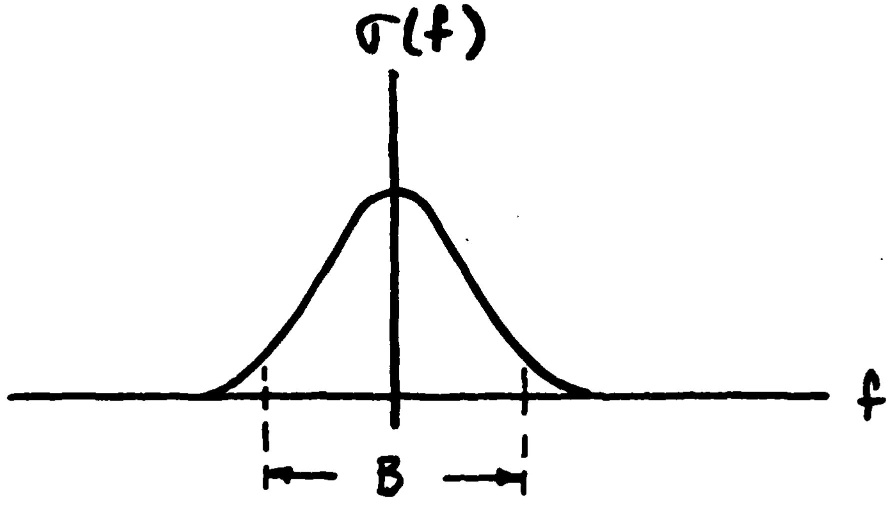


(a) F-layer Multipath

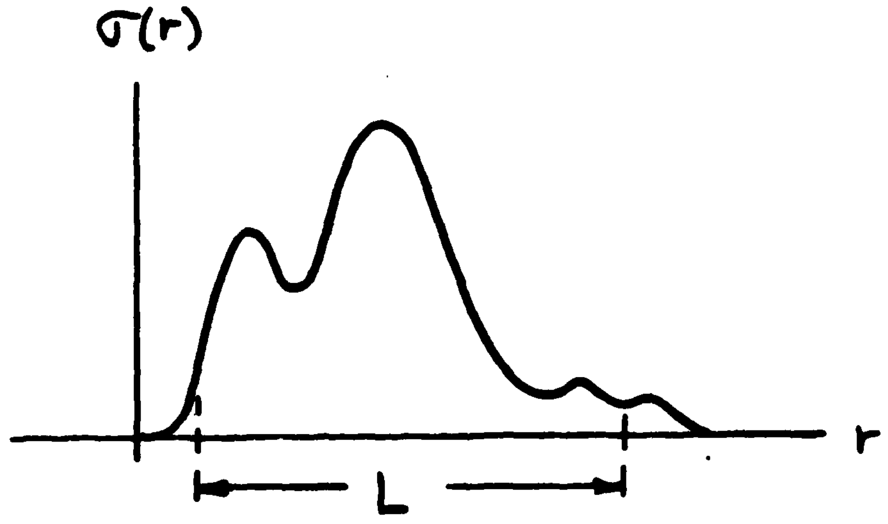


(b) Scattering function of West Ford dipole belt, May 1963.

FIGURE 3. Scatter Functions



(a) Doppler Spread Function



(b) Multipath Spread Function

FIGURE 4. Spread Functions

the range of delays and doppler shifts. For this uniform scatter function case we have

$$\sigma(r,f)=\begin{cases}\alpha\ , & 0 \leq r \leq L,\ |f| \leq \frac{B}{2}\\ 0\ , & \text{elsewhere}\ ,\end{cases}$$

$$\sigma(f)\ =\begin{cases}\alpha L\ , & |f| \leq \frac{B}{2}\\ 0\ , & \text{elsewhere}\end{cases}$$

$$\sigma(r)\ =\begin{cases}\alpha B\ , & 0 \leq r \leq L\\ 0\ , & \text{elsewhere}\end{cases}$$

Typical range of values for L and B are shown in Table 1 for various radio fading dispersive channels. Generally the actual scatter function for these channels are time varying and difficult to measure. From a practical viewpoint we can at best have approximate measurements of the basic parameters B and L. Namely, we can at best, measure the range of doppler spread and multipath delays.

III. Relationships Between Channel and Signal Parameters

The signal $x(t)$ is characterized by two basic parameters

T second (duration)

W Hz (bandwidth)

while the channel is characterized by a scatter function $\sigma(r,f)$ with basic parameters

L second (multipath spread)

B Hz (doppler spread)

We now examine the signal distortion caused by the channel for various relationships between signal parameters T,W and channel parameters L,B.

Nondispersive Channel

For the case where

$$B = 0\ ,\ L = 0$$

we have the channel output signal

CHANNEL	L	B	BL
Ionospheric Propagation (HF) 2MHz - 30MHz	10^{-3} to 10^{-2}	10^{-1} to 1 10 to 100 (disturbed auroral)	10^{-4} to 10^{-2} 10^{-2} to 1
Ionospheric Forward Scatter (VHF) 30MHz - 300MHz	10^{-4}	10	10^{-3}
Tropospheric Scatter (SHF) 3GHz - 30GHz	10^{-6}	10	10^{-5}

Table 1. Typical Values of B and L

$$z(t) = \sum_i \rho_i A(t) \cos[w_o t + \psi(t) + \theta_i]$$
$$= \rho\, A(t) \cos[w_o t + \psi(t) + \theta]$$

where ρ and Θ are independent random variables with

$$\Theta \sim \text{Uniform over} \quad [0,2\pi]$$

$$\rho \sim \text{Rayleigh}$$

Here we have the condition that multipath delays and the doppler spread of the channel is negligable and only a constant (random) amplitude and phase shift of the signal occurs. This is the case referred to as "flat-flat" fading since there is no time or frequency variation of the fading channel.

Dispersion in Time Only

Next suppose

$$B = 0\ ,\ L > 0$$

and consider a simple example where the component cos wt is transmitted and this signal is received together with a delay of L seconds. The received signal is thus,

$$z(t;w) = \cos wt + \cos[w(t-L)]$$
$$= (1 + \cos wL) \cos wt + \sin wL \sin wt$$
$$= A(wL) \cos[wt + \phi(wL)]$$

where

$$A(x) = \sqrt{(1 + \cos x)^2 + \sin^2 x}$$

$$\phi(x) = \tan^{-1}\left(\frac{\sin x}{1+ \cos x}\right)$$

Figure 5 shows A(x) versus x. For $wL \leq 1$ we have

$$1.76 = A(1) \leq A(wL) \leq A(0) = 2$$

and the approximation

$$\cos[w(t-L)] \simeq \cos wt$$

Thus in general if $wL \leq 1$ we have the approximation

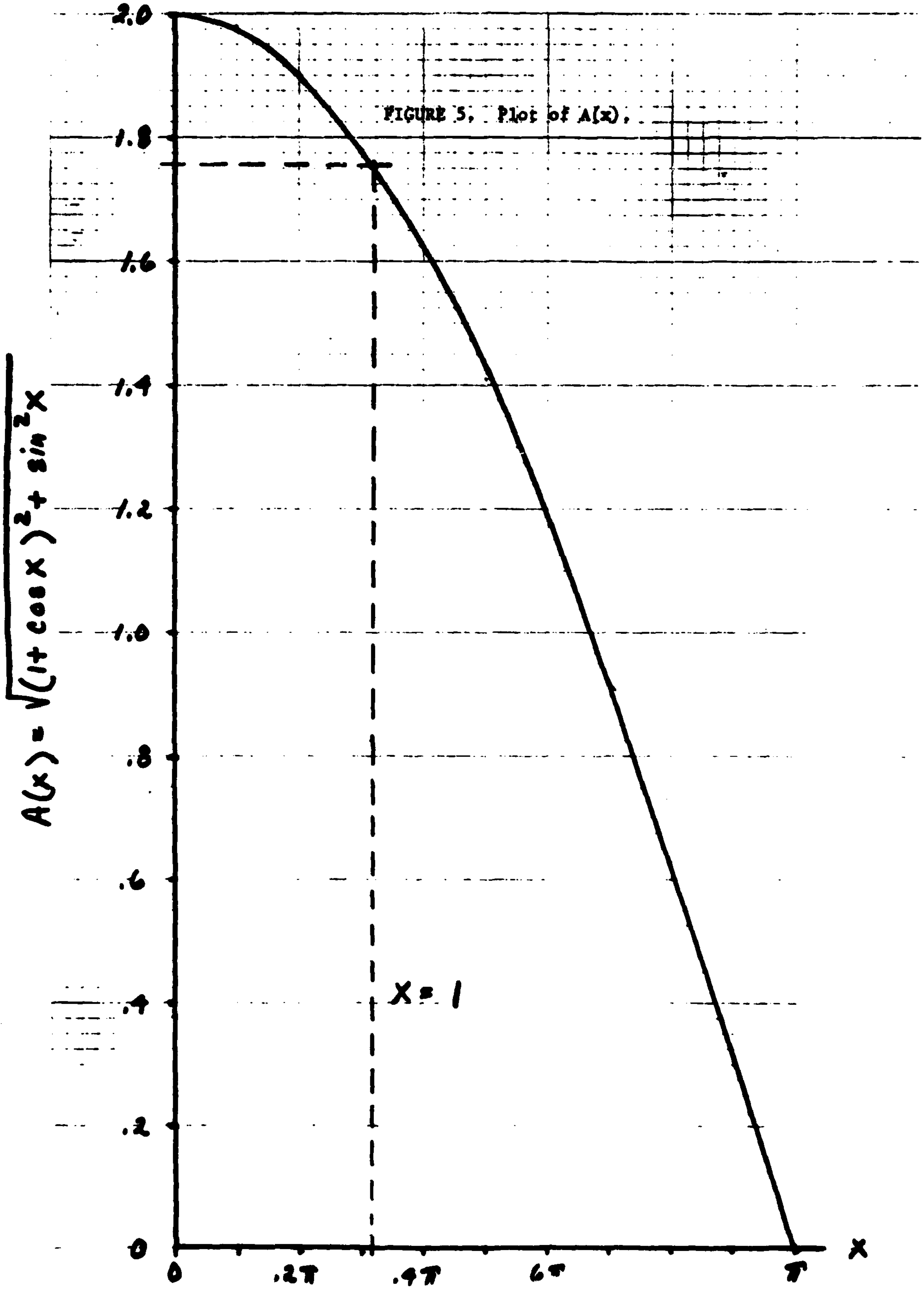


FIGURE 5. Plot of A(x).

$$z(t;w) = \sum_i \rho_i \cos [wt - wr_i + \Theta_i]$$
$$\cong \sum_i \rho_i \cos [wt + \Theta_i]$$
$$= \rho \cos [wt + \Theta]$$

Here we take $r_i \leq L$ so that $wr_i \leq 1$ too .

In general if

$$x(t) = A(t) \cos [w_o t + \psi(t)]$$

has signal bandwidth W Hz where

$$2\pi WL \leq 1$$

then we have the approximation

$$z(t) = \sum_i \rho_i A(t-r_i) \cos [w_o(t-ri) + \psi(t-r_i) + \Theta_i]$$
$$\cong \sum_i \rho_i A(t) \cos [w_o t + \psi(t) + \Theta_i]$$
$$= \rho A(t) \cos [w_o t + \psi(t) + \Theta]$$

which is again the "flat-flat" fading case.

Next consider the response to $\cos w_1 t$ denoted

$$z(t;w_1) = \sum_i \rho_i \cos[w_1 t - w_1 r_i + \Theta_i]$$

and the channel response to $\cos w_2 t$ denoted

$$z(t;w_2) = \sum_i \rho_i \cos[w_2 t - w_2 r_i + \Theta_i]$$

The cross correlation between these two different frequency components is,

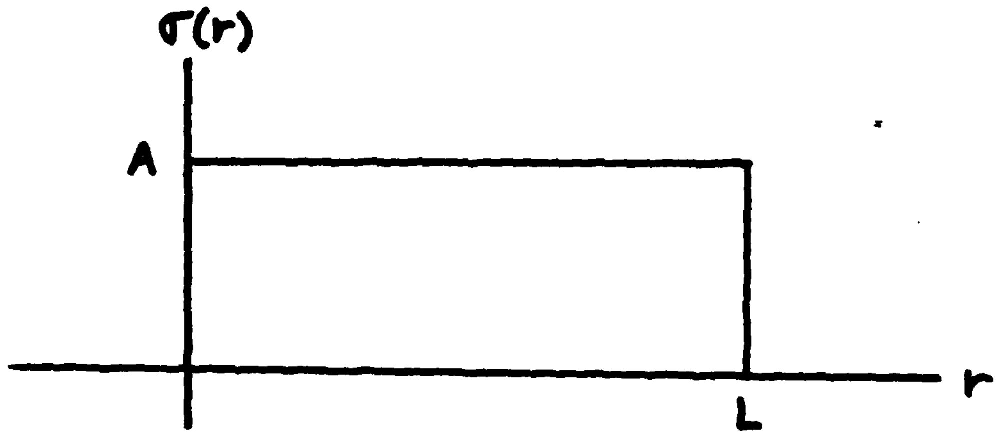
$$C(t;w_1,w_2) = E\{z(t;w_1)\, z(t;w_2)\}$$
$$= \tfrac{1}{2}\sum_i E\{\rho_i^2\} \cos [(w_1 - w_2)t - (w_1-w_2)r_i]$$

Suppose w_1,w_2 satisfies

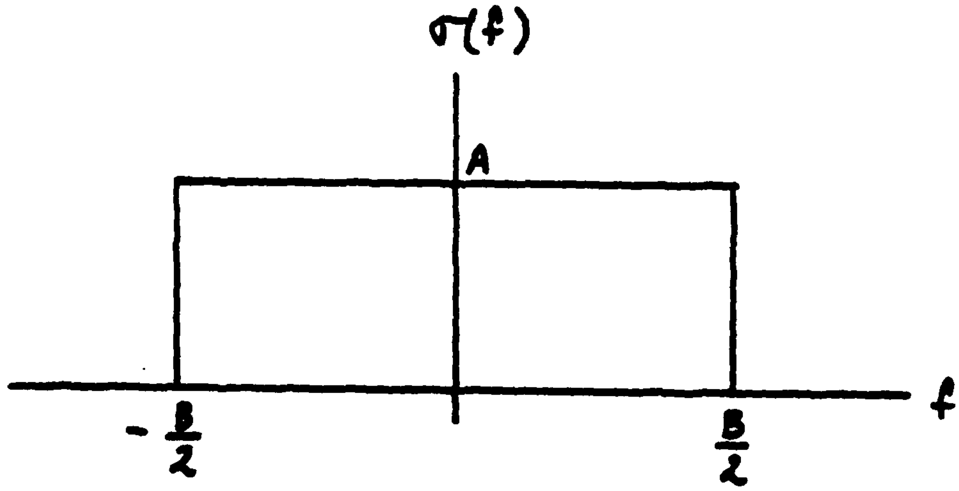
$$(w_1-w_2)L = 2\pi$$

and as shown in Figure 6(a) $\{r_i\}$ are spread uniformly over [0,L]. That is,

$$\sigma(r) = \begin{cases} A & , \quad 0 \leq r \leq L \\ 0 & , \quad \text{otherwise} \end{cases}$$



(a) Uniform Multipath Spread Function



(b) Uniform Doppler Spread Function

FIGURE 6. Uniform Spread Functions

Then

$$C(t;w_1,w_2) = \tfrac{1}{2}\int_0^L A \cos [(w_1-w_2)t - (w_1-w_2)r]dr$$

$$= \frac{A}{2}\int_0^L \cos [(w_1-w_2)t - \frac{2\pi}{L} r]dr$$

$$= 0$$

This correlation is equal to zero for all sine and cosine signals separated by $\frac{1}{L}$ Hz. This motivates the definition of

$$W_c = \frac{1}{L} \text{ Hz}$$

as the <u>channel coherence frequency</u>.

In general if a signal of bandwidth W has

$$W \leq \frac{Wc}{2\pi} = .16 W_c$$

we have "flat-flat" fading while for

$$W \geq W_c$$

we have "frequency selective" fading where the fading due to the channel depends on the frequency components of the transmitted signal. Roughly speaking, if we have a signal of bandwidth $W > W_c$ then the frequency components of the received signal spaced apart W_c Hz or more will have uncorrelated effects due to the randomness of the channel. Figure 7 illustrates the cases for this "frequency selective" fading due to channels with dispersion in time only where $B = 0$ and $L > 0$.

<u>Dispersion in Frequency Only</u>

Suppose next

$$B > 0, L = 0$$

and again consider the example where component cos wt is transmitted. The received signal is assumed to be the transmitted signal plus its frequency shift by B Hz given by,

Waveform Characteristics for Channels Dispersive only in Time

WL	L/T	Distorted	Dispersed	Remarks
$\ll 1$	$\ll 1$	No	No	Implied by $WL \ll 1$ since $TW \geq 1$
$\gg 1$	$\ll 1$	Yes	No	Implies $TW \gg 1$
$\gg 1$	$\gg 1$	Yes	Yes	Implied by $L \gg T$ since $TW \geq 1$

$$B = 0,\ L > 0,\ W_c = \frac{1}{L}$$

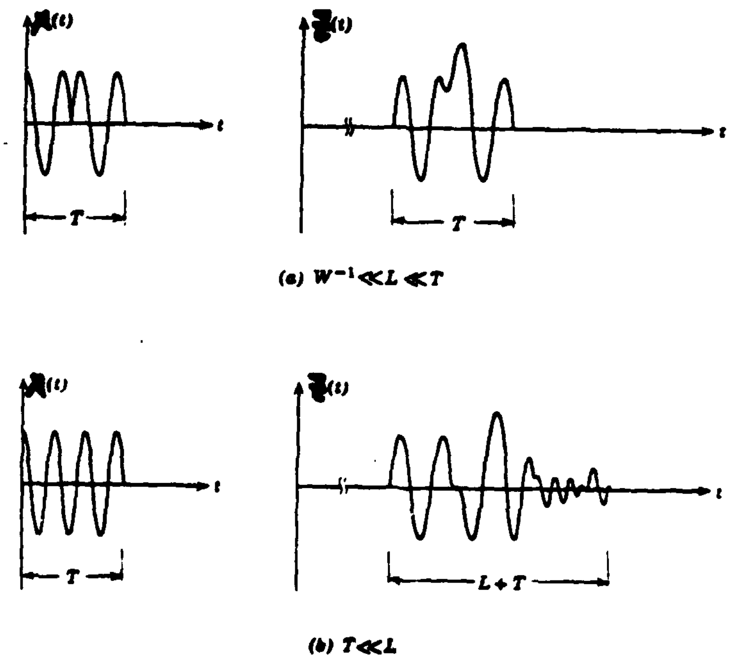


FIGURE 7 Dispersion in Time Only:
Frequency Selective Fading

$$z(t;w) = \cos wt + \cos[(w-2\pi B)t]$$

$$= A(2\pi Bt)\cos[wt + \phi(2\pi Bt)]$$

Again $A(x)$ is given by Figure 5. For

$$2\pi Bt \leq 1$$

we have the approximation

$$\cos[(w-2\pi B)t] \cong \cos wt.$$

Thus in general if

$$x(t) = A(t)\cos[w_o t + \psi(t)]$$

has duration T seconds where

$$2\pi BT \leq 1$$

then we have the approximation

$$z(t) = \sum_i \rho_i A(t) \cos[(w_o - w_i)t + \psi(t) + \theta_i]$$

$$\cong \sum_i \rho_i A(t) \cos[w_o t + \psi(t) + \theta_i]$$

$$= \rho A(t)\cos[w_o t + \psi(t) + \theta]$$

which is again the "flat-flat" fading case.

Consider now two time samples of the channel response due to cos wt denoted

$$z(t_k;w) = \sum_i \rho_i \cos[(w-w_i)t_k + \theta_i]$$

$$k = 1,2.$$

The correlation

$$C(t_1,t_2;w) = E\{z(t_1;w)z(t_2;w)\}$$

$$= \tfrac{1}{2}\sum_i E\{\rho_i^2\}\cos[(w-w_i)(t_1-t_2)]$$

$$= \tfrac{1}{2}\sum_i E\{\rho_i^2\}\cos[w(t_1-t_2) - w_1(t_1-t_2)]$$

Now suppose t_1 and t_2 satisfy

$$B(t_1-t_2) = 1$$

and as shown in Figure 6(b) $\{w_i\}$ are spread uniformly over $[-\pi B, \pi B]$.

That is,

$$\sigma(f) = \begin{cases} A & , \quad |f| \leq \frac{B}{2} \\ 0 & , \quad \text{otherwise} \end{cases}$$

Then

$$C(t_1,t_2;w) = \tfrac{1}{2}\int_{-\frac{B}{2}}^{\frac{B}{2}} A \cos[w(t_1-t_2)-2\pi f(t_1-t_2)]\, df$$

$$= \frac{A}{2}\int_{\frac{B}{2}}^{\frac{B}{2}} \cos[w(t_1-t_2)- \frac{2\pi}{B} f]\, df$$

$$= 0$$

This correlation is equal to zero for all sine and cosine input signals whose channel outputs are correlated with time delay $\frac{1}{B}$ seconds. This motivates the definition

$$T_c = \frac{1}{B} \text{ seconds}$$

as the <u>channel coherence time</u>.

In general if a signal of duration T has

$$T \leq \frac{T_c}{2\pi} = .16T_c$$

we have "flat-flat" fading while for

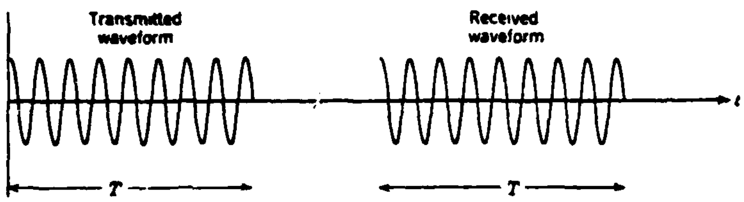
$$T \geq T_c$$

we have "time selective" fading where the fading due to the channel varies with time. Roughly speaking, if we have a signal of duration $T > T_c$ then time samples of the received signal spaced T_c seconds or more will have uncorrelated effects due to the randomness of the channel. Figure 8 illustrates the cases for this "time selective" fading due to channels with dispersion in frequency only where $B > 0$ and $L = 0$.

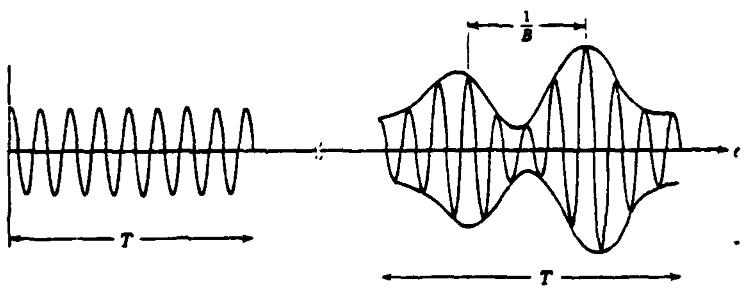
Waveform Characteristics for Channels Dispersive only in Frequency

BT	B/W	Distorted	Dispersed	Remarks
$\ll 1$	$\ll 1$	No	No	Implied by $BT \ll 1$ since $TW \geq 1$
$\gg 1$	$\ll 1$	Yes	No	Implies $TW \gg 1$
$\gg 1$	$\gg 1$	Yes	Yes	Implied by $B \gg W$ since $TW \geq 1$

$$B > 0, \; L = 0, \; T_c = \frac{1}{B}$$



(a) $BT < 1$



(b) $BT > 1$

FIGURE 8. Dispersion in Frequency Only: Time Selective Fading

Dispersion in Time and Frequency

In general with

$$B > 0, \; L > 0$$

we have both time and frequency fading. Figure 9 summarizes the various relationships between the signal parameters T,W and the channel parameters L,B or equivalently

$$W_c = \frac{1}{L}$$

and

$$T_c = \frac{1}{B}$$

IV. Diversity and Interleaving

In general a signal x(t) with bandwidth

$$W > W_0 = \frac{1}{L}$$

and duration

$$T > T_c = \frac{1}{B}$$

has *roughly*

$$D = \frac{\;}{_c}\Big)\Big(\frac{T}{T_c}\Big)$$

$$= (BL)\ (WT)$$

independent variations or diversity. That is, there are roughly D samples of the signal where the randomness of the channel are independent. A good receiver would exploit this available diversity in the received signal.

DS/BPSK

For direct sequence spread coherent BPSK signals an optimum receiver consists of a time varying matching filter realized by a tapped delay line with adaptively adjusted (using recursive filters) weighing terms. This is the "Rake Receiver" system and is very complex.

Waveform Characteristics for Doubly Dispersive Channels

					Dispersed in		
BT	WL	B/W	L/T	Distorted	Time	Frequency	Remarks
$\ll 1$	$\ll 1$	$\ll 1$	$\ll 1$	No	No	No	Implies $BL \ll 1$
$\ll 1$	$\gg 1$	$\ll 1$	$\ll 1$	Yes	No	No	Implies $BL \ll 1$, and $TW \gg 1$
$\ll 1$	$\gg 1$	$\ll 1$	$\gg 1$	Yes	Yes	No	
$\gg 1$	$\ll 1$	$\ll 1$	$\ll 1$	Yes	No	No	Implies $BL \ll 1$, and $TW \gg 1$
$\gg 1$	$\gg 1$	$\ll 1$	$\ll 1$	Yes	No	No	Implies $TW \gg 1$
$\gg 1$	$\gg 1$	$\ll 1$	$\gg 1$	Yes	Yes	No	Implies $BL \gg 1$, and $TW \gg 1$
$\gg 1$	$\ll 1$	$\gg 1$	$\ll 1$	Yes	No	Yes	
$\gg 1$	$\gg 1$	$\gg 1$	$\ll 1$	Yes	No	Yes	Implies $BL \gg 1$, and $TW \gg 1$
$\gg 1$	$\gg 1$	$\gg 1$	$\gg 1$	Yes	Yes	Yes	Implies $BL \gg 1$

$$B > 0,\ L > 0,\ W_c = \frac{1}{L},\ R_c = \frac{1}{B}$$

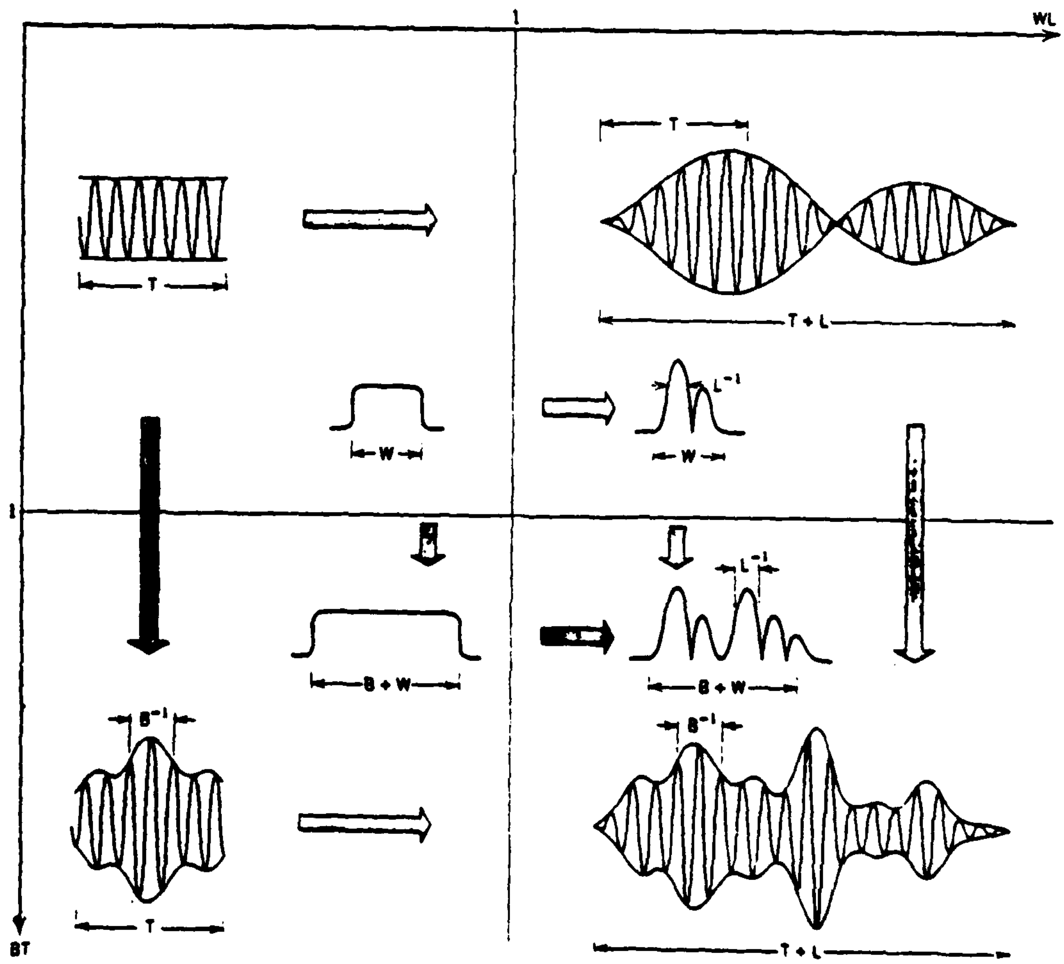


FIGURE 9. Dispersion in Time and Frequency

FH/ MFSK

Frequency hopped noncoherent MFSK signals can achieve good performance if diversity is used. If each MFSK diversity chip is of duration

$$T < T_c$$

and bandwidth

$$W < W_c$$

then we can have independent flat-flat fading of each diversity MFSK chip by sending only one MFSK chip in each $W_c T_c$ bandwidth and time rectangle.

Since independence of each diverstiy chip applies to each MFSK signal with say m diversity, diversity chips of different MFSK signals can share the same $W_c T_c$ bandwidth and time rectangle. This may be necessary if the hopping rate is less than $\frac{1}{T_c}$ hops per second. To achieve this sharing of the $W_c T_c$ rectangle, interleaving is required. Figure 10 illustrates an example where $T = 3T_c$ and three 4 -ary FSK chip sequences are interleaved in time to insure independent diversity chip fade for each MFSK diversity chip sequence. This illustrates the natural tradeoff between hopping rates and interleaving to achieve the advantages of diversity. The same principle applies with partial band jamming when we want to achieve independent jamming statistics in each MFSK chip.

V. Analysis of Noncoherent MFSK Signals

We analyze here the performance of MFSK signals using m diversity chips where we assume each chip has independent flat-flat fading. The transmitter selects symbol

$$\ell \in \{1, 2, \ldots, M\}$$

where

$$M = 2^K$$

This symbol ℓ is sent using m MFSK diversity chips corresponding to symbol

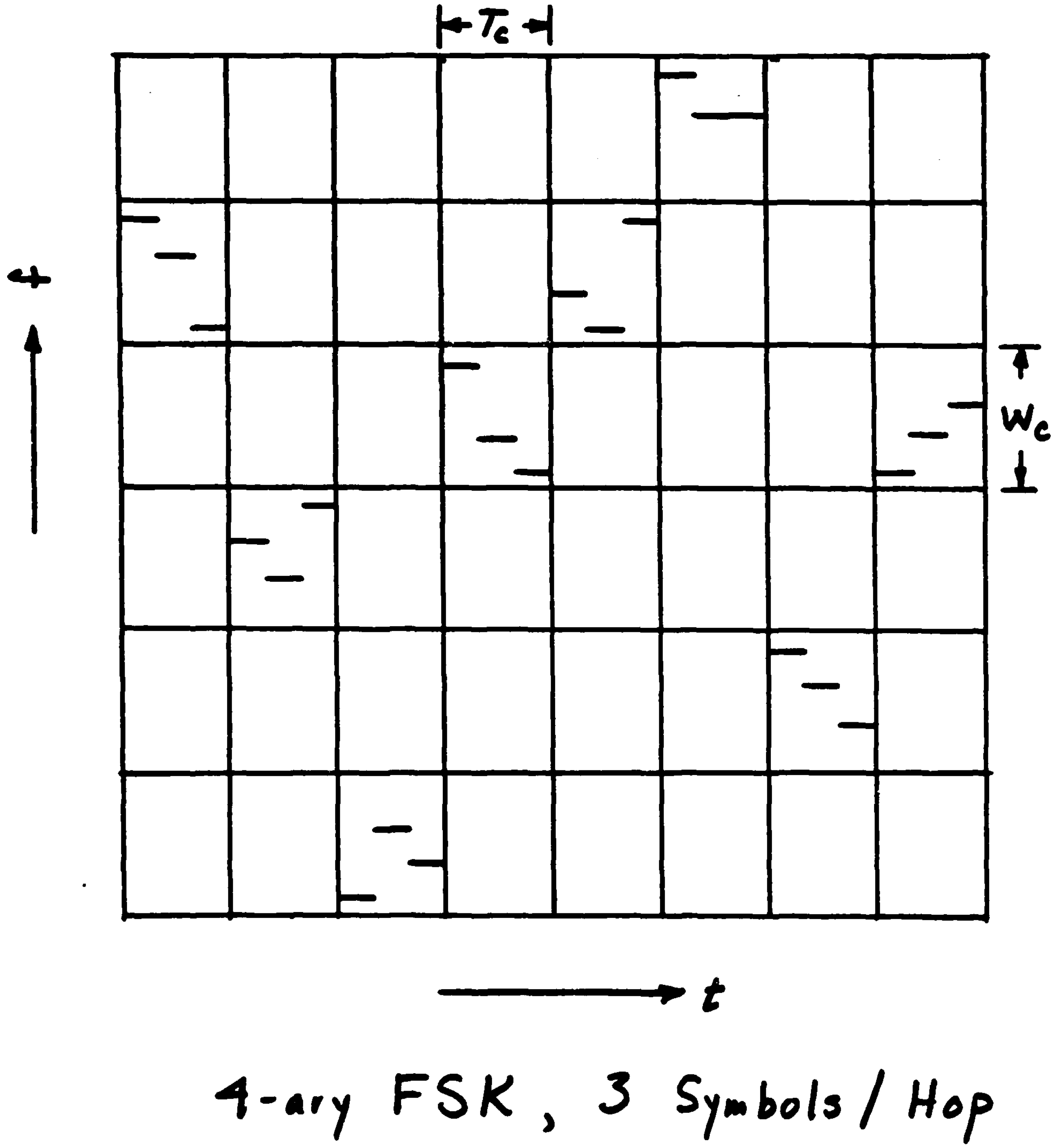


FIGURE 10. Time - Frequency Diversity

ℓ. Each MFSK chip has energy E_c so that we have

$$E_b = \frac{m}{K} E_c \text{ energy/bit.}$$

For each diversity chip the receiver examines the received chip energy at each of the M detector outputs. Denote

$y_k^{(\hat{\ell})}$ = energy of the $k^{\underline{th}}$ received chip interval of the $\hat{\ell}$th symbol

$\hat{\ell} = 1, 2, \ldots, M = 2^K$

$k = 1, 2, \ldots, m$

Each MFSK chip ends up at the receiver as a Gaussian process (Rayleigh fading with uniformly distributed phase) with energy denoted $\overline{E}_c$. Assuming additive white Gaussian noise with single sided spectral density of N_o also at the receiver we have

$$P(y_k^{(\hat{\ell})} \mid \ell) = \begin{cases} \dfrac{1}{N_o} e^{-\frac{y_k^{(\hat{\ell})}}{N_o}} , & \hat{\ell} \neq \ell \\[2ex] \dfrac{1}{N_o + \overline{E}_c} e^{-\frac{y_k^{(\hat{\ell})}}{N_o + \overline{E}_c}} , & \hat{\ell} = \ell \end{cases}$$

$$y_k^{(\hat{\ell})} \geq 0$$

When m chips are sent the receiver has N = Mn chip energy detector outputs

$$\underline{y} = (\underline{y}_1, \underline{y}_2, \ldots, \underline{y}_m)$$

where

$$\underline{y}_k = (y_k^{(1)}, y_k^{(2)}, \ldots, y_k^{(m)})$$

$k = 1, 2, \ldots, m.$

Since all chip statistics at the receiver are assumed independent we have

$$P_N(\underline{y}|\ell) = \prod_{k=1}^{m} P_M(\underline{y}_k|\ell)$$

$$= \prod_{k=1}^{m} \prod_{\hat{\ell}=1}^{M} P(y_k^{(\hat{\ell})}|\ell)$$

Defining

$$P_N(\underline{y}) = \prod_{k=1}^{m} \prod_{\hat{\ell}=1}^{M} \frac{1}{N_o} e^{-\frac{y_k^{(\ell)}}{N_o}}$$

we have

$$P_N(\underline{y}|\ell) = P_N(\underline{y}) \prod_{k=1}^{m} \frac{No}{N_o+\overline{E}_c} e^{(\frac{1}{N_o} - \frac{1}{N_o+\overline{E}_c})y_k^{(\ell)}}$$

$$= P_N(\underline{y}) (\frac{No}{N_o+\overline{E}_c})^m \; e^{(\frac{\overline{E}_c}{N_o(N_o+\overline{E}_c)}) \sum_{k=1}^{m} y_k^{(\ell)}}$$

The maximum likelihood metric is thus

$$\lambda(\underline{y},\ell) = \sum_{k=1}^{m} y_k^{(\ell)}$$

If each chip had a different average energy at the receiver and the noise level was also different for each chip interval (noise is flat in each MFSK band but possible different from band to band) then

$$P(y_k^{(\hat{\ell})}|\ell) = \begin{cases} \dfrac{1}{N_{ok}} \; e^{-\frac{y_k^{(\hat{\ell})}}{N_{ok}}} & , \quad \hat{\ell} \neq \ell \\[2ex] \dfrac{1}{N_{ok}+\overline{E}_{ck}} \; e^{-\frac{y_k^{(\hat{\ell})}}{N_o+\overline{E}_{ck}}} & , \quad \hat{\ell} = \ell \end{cases}$$

$$y_k^{(\hat{\ell})} \geq 0$$

Here the maximum likelihood metric is then

$$\lambda(\underline{y};\ell) = \sum_{k=1}^{m} \frac{\gamma_k}{1+\gamma_k} \cdot \frac{y_k^{(\ell)}}{N_{ok}}$$

where

$$\gamma_k = \frac{\overline{E}_{ck}}{N_{ok}}$$

Returning now to the case where

$$\overline{E}_{ck} = \overline{E}_c$$

$$N_{ok} = N_o$$

$$k = 1, 2, \ldots, m$$

we assume the maximum likelihood receiver and use the Bhattacharyya bound on the pairwise error,

$$P(\ell \to \hat{\ell}) \leq \int \sqrt{P_N(\underline{y}|\ell) P_N(\underline{y}|\hat{\ell}) d\underline{y}}$$

$$= \prod_{k=1}^{m} \int \sqrt{P_M(\underline{y}_k|\ell) P_M(\underline{y}_k|\hat{\ell})} d\underline{y}_k$$

$$= \prod_{k=1}^{m} \left\{ \int_0^\infty \frac{1}{\sqrt{N_o(N_o + \overline{E}_c)}} e^{-\frac{1}{2}\left[\frac{1}{N_o+\overline{E}_c} + \frac{1}{N_o}\right]y} dy \right\}^2$$

$$= \left\{ \frac{1 + \frac{\overline{E}_c}{N_o}}{\left[1 + \frac{\overline{E}_c}{2N_o}\right]^2} \right\}^m$$

The symbol error is union bounded as

$$P_E \leq \tfrac{1}{2}(M-1) \left\{ \frac{1 + \frac{\overline{E}_c}{N_o}}{\left[1 + \frac{\overline{E}_c}{2N_o}\right]^2} \right\}^m$$

For MFSK with $K = \log_2 M$ the bit error is

$$P_b = \frac{\frac{1}{2} M}{M - 1} \; P_E$$

or using the above bounds

$$P_b \leq 2^{K-2} \left\{ \frac{1 + \frac{K}{m} \left(\frac{\bar{E}_b}{N_o}\right)}{\left[1 + \frac{K}{2m}\left(\frac{\bar{E}_b}{N_o}\right)\right]^2} \right\}$$

Figures 11, 12, and 13 show those bounds versus $\bar{E}_b/N_o$ for various choices of diversity m. Note that with Rayleigh fading there is little combining loss due to large values of diversity m.

VI. Coded BPSK - Partial Interleaving

To illustrate the impact of diversity and interleaving further we next consider coded coherent BPSK signals. This does not include any direct sequence spreading and we assume ideal phase coherence.

At any time let R denote the random fade envelope due to scatterers in a flat-flat fading case. Here $\rho = R \sqrt{E_c}$ is the envelope of the received signal for each coded BPSK symbol. Here we assume each BPSK symbol has duration

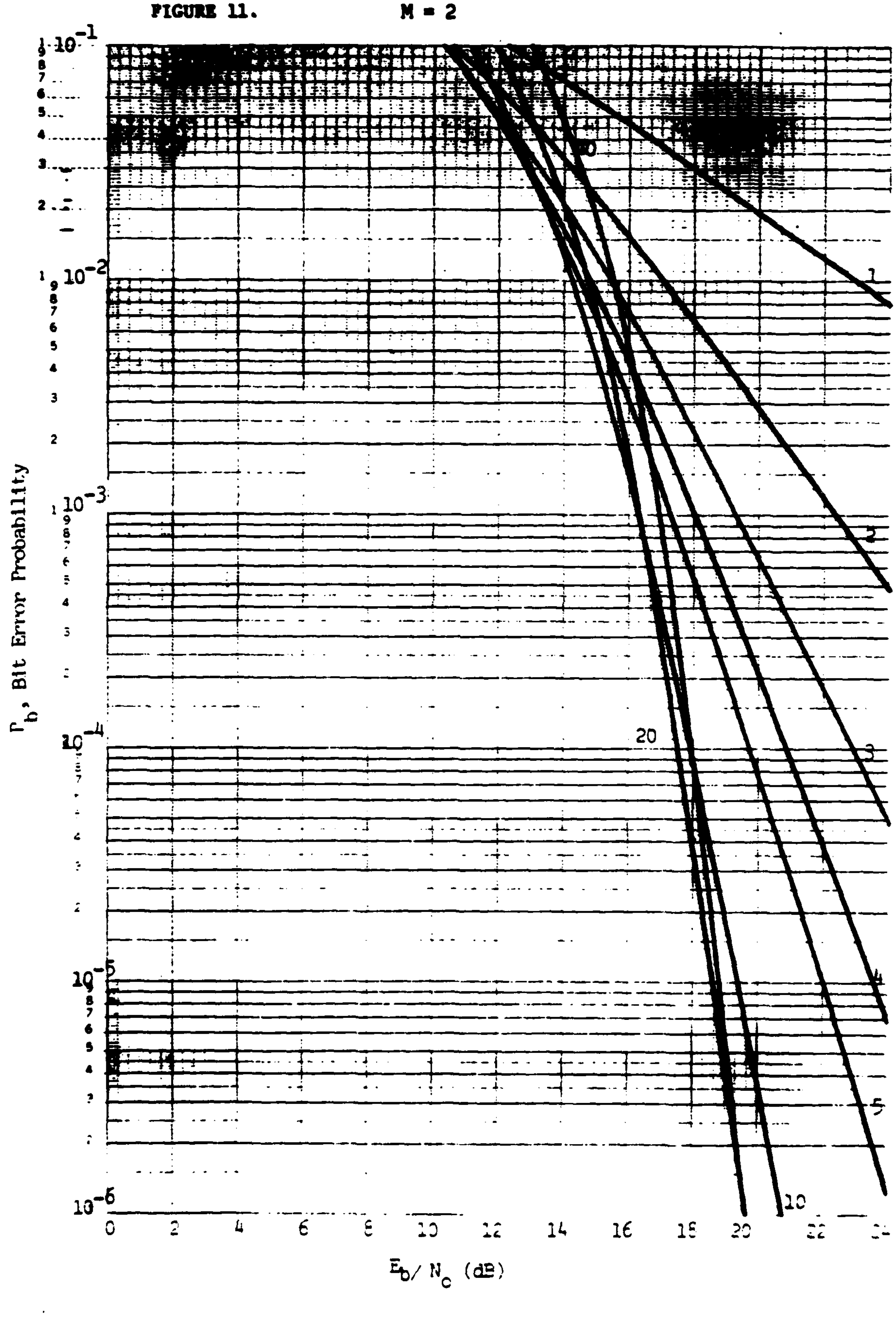
$$T < T_c$$

and bandwidth

$$W < W_c.$$

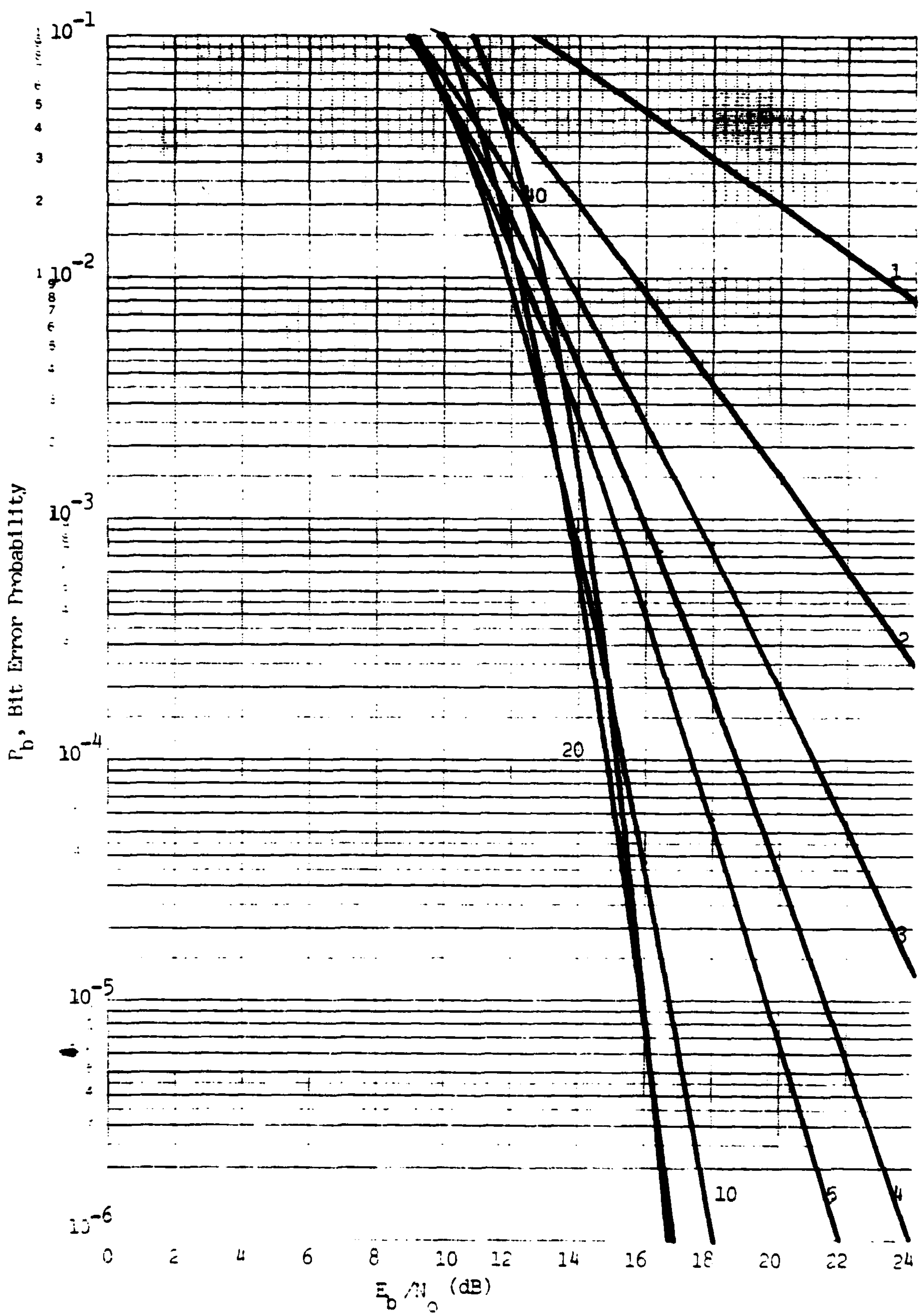
and make the assumption that each BPSK signal experiences flat-flat fading. The phase variations due to the channel is assumed to be slow enough to be tracked perfectly.

FIGURE 11. M = 2



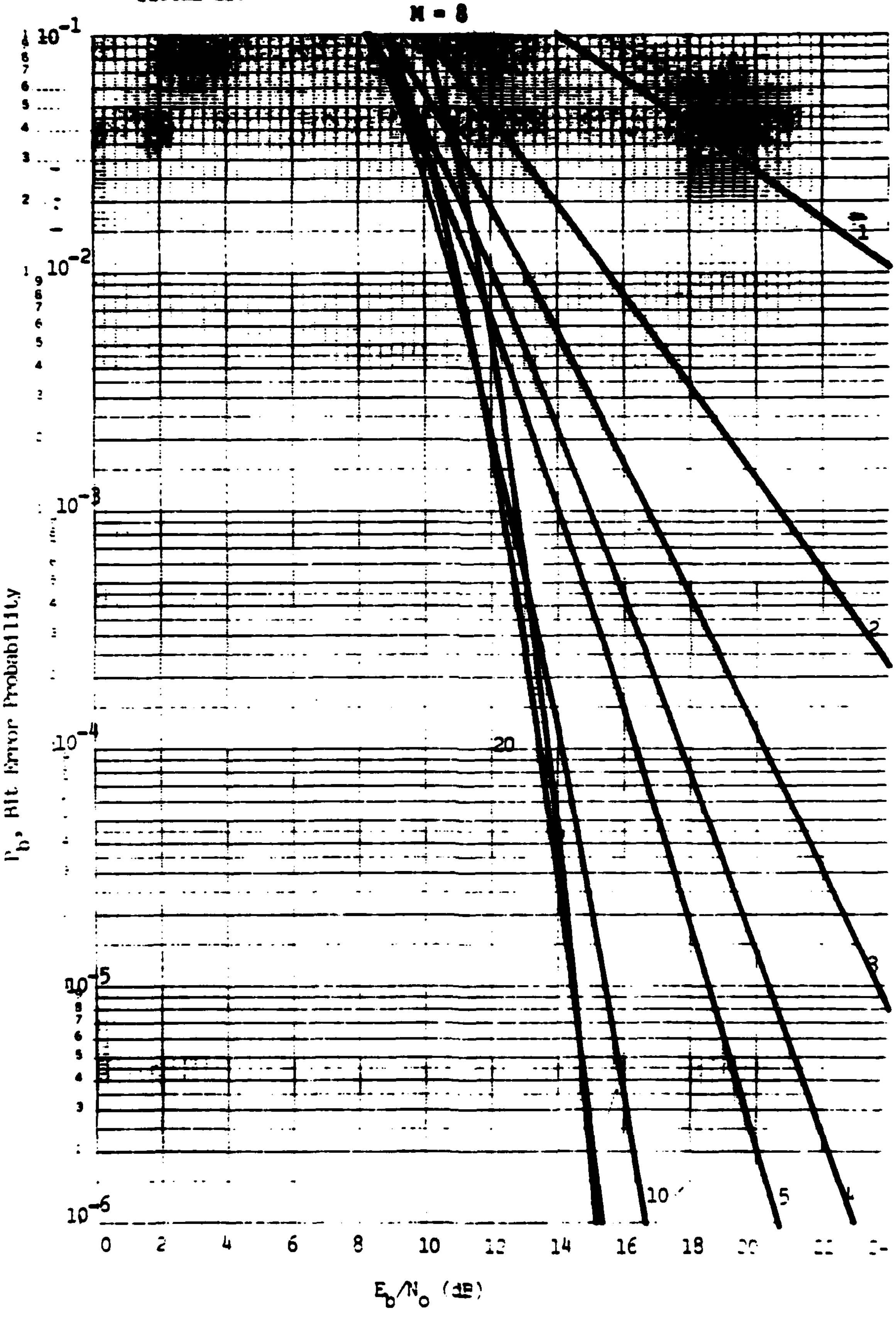
P_b, Bit Error Probability
E_b/N_o (dB)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
2
4
6
8
10
12
14
16
18
20
22
24
1
2
3
4
5
10
20

FIGURE 12. M = 4



46 6290

FIGURE 13.



M = 8
P_b, Bit Error Probability
E_b/N_o (dB)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
2
4
6
8
10
14
16
18
1
2
3
4
5
10
20

Without any coding we have $E_b = E_c$ and for a given R

$$P_b(R) = Q\left(R\sqrt{\frac{2E_b}{N_o}}\right)$$

$$< \tfrac{1}{2}\, e^{-R^2\left(\frac{E_b}{N_o}\right)}$$

Averaging this over the Rayleigh density

$$f_R(r) = 2r\, e^{-r^2} \qquad r \geq 0$$

where we normailzed to have

$$E\,\{R^2\} = 1$$

we have

$$\overline{P}_b = E\,\{P_b(R)\}$$

$$\leq \frac{\tfrac{1}{2}}{1 + \frac{E_b}{N_o}}$$

For coded BPSK signals we take the K=7, r=½ convolutional code where the bit error bound without fading is given by

$$P_b \leq \tfrac{1}{2} \sum_{k=d_{min}}^{\infty} N_k\, e^{-r\left(\frac{E_b}{N_o}\right)k}$$

where

$d_{min} = 10$

$N_{10} = 36$	$N_{14} = 1404$
$N_{11} = 0$	$N_{15} = 0$
$N_{12} = 211$	$N_{16} = 11633$
$N_{13} = 0$	$N_{17} = 0$

Suppose all the coded BPSK signal experience independent flat-flat fading characterized by independent Rayleigh random variables

$$\underline{R} = (R_1, R_2, \ldots)$$

Then the conditional bit error probability is bounded by

$$P_b(\underline{R}) \leq \tfrac{1}{2} \sum_{k=d_{min}}^{\infty} N_k \prod_{\ell=1}^{k} e^{-rR_\ell^2 (\frac{E_b}{N_o})}$$

Averaging this over the same Rayleigh statistics we have

$$\overline{P}_b \leq \tfrac{1}{2} \sum_{k=d_{min}}^{\infty} N_k \left(\frac{1}{1+r(\overline{E}_b/N_o)}\right)^k$$

or defining

$$D(x) = \frac{1}{1+x}$$

for the specific K=7, r=½ convolutional code,

$$\overline{P}_b \leq \tfrac{1}{2} \left\{ 36D(.5 \frac{\overline{E}_b}{N_o})^{10} + 211D(.5 \frac{\overline{E}_b}{N_o})^{12} + 1404D(.5 \frac{\overline{E}_b}{N_o})^{14} + 11633D(.5 \frac{\overline{E}_b}{N_o})^{16} + \ldots \right\}$$

Suppose next each pair of symbols experience the same fade. That is, $R_1=R_2, R_3=R_4, R_5=R_6$, etc. Then Averaging over the fade statistics yeilds

$$\overline{P}_b \leq \tfrac{1}{2} \left\{ 36D(\frac{\overline{E}_b}{N_o})^5 + 211D(\frac{\overline{E}_b}{N_o})^6 + 1404D(\frac{\overline{E}_b}{N_o})^7 + 11633D(\frac{\overline{E}_b}{N_o})^8 + \ldots \right\}$$

Next if each group of three symbols experience the same fade where $R_1 = R_2 = R_3$, $R_4 = R_5 = R_6$, etc. then the average bit error bound becomes

$$\overline{P}_b \leq \tfrac{1}{2} \left\{ 36D(1.5 \frac{\overline{E}_b}{N_o})^3 D(.5 \frac{\overline{E}_b}{N_o}) + 211D(1.5 \frac{E_b}{N_o})^4 + 1404D(1.5 \frac{\overline{E}_b}{N_o})^4 D(\frac{\overline{E}_b}{N_o}) + 11633D(1.5 \frac{\overline{E}_b}{N_o})^5 D(.5 \frac{E_b}{N_o}) \right\}$$

Here we assume each group fades independently. This is an approximation to time selective fading channels where $T_c = \frac{1}{B}$ is equal to the time to transmit a group of symbols that we assume experience the same fade. Figure 14 shows the uncoded BPSK bound which is the same as uncoded

DBPSK. Also shown are the bounds on the K=7, r=½ convolutionally coded bit error probability where we assume

$$T_c = LT_s$$

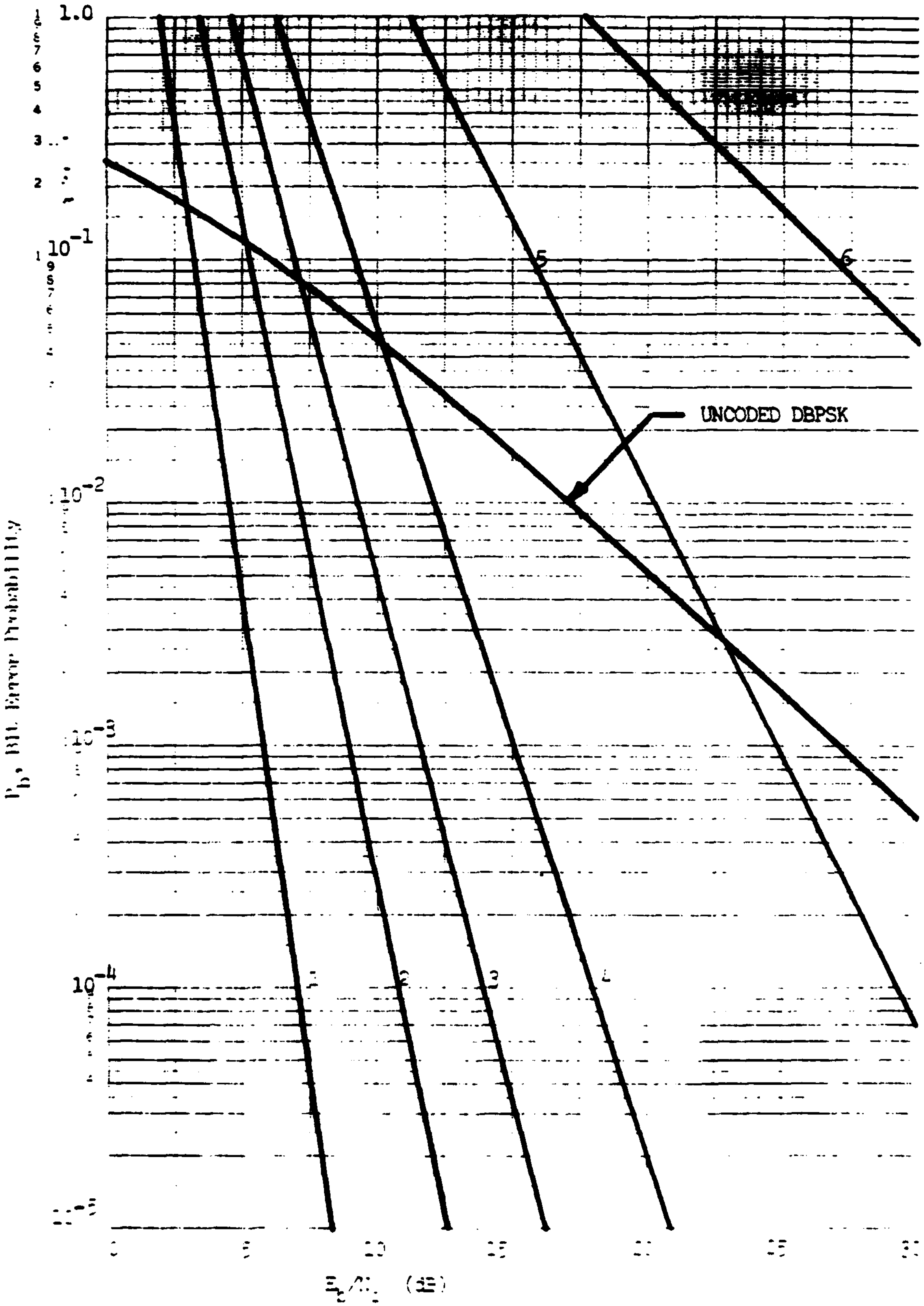
where

$$T_s = \text{coded symbol time}$$

$$L = 1, 2, 3, 4, 5, 6$$

FIGURE 14. PARTIAL INTERLEAVING WITH A

K=7, r=1/2 Convolutional Code - BPSK



APPENDIX: A Filter Approach

Suppose the signal $x(t)$ is a narrow band signal at carrier frequency $f_o = \frac{w_o}{2\pi}$, of the general form

$$x(t) = A(t) \cos [w_o t + \psi(t)]$$

$$= \text{Re}\, \{X(t)\, e^{jw_o t}\}$$

where

$$X(t) = A(t)\, e^{j\psi(t)}$$

is the complex envelope which is slowly varying compared to the carrier $e^{jw_o t}$.

We think of the fading dispersive channel as a filter with impulse response

$h(t;r)$ = impulse response at time t due to those scatter points at delay r.

where

$$h(t;r) = \text{Re}\, \{2H(t;r)e^{jw_o t}\}$$

$H(t;r)$ = complex envelope of the impulse response at time t due to those scatter points at delay r.

Here $H(t;r)$ is due to many scatter points and can thus be taken as a complex Gaussian process. Also, assuming scatterers are independent

$$E\{H(t_1;r_1)H^*(t_2;r_2)\} = 0 \quad \text{all } r_1 \neq r_2$$

We also let for $r_1 = r_2 = r$

$$E\{H(t_1;r)H^*(t_2;r)\} = \beta(t_1-t_2;r).$$

This is the effect of the same scatterers at different times.

The total signal out of the channel is

$$z(t) = \text{Re}\, \{Z(t)\, e^{jw_o t}\}$$

where

$$Z(t) = \int H(t;r)X(t-r)dr$$

and

$$E\{H(t_1;r_1)H^*(t_2;r_2)\} = \delta(r_1-r_2)\beta(t_1-t_2;r_1)$$

Next consider

$$R_z(t_1;t_2) = E\{z(t_1)z(t_2)\}$$

$$= \tfrac{1}{2}\,\mathrm{Re}\{\iint E[H(t_1;r_1)H^*(t_2;r_2)]$$

$$\cdot X(t_1-r_1)X^*(t_2-r_2)dr_1dr_2\cdot e^{jw_o(t_1-t_2)}\}$$

$$= \tfrac{1}{2}\,\mathrm{Re}\{\int\beta(t_1-t_2;r)X(t_1-r)X^*(t_2-r)dr\cdot e^{jw_o(t_1-t_2)}\}$$

Define scatter function

$$\sigma(r,f) = \int\beta(t;r)e^{+2\pi jft}dt$$

and so

$$\beta(t;r) = \int\sigma(r,f)e^{-2\pi jft}df$$

Hence,

$$\int\beta(t_1-t_2;r)X(t_1-r)X^*(t_2-r)e^{jw_o(t_1-t_2)}dr$$

$$= \iint \sigma(r,f)X(t_1-r)X^*(t_2-r)e^{jw_o(t_1-t_2)}\quad e^{jw(t_1-t_2)}dfdr$$

$$= \iint \sigma(r,f)X(t_1-r)X^*(t_2-r)e^{j[w_o-w)(t_1-t_2)]}drdf$$

or

$$R_z(t_1,t_2) = \mathrm{Re}\{\tfrac{1}{2}\iint\sigma(r,f)X(t_1-r)X^*(t_2-r)\quad e^{j[(w_o-w)(t_1-t_2)]}drdf\}$$

Define

$$\sigma(r,f) = \int\beta(t;r)e^{+2\pi jft}dt$$

and so

$$\beta(t;r) = \int\sigma(r,f)e^{-2\pi jft}df$$

Hence,

$$\int\beta(t_1-t_2;r)X(t_1-r)X^*(t_2-r)e^{jw_o(t_1-t_2)}\ dr$$

$$= \iint\sigma(r,f)X(t_1-r)X^*(t_2-r)e^{jw_o(t_1-t2)}\quad e^{jw(t_1-t_2)}\ dfdr$$

$$= \iint\sigma(r,f)X(t_1-r)X^*(t_2-r)e^{j[(w_o-w)(t_1-t_2)]}\ drdf$$

or

$$R_z(t_1,t_2) = \mathrm{Re}\{\tfrac{1}{2}\iint\sigma r,f)X(t_1-r)X^*(t_2-r)\quad e^{j[(w_o-w)(t_1-t_2)]}\ drdf\}$$

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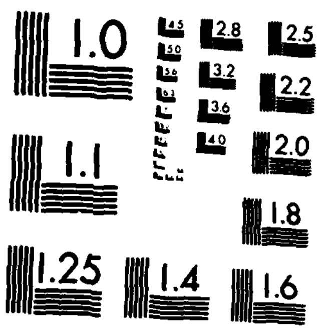

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Note No. 6

ANTI-JAM ANALYSIS OF FH/MFSK SYSTEMS IN RAYLEIGH FADING CHANNELS

to

NAVAL RESEARCH LABORATORY

(Contract Award No. N00014-80-K-0935)

for

HF COMMUNICATION NETWORK SIGNALS

USING CHANNEL EVALUATION DATA

Principal Investigator

Jim K. Omura
Professor
System Science Department
University of California
Los Angeles, California

March, 1981

ANTI-JAM ANALYSIS OF FH/MFSK SYSTEMS IN RAYLEIGH FADING CHANNELS

by

Dan Avidor
Jim Omura
Department of System Science
University of California
Los Angeles, California

I. INTRODUCTION

In this note we examine several cases where we have FH/MFSK signals in Rayleigh fading channels. In all our cases we assume the worst case partial band noise jammer where we define

$$N_J = \frac{J}{W}$$

where J is the total jammer power and W is the total spread spectrum bandwidth. The jammer has the option of distributing this power in any way across the bandwidth W.

An MFSK chip of duration T_c seconds occupies one of M tone positions in a sub-band of bandwidth roughly M/T_c. Each of the

$$L = \frac{W}{M/T_c} = \frac{WT_c}{M}$$

M tone sub-bands is assumed to have independent Rayleigh fading. Thus assuming each chip is frequency hopped independently among the M tone sub-bands, any sequence of MFSK chips experiences independent fading and jamming noise in each chip. In each sub-band the fading is assumed to be slow and uniform across the sub-band. Since we have Rayleigh fading a multitone jammer also experiencing the same type of Rayleigh fading will appear as Gaussian noise at the receiver. Thus we feel that the worst case partial band jamming is close to the worst type of jammer among all jamming waveforms.

For the above assumptions we consider several channel models with parameters:

- Average received signal power distribution across the bandwidth W.
- Jammer propagation loss distribution across the bandwidth W.
- Noise power and interference signals distribution across the bandwidth W.

The simplest channel that we examine first, has no background noise and both the average received signal power and jammer propagation loss are uniform across the band. Then we proceed to consider more realistic channel models that allow arbitrary distribution of these parameters.

We assume that the channel parameters mentioned above can be measured using "long-term observations." We also consider "short-term observations" which depend on the ability of the receiver to detect the presence or absence of the jammer signal during each chip time-interval, and modify the metric, used by the decoder, accordingly. Receivers having channel parameter information will be referred to as having "Channel State Information" [C.S.I.], whereas receivers having also "Jammer Presence Information" will be referred to as having jammer state information [J.S.I.]. For receivers/transmitters having C.S.I. we introduce the option of using nonuniform frequency hopping among M tone sub-bands. Coded communication systems will be discussed which use this information for establishing the optimal hopping strategy and also in the decoding process.

Out of many possible receiver structures, that may seem appropriate, we will examine the following:

- Hard decision with J.S.I.
- Hard decision with no J.S.I.
- Soft decision with J.S.I.
- Soft decision with no J.S.I.

Note that all 4 receivers are assumed to have C.S.I.

Several examples will be presented to illustrate the potential advantage of providing C.S.I. and having J.S.I. In this note we consider noise jamming only. When considering a uniform channel, we will assume partial-band jamming. In this case we denote by W_J the band "covered" by jammer noise, and define:

$$\rho \triangleq \frac{W_J}{W} \tag{1}$$

where ρ is the jammed fraction of the total spread spectrum bandwidth W. Alternatively, we could regard ρ as being the duty cycle of a pulsed jammer. Performance-wise these two methods are equivalent due to the ideal interleaving that we assume. For nonuniform channels ρ can only be viewed as the duty cycle of a pulsed jammer. For each receiver considered, we carry out the analysis for arbitrary ρ, and finally find the performance under the "worst case ρ." (Recall from note 4, that for non-fading channels, the worst case ρ is in general lower than 1.)

II. DEFINITIONS

We consider conventional MFSK signaling over a Rayleigh fading channel with additive white Gaussian noise of single-sided spectral density N_0. Let H_ℓ be the hypothesis that the ℓ^{th} tone is sent and assume the average signal energy at the receiver is $\bar{E}_c$. The square roots of the M energy detector outputs:

$$y_1, y_2, \ldots, y_M$$

are independent random variables with probability density functions:

$$P(y_\ell/H_\ell) = \frac{2y_\ell}{N_0+\bar{E}_c} \exp\left\{-\frac{y_\ell^2}{N_0+\bar{E}_c}\right\} \tag{2}$$

and

$$P(y_i/H_\ell) = \frac{2y_i}{N_0} \exp\left\{-\frac{y_i^2}{N_0}\right\} \quad ; \; i \neq \ell. \tag{3}$$

For such a receiver we have:

$$P(y_\ell \leq y_i/H_\ell) = \frac{1}{2 + \frac{\bar{E}_c}{N_0}} \quad ; \quad i \neq \ell \tag{4}$$

which is the symbol (chip) error probability for $M = 2$. For general M we have:

$$P_s = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1 + \frac{\bar{E}_c}{N_0}\right)} \tag{5}$$

Next we introduce a set of parameters indexed by the L sub-bands. The received signal at the ℓ^{th} sub-band is:

$$A_\ell \cos(\omega_{\ell i} t + \theta_{\ell i}) \quad ; \quad \ell=1,\ldots,L \quad ; \quad i=1,\ldots,M \tag{6}$$

where A_ℓ is a Rayleigh distributed random variable:

$$P_{A_\ell}(a) = \frac{a}{\sigma_\ell^2} \exp\left\{-\frac{a^2}{2\sigma_\ell^2}\right\} \quad ; \quad \ell=1,\ldots,L \tag{7}$$

which implies:

$$E\left\{A_\ell^2\right\} = 2\sigma_\ell^2 \tag{8}$$

and

$$P_{\theta_{\ell i}}(\alpha) = \begin{cases} \frac{1}{2\pi} & ; \quad 0 \leq \alpha < 2\pi \\ 0 & ; \quad \text{elsewhere} \end{cases} \quad ; \quad \ell=1,\ldots,L \quad i=1,\ldots,M \tag{9}$$

Hence, the average received energy per chip, when the ℓ^{th} sub-band is used is:

$$\bar{E}_\ell = \sigma_\ell^2 T_c \tag{10}$$

when T_c is the chip duration. The noise distribution is given by:

$$\underline{N} = (N_1, N_2, \ldots, N_L)$$

where N_k is the single-sided spectral density for the k^{th} sub-band.

The jammer distributes total power $\underline{J}$ over the L sub-bands with distribution:

$$\underline{J} = (J_1, J_2, \ldots, J_L)$$

where

$$\sum_{\ell=1}^{L} J_\ell = J. \tag{11}$$

The jammer's propagation loss, c_ℓ, also depends on ℓ. Hence, the contribution of the jammer to the noise power density of the ℓ^{th} sub-band, denoted $N_{J\ell}$ is

$$N_{J\ell} \triangleq J_\ell \cdot c_\ell \ . \tag{12}$$

Whether pulsed jamming or partial-band jammer is assumed, some received chips will be hit by the jammer and some will not.

The binary sequence:

$$\underline{Z} = (Z_1, \ldots, Z_m)$$

where

$$Z_i = \begin{cases} 0 & , \quad \text{the } i^{th} \text{ chip is not hit} \\ 1 & , \quad \text{the } i^{th} \text{ chip is hit by jammer} \end{cases}$$

specifies the jammed chips.

When hopping across the band, the hopping pattern will be defined by the vector $\underline{L}$:

$$\underline{L} = (\ell_1, \ell_2, \ldots, \ell_m)$$

where

$$\ell_j \epsilon \{1, 2, \ldots, L\} \quad ; \quad j = 1, \ldots, m.$$

i.e., ℓ_j specifies the sub-band used for the j^{th} chip. Note that for any transmitted sequence $\underline{x}$, consisting of m MFSK chips, and denoted:

$$\underline{x} = (x^{(1)}, x^{(2)}, \ldots, x^{m})$$

there will be m sets of energy detector outputs:

$$\underline{y} = (\underline{y}^{(1)}, \underline{y}^{(2)}, \ldots, \underline{y}^{(m)})$$

where

$$\underline{y}^{(n)} = (y_1^{(n)}, y_2^{(n)}, \ldots, y_M^{(n)}) \quad ; \quad n=1,\ldots,m$$

and $y_k^{(n)}$ is the k^{th} detector output at the end of the n^{th} chip time.

III. UNIFORM CHANNELS WITH NEGLIGIBLE BACKGROUND NOISE

A. Soft Decision Receiver with J.S.I.

Our basic assumption is that the receiver is capable of detecting $\underline{Z}$ and is using it in the decoding process. Let J denote, in this uniform case, the total received jammer power, and let:

$$N_J \triangleq \frac{J}{W} . \tag{13}$$

Then, the conditional density function of $\underline{y}$ given $\underline{x}$ and $\underline{Z}$ is:

$$P_{mM}(\underline{y}/\underline{x},\underline{Z}) = \prod_{n=1}^{m} P_M(\underline{y}^{(n)}/x^{(n)}, Z_n) \tag{14}$$

and

$$P_M(\underline{y}^{(n)}/x^{(n)}, Z_n) = \prod_{k=1}^{M} P(y_k^{(n)}/x^{(n)}, Z_n) \tag{15}$$

where

$$P(y_k^{(n)}/x^{(n)}, Z_n=1) = \begin{cases} \dfrac{2y_k^{(n)}}{N_J/\rho+\bar{E}_c} \exp\left\{-\dfrac{{y_k^{(n)}}^2}{N_J/\rho+\bar{E}_c}\right\} & ; \; x^{(n)}=k \\[2ex] \dfrac{2y_k^{(n)}}{N_J/\rho} \exp\left\{-\dfrac{{y_k^{(n)}}^2}{N_J/\rho}\right\} & ; \; x^{(n)}\neq k \end{cases} \tag{16}$$

and

$$P(y_k^{(n)}/x^{(n)}, Z_n=0) = \begin{cases} \dfrac{2y_k^{(n)}}{\bar{E}_c} \exp\left\{-\dfrac{{y_k^{(n)}}^2}{\bar{E}_c}\right\} & ; \; x^{(n)}=k \\[2ex] \delta\left(y_k^{(n)}\right) & ; \; x^{(n)}\neq k. \end{cases} \tag{17}$$

Next define:

$$G(\underline{y},\underline{Z}) = \prod_{\substack{n=1 \\ n:Z_n=1}}^{m} \frac{N_J/\rho}{N_J/\rho+\bar{E}_c} \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_J/\rho} \exp\left\{-\frac{{y_k^{(n)}}^2}{N_J/\rho}\right\} \tag{18}$$

$$\Delta(\underline{y};\underline{x},\underline{Z}) \triangleq \sum_{\substack{n=1 \\ n:Z_n=1}}^{m} \frac{\bar{E}_c}{N_J/\rho(N_J/\rho+\bar{E}_c)} \, {y_{x^{(n)}}^{(n)}}^2 = \tag{19}$$

$$= \sum_{n=1}^{m} Z_n a \, {y_{x^{(n)}}^{(n)}}^2 \tag{20}$$

where

$$a \triangleq \frac{1}{N_J/\rho} \; \frac{\dfrac{\bar{E}_c}{N_J/\rho}}{1 + \dfrac{\bar{E}_c}{N_J/\rho}} \tag{21}$$

Also let

$$F(\underline{y};\underline{x},\underline{Z}) \triangleq \prod_{\substack{n=1 \\ n:Z_n=0}}^{m} \frac{2y_{x^{(n)}}^{(n)}}{\bar{E}_c} \exp\left\{- \frac{{y_{x^{(n)}}^{(n)}}^2}{\bar{E}_c}\right\} \prod_{\substack{k=1 \\ k\neq x^{(n)}}}^{M} \delta\left(y_k^{(n)}\right) \tag{22}$$

Then

$$P_{mM}(\underline{y}/\underline{x},\underline{Z}) = G(\underline{y},\underline{Z}) \exp\{\Delta(\underline{y};\underline{x},Z)\} \cdot F(\underline{y};\underline{x},\underline{Z}). \tag{23}$$

The maximum likelihood (M.L.) receiver uses the total metric:

$$m(\underline{y};\underline{x}/\underline{Z}) = \ln P_{mM}(\underline{y}/\underline{x},\underline{Z}) = \tag{24}$$

$$= \ln G(\underline{y},\underline{Z}) + \Delta(\underline{y};\underline{x},\underline{Z}) + \ln F(\underline{y};\underline{x},\underline{Z}). \tag{25}$$

But $G(\underline{y},\underline{Z})$ does not depend on $\underline{x}$. Furthermore, for all $\hat{\underline{x}}$ such that $\hat{\underline{x}}^{(n)} \neq x^{(n)}$:

$$\ln\left[\frac{2y_{\hat{\underline{x}}^{(n)}}^{(n)}}{\bar{E}_c} \exp\left\{- \frac{{y_{\hat{\underline{x}}^{(n)}}^{(n)}}^2}{\bar{E}_c}\right\} \prod_{\substack{k=1 \\ k\neq\hat{\underline{x}}^{(n)}}}^{M} \delta\left(y_k^{(n)}\right)\right] = -\infty.$$

Hence, it suffices to compute:

$$\bar{m}(\underline{y};\underline{x}/\underline{Z}) = \sum_{n=1}^{m} Z_n a \, {y_{x^{(n)}}^{(n)}}^2 + \sum_{\substack{n=1 \\ n:Z_n=0}}^{m} \ln \prod_{\substack{k=1 \\ k\neq x^{(n)}}}^{M} \delta\left(y_k^{(n)}\right) \tag{26}$$

for every sequence $\underline{x} \epsilon C$ to determine the maximum likelihood sequence.

To find the performance of the receiver, we start with the Chernoff bound:

$$P(\underline{x} \to \hat{\underline{x}}) \leq E\left[\exp\left\{\lambda \sum_{n=1}^{m} [m(\underline{y}^{(n)};\hat{x}^{(n)}/Z_n) - m(\underline{y}^{(n)};x^{(n)}/Z_n)]\right\} \Big/ \underline{x}\right] =$$

$$= \prod_{n=1}^{m} E\left[\exp\left\{\lambda[m(\underline{y}^{(n)};\hat{x}^{(n)}/Z_n) - m(\underline{y}^{(n)};x^{(n)}/Z_n)]\right\} \Big/ x^{(n)}\right] =$$

$$= \prod_{n=1}^{m} D(\hat{x}^{(n)},x^{(n)};\lambda) \tag{27}$$

where

$$D(\hat{x}^{(n)},x^{(n)};\lambda) \triangleq E[\exp\{\lambda[m(\underline{y}^{(n)};\hat{x}^{(n)}/Z_n) - m(\underline{y}^{(n)};x^{(n)}/Z_n)]\}/x^{(n)}]. \tag{28}$$

Since the M.L. receiver uses the metric:

$$m(\underline{y}^{(n)};x^{(n)}/Z_n) = \ln P_M(\underline{y}^{(n)}/x^{(n)},Z_n) \tag{29}$$

we obtain

$$D(\hat{x}^{(n)},x^{(n)};\lambda) = E[\exp\{\lambda[\ln P_M(\underline{y}^{(n)}/\hat{x}^{(n)},Z_n) - \ln P_M(\underline{y}^{(n)}/x^{(n)},Z_n)]\}/x^{(n)} =$$

$$= E\left\{\left[\frac{P_M(\underline{y}^{(n)}/\hat{x}^{(n)},Z_n)}{P_M(\underline{y}^{(n)}/x^{(n)},Z_n)}\right]^{\lambda} \Big/ x^{(n)}\right\} =$$

$$= E\left\{E\left\{\left[\frac{P_M(\underline{y}^{(n)}/\hat{x}^{(n)},Z_n)}{P_M(\underline{y}^{(n)}/x^{(n)},Z_n)}\right]^{\lambda} \Big/ x^{(n)},Z_n\right\} \Big/ x^n\right\} =$$

$$= E\left\{\int_0^{\infty}\cdots\int_0^{\infty} \frac{P_M(\underline{y}^{(n)}/\hat{x}^{(n)},Z_n)^{\lambda}}{P_M(\underline{y}^{(n)}/x^{(n)},Z_n)^{\lambda}} \; P_M(\underline{y}^{(n)}/x^{(n)},Z_n)dy_1^{(n)}\cdots dy_M^{(n)} \Big/ x^{(n)}\right\} \tag{30}$$

The value of λ which minimizes this bound is $\lambda = \frac{1}{2}$.

Hence:

$$\min_{0<\lambda} D(\hat{x}^{(n)},x^{(n)};\lambda) = D\left(\hat{x}^{(n)},x^{(n)};\tfrac{1}{2}\right) =$$

$$= E\left\{\int_0^{\infty}\cdots\int_0^{\infty}\sqrt{P(\underline{y}^{(n)}/\hat{x}^{(n)},Z_n)P(\underline{y}^{(n)}/x^{(n)},Z_n)}\quad dy_1^{(n)}\ldots dy_M^{(n)}\Big/\,x^{(n)}\right\} \tag{31}$$

which is the Bhattacharyya bound.

But since

$$P_{Z_n}(k)=\begin{cases}\rho & ; \quad k=1\\ 1-\rho & ; \quad k=0\end{cases}$$

this can be further reduced to:

$$D\left(\hat{x}^{(n)},x^{(n)};\tfrac{1}{2}\right)=\begin{cases}1 & ; \quad x^{(n)}=\hat{x}^{(n)}\\ D & ; \quad x^{(n)}\neq x^{(n)}\end{cases} \tag{32}$$

where D =

$$= (1-\rho)\int_0^{\infty}\cdots\int_0^{\infty}\sqrt{P(\underline{y}^{(n)}/\hat{x}^{(n)}0)P(\underline{y}^{(n)}/x^{(n)},0)}\quad dy_1^{(n)}\ldots dy_M^{(n)}\;+$$

$$+\,\rho\int_0^{\infty}\cdots\int_0^{\infty}\sqrt{P(\underline{y}^{(n)}/\hat{x}^{(n)}1)P(\underline{y}^{(n)}/\hat{x}^{(n)},1)}\quad dy_1^{(n)}\ldots dy_M^{(n)} \tag{33}$$

Substituting the conditional probabilities that we have in this case, we obtain:

D =

$$= (1-\rho)\int_0^{\infty}\cdots\int_0^{\infty}\sqrt{\frac{4y^{(n)}_{x^{(n)}}y^{(n)}_{\hat{x}^{(n)}}}{\bar{\bar{E}}_c^2}\exp\left\{-\frac{{y^{(n)}_{x^{(n)}}}^2}{\bar{\bar{E}}_c}-\frac{{y^{(n)}_{\hat{x}^{(n)}}}^2}{\bar{\bar{E}}_c}\right\}}\;\times$$

$$\sqrt{\left[\prod_{\substack{k=1\\ k\neq x^{(n)}}}^{M}\delta\left(y_k^{(n)}\right)\right]\left[\prod_{\substack{k=1\\ k\neq \hat{x}^{(n)}}}^{M}\delta\left(y_k^{(n)}\right)\right]}\;dy_1^{(n)}\ldots dy_M^{(n)}\;+$$

$$+ \rho \int \cdots \int \sqrt{\frac{4y^{(n)}_{x^{(n)}}}{(\bar{E}_c+N_J/\rho)^2} \exp\left\{-\frac{y^{(n)2}_{x^{(n)}}+y^{(n)2}_{x^{(n)}}}{\bar{E}_c+N_J/\rho}\right\}\left[\prod_{\substack{k=1\\k\neq x^{(n)}}}^{M} \frac{2y_k^{(n)}}{N_J/\rho} \exp\left\{-\frac{y_k^{(n)2}}{N_J/\rho}\right\}\right]} \times$$

$$\sqrt{\left[\prod_{\substack{j=1\\j\neq \hat{x}^{(n)}}}^{M} \frac{2y_j^{(n)}}{N_J/\rho} \exp\left\{-\frac{y_j^{(n)2}}{N_J/\rho}\right\}\right]}\, dy_1^{(n)} \ldots dy_M^{(n)} \qquad (34)$$

The first integral is clearly zero. The second can be integrated over $y_i^{(n)}$

$$i=1,\ldots,M \quad ; \quad i\neq x^{(n)} \quad ; \quad i=\hat{x}^{(n)}$$

and finally reduces to:

$$\rho\left\{\int_0^\infty \frac{2y}{\sqrt{\frac{N_J}{\rho}\left(\bar{E}_c+\frac{N_J}{\rho}\right)}} \exp\left\{-\frac{y^2}{2}\left(\frac{1}{N_J/\rho+\bar{E}_c}+\frac{1}{N_J/\rho}\right)\right\} dy\right\}^2 =$$

$$= \rho \frac{4N_J/\rho(N_J/\rho+\bar{E}_c)}{(2N_J/\rho+\bar{E}_c)^2} = \rho \frac{4\left(1+\frac{\bar{E}_c}{N_J/\rho}\right)}{\left(2+\frac{\bar{E}_c}{N_J/\rho}\right)^2} \qquad (35)$$

Therefore

$$P(\underline{x} \to \hat{x}) \leq D^{W(\underline{x};\hat{\underline{x}})} =$$

$$= \left[\frac{4\rho\left(1+\frac{\bar{E}_c}{N_J/\rho}\right)}{\left(2+\frac{\bar{E}_c}{N_J/\rho}\right)^2}\right]^{W(\underline{x};\hat{\underline{x}})} \qquad (36)$$

It is easily verified that $\rho = 1$ maximizes this bound, i.e., continuous jamming (broadband-jamming) is the worst case jamming for this receiver.

Or

$$D_{Wc} = \max_{0<\rho\leq 1} D = \frac{4\left(1+\frac{\bar{E}_c}{N_J}\right)}{\left(2+\frac{\bar{E}_c}{N_J}\right)^2} \qquad (37)$$

B. Soft Decision Receiver with no J.S.I.

Since we have just seen that continuous jamming is the "worst case jamming" for the receiver analyzed in A, and since the receiver we presently consider does not have J.S.I., we are tempted to try the simple metric:

$$m(\underline{y};\underline{x}) = \sum_{n=1}^{n} y_{x^{(n)}}^{(n)^2} \tag{38}$$

which is just an equal weight summation of the energy detector outputs, which correspond to a sequence $\underline{x}$.

Hence

$$P(\underline{x} \to \hat{\underline{x}}) \leq E\left[\exp\left\{\lambda \sum_{n=1}^{m}\left(y_{\hat{x}^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)^2}\right)\right\} \Big/ \underline{x}\right] =$$

$$= \prod_{n=1}^{m} E\left[\exp\left\{\lambda\left(y_{\hat{x}^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)^2}\right)\right\} \Big/ x^{(n)}\right] \tag{39}$$

$$\therefore \quad D(\hat{x}^{(n)},x^{(n)};\lambda) = E\left[\exp\left\{\lambda\left(y_{\hat{x}^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)}\right) \Big/ x^{(n)}\right]\right. = \tag{40}$$

$$= (1-\rho)E\left[\exp\left\{\lambda\left(y_{\hat{x}^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)^2}\right)\right\} \Big/ x^{(n)}, Z_n=0\right] + \rho E\left[\exp\left\{\lambda\left(y_{\hat{x}^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)^2}\right)\right\} \Big/ x^{(n)}, Z_n=1\right].$$

But given $x^{(n)}$, $\hat{x}^{(n)} \neq x^{(n)}$ and Z_n ; $y_{\hat{x}^{(n)}}^{(n)}$ & $y_{x^{(n)}}^{(n)}$ are statistically independent.

Hence; for $\hat{x}^{(n)} \neq x^{(n)}$,

$$D(\hat{x}^{(n)},x^{(n)};\lambda) =$$

$$= (1-\rho)\ E\left[\exp\left\{\lambda y_{\hat{x}^{(n)}}^{(n)^2}\right\} \Big/ x^{(n)}, Z_n=0\right] \cdot E\left[\exp\left\{-\ \lambda y_{x^{(n)}}^{(n)^2}\right\} \Big/ x^{(n)}, Z_n=0\right] +$$

$$+ \rho E\left[\exp\left\{\lambda y_{\hat{x}^{(n)}}^{(n)^2}\right\} \Big/ x^{(n)}, Z_n=1\right] \cdot E\left[\exp\left\{-\ \lambda y_{x^{(n)}}^{(n)^2}\right\} \Big/ x^{(n)}, Z_n=1\right]. \tag{41}$$

We now use the fact that for a Gaussian random variable $x \sim N\left(0,\sigma_x^2\right)$:

$$E[\exp\{\lambda x^2\}] = \frac{1}{\sqrt{1-2\lambda\sigma_x^2}} \quad ; \quad \lambda < \frac{1}{2\sigma_x^2}$$

$$E[\exp\{-\lambda x^2\}] = \frac{1}{\sqrt{1+2\lambda\sigma_x^2}} \quad ; \quad \lambda > -\frac{1}{2\sigma_x^2} .$$

Since

$$y_k^{(n)^2} = r_{ck}^{(n)^2} + r_{sk}^{(n)^2}$$

$r_{ck}^{(n)}$ & $r_{sk}^{(n)}$ being Gaussian and statistically independent (see Figure 3), we obtain:

$$E\left[\exp\left\{\lambda y_k^{(n)^2}\right\}\right] = E\left[\exp\left\{\lambda\left(r_{ck}^{(n)^2} + r_{sk}^{(n)^2}\right)\right\}\right] =$$

$$= E\left[\exp\left\{\lambda r_{ck}^{(n)^2}\right\}\right] \cdot E\left[\exp\left\{\lambda r_{sk}^{(n)^2}\right\}\right] \tag{42}$$

$$\therefore D(\hat{x}^{(n)},x^{(n)};\lambda) =$$

$$= \frac{1-\rho}{(1-\lambda\bar{E}_c} + \frac{\rho}{(1-\lambda N_J/\rho)[1+\lambda(N_J/\rho+\bar{E}_c)]} = D(\lambda) \tag{43}$$

$$0 < \lambda < \frac{\rho}{N_J}$$

Now we want to find the worst case D by taking the maximum over ρ, $0 < \rho \leq 1$ and the minimum over λ: $0 < \lambda < \frac{\rho}{N_J}$

i.e.,

$$D_{Wc} = \max_{0<\rho\leq 1} \min_{0<\lambda<\frac{\rho}{N_J}} D(\lambda). \tag{44}$$

Note, however, that the condition:

$$\lambda < \frac{\rho}{N_J}$$

implies that for any allowable value of λ, the first term, namely:

$$\frac{1-\rho}{1+\lambda\bar{E}_c}$$

approaches 1 as $\rho \to 0$, and therefore,

$$\min_{0<\lambda<\frac{\rho}{N_J}} D(\lambda) \xrightarrow{\rho \to 0} 1 \tag{45}$$

We expect that in general, the soft decision receiver using this metric has poor performance under a low duty cycle jammer. Recall that the same conclusion was stated above for the non-fading channel.

Although the receiver now considered does not have J.S.I., it may still know ρ. In such a case the receiver can use the ML metric. We want now to find this metric and to analyze the performance of a Soft Decision receiver using the ML metric. The conditional probability of $\underline{y}$ given the transmitted sequence $\underline{x}$ is:

$$P_{mM}(\underline{y}/\underline{x}) = \prod_{n=1}^{m} P_M(\underline{y}^{(n)}/x^{(n)}) =$$

$$= \prod_{n=1}^{m}\left[\sum_{k=0}^{1} P_M(\underline{y}^{(n)}/x^{(n)},k)P_{Z_n}(k)\right] =$$

$$= \prod_{n=1}^{m}\left[P_M(\underline{y}^{(n)}/x^{(n)},0)(1-\rho) + P_M(\underline{y}^{(n)}/x^{(n)},1)\rho\right]. \tag{46}$$

But

$$P_M(\underline{y}^{(n)}/x^{(n)},0) = \frac{2y_{x^{(n)}}^{(n)}}{\bar{E}_c} \exp\left\{-\frac{{y_{x^{(n)}}^{(n)}}^2}{\bar{E}_c}\right\} \prod_{\substack{k=1 \\ k\neq x^{(n)}}}^{M} \delta\left(y_k^{(n)}\right) \tag{47}$$

and

$$P_M(\underline{y}^{(n)}/x^{(n)},1) = \frac{2y_{x^{(n)}}^{(n)}}{\bar{E}_c+N_J/\rho} \exp\left\{-\frac{{y_{x^{(n)}}^{(n)}}^2}{N_J/\rho+E_c}\right\} \prod_{\substack{k=1 \\ k\neq x^{(n)}}}^{M} \frac{2y_k^{(n)}}{N_J/\rho} \exp\left\{-\frac{{y_k^{(n)}}^2}{N_J/\rho}\right\} \tag{48}$$

Hence:

$$P_{mM}(\underline{y}/\underline{x}) =$$

$$= \prod_{n=1}^{m}\left[(1-\rho)\,\frac{2y_{x^{(n)}}^{(n)}}{\bar{E}_c}\exp\left\{-\frac{{y_{x^{(n)}}^{(n)}}^2}{\bar{E}_c}\right\}\prod_{\substack{k=1\\k\neq x^{(n)}}}^{M}\delta\left(y_k^{(n)}\right)+\right.$$

$$\left.+\,\rho\,\frac{N_J/\rho}{\bar{E}_c+N_J/\rho}\exp\left\{\frac{{y_{x^{(n)}}^{(n)}}^2\,\bar{E}_c}{N_J/\rho(\bar{E}_c+N_J/\rho)}\right\}\prod_{k=1}^{M}\frac{2y_k^{(n)}}{N_J/\rho}\exp\left\{-\frac{{y_k^{(n)}}^2}{N_J/\rho}\right\}\right] \tag{49}$$

Let

$$G(\underline{y}^{(n)}) \triangleq \prod_{k=1}^{M}\frac{2y_k^{(n)}}{N_J/\rho}\exp\left\{-\frac{{y_k^{(n)}}^2}{N_J/\rho}\right\}$$

Then

$$P_{mM}(\underline{y}/\underline{x}) =$$

$$= \prod_{n=1}^{m}\left[(1-\rho)\,\frac{2y_{x^{(n)}}^{(n)}}{\bar{E}_c}\exp\left\{-\frac{{y_{x^{(n)}}^{(n)}}^2}{\bar{E}_c}\right\}\prod_{\substack{k=1\\k\neq x^{(n)}}}^{M}\delta\left(y_k^{(n)}\right)+\rho\,\frac{N_J/\rho}{N_J/\rho+\bar{E}_c}\,G(\underline{y}^{(n)})\exp\left\{\frac{{y_{x^{(n)}}^{(n)}}^2\,\bar{E}_c}{N_J/\rho(\bar{E}_c+N_J/\rho)}\right\}\right] \tag{50}$$

Now we take:

$$m(\underline{y};\underline{x}) = \ln P_{mM}(\underline{y}/x) =$$

$$= \sum_{n=1}^{m}\ln\left[(1-\rho)\,\frac{2y_{x^{(n)}}^{(n)}}{\bar{E}_c}\exp\left\{-\frac{{y_{x^{(n)}}^{(n)}}^2}{\bar{E}_c}\right\}\prod_{\substack{k=1\\k\neq x^{(n)}}}^{M}\delta\left(y_k^{(n)}\right)+\rho\,\frac{N_J/\rho}{\bar{E}_c+N_J/\rho}\,G(\underline{y}^{(n)})\,\exp\left\{\frac{{y_{x^{(n)}}^{(n)}}^2\,\bar{E}_c}{N_J/\rho(\bar{E}_c+N_J/\rho)}\right\}\right] \tag{51}$$

i.e., the receiver uses the M.L. metric. Therefore, as we have seen above, the Chernoff bound reduces to the Bhattacharyya bound:

$$P_r(\underline{x}\to\hat{\underline{x}}) \le D^{W(\underline{x};\hat{\underline{x}})} \quad ; \quad \text{where}$$

$$D \triangleq \int_0^{\infty}\cdots\int_0^{\infty}\sqrt{P_M(\underline{y}^{(n)}/x^{(n)})P_M(\underline{y}^{(n)}/\hat{x}^{(n)})}\;dy_1^{(n)}\ldots dy_M^{(n)}$$

Substituting $P_M(\underline{y}^{(n)}/x^{(n)})$ & $P_M(y^{(n)})/\hat{x}^{(n)})$ we obtain:

$$= \int_0^\infty \cdots \int_0^\infty \rho \, \frac{N_J/\rho}{N_J/\rho+\bar{E}_c} \, G(\underline{y}^{(n)}) \, \exp\left\{ \frac{y_{x^{(n)}}^{(n)2} + y_{\hat{x}^{(n)}}^{(n)2}}{2} \cdot \frac{\bar{E}_c}{N_J/\rho(N_J/\rho+\bar{E}_c)} \right\} dy_1^{(n)} \cdots dy_M^{(n)} \tag{52}$$

Carrying out the integration over $y_i^{(n)}$, $i=1,\ldots,M$

$i \neq x^{(n)}; \; i \neq \hat{x}^{(n)}$

we get the double integral:

$$D = \rho\left[\int_0^\infty \frac{2y}{\sqrt{N_J/\rho(N_J/\rho+\bar{E}_c)}} \exp\left\{ y^2 \left(-\frac{1}{N_J/\rho} + \frac{\bar{E}_c}{2N_J/\rho(N_J/\rho+\bar{E}_c)} \right) \right\} dy \right]^2 =$$

$$= \rho\left[\int_0^\infty \frac{2y}{\sqrt{N_J/\rho(N_J/\rho+\bar{E}_c)}} \exp\left\{ -y^2 \, \frac{\bar{E}_c+2N_J/\rho}{2N_J/\rho(N_J/\rho+\bar{E}_c)} \right\} dy \right]^2$$

$$= \rho \, \frac{4\left(1 + \frac{\bar{E}_c}{N_J/\rho}\right)}{\left(2 + \frac{\bar{E}_c}{N_J/\rho}\right)^2} \tag{53}$$

This is exactly the value of D obtained above for the Soft Decision receiver having J.S.I. We conclude that a Soft Decision receiver with no J.S.I., but knowing ρ, performs exactly as well as a Soft Decision receiver having J.S.I., provided the background noise is negligible.

As before, the worst case ρ is $\rho = 1$, in which case:

$$\max_{0<\rho<1} D = \frac{4\left(1 + \frac{\bar{E}_c}{N_J}\right)}{\left(2 + \frac{\bar{E}_c}{N_J}\right)^2} \tag{54}$$

C. Hard Decision receiver with J.S.I.

The input and output alphabets of the channel are:

$$x = Y\epsilon\{1,2,\ldots,M\} \tag{55}$$

and the conditional probability

$$P(y/x,Z) = \begin{cases} 1 & ; \quad y = x \quad ; \quad Z = 0 \\ 0 & ; \quad y \neq x \quad ; \quad Z = 0 \\ 1-\epsilon & ; \quad y = x \quad ; \quad Z = 1 \\ \frac{\epsilon}{M-1} & ; \quad y \neq x \quad ; \quad Z = 1 \end{cases} \tag{56}$$

where

$$\epsilon(\rho) = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1 + \frac{\rho\bar{E}_c}{N_J}\right)}. \tag{57}$$

Since the channel is memoryless

$$P_m(\underline{y}/\underline{x},\underline{Z}) = \prod_{n=1}^{m} P(y_n/x_n,Z_n).$$

We choose the metric

$$m(y_n;x_n/Z_n) = \ln P(y_n/x_n,Z_n)$$

which is the M.L. metric.

Hence, the Chernoff bound reduces to the Bhattacharyya bound:

$$P(\underline{x} \to \hat{x}) \leq D^{W(\underline{x};\underline{\hat{x}})}$$

where

$$D \triangleq E\left\{\sum_y \sqrt{P(y_n/\hat{x}_n,Z_n)P(y_n/x_n,Z_n)} \Big/ x_n\right\} =$$

$$= (1-\rho)\sum_y \sqrt{P(y_n/\hat{x}_n,0)P(y_n/x_n,0)} + \rho \sum_y \sqrt{P(y_n/\hat{x}_n,1)P(y_n/x_n,1)} \qquad \hat{x}_n \neq x_n. \tag{58}$$

But, for $\hat{x}_n \neq x_n$,

$$P(y_n/x_n,0) \neq 0 \Rightarrow P(y_n/\hat{x}_n,0) = 0.$$

$$\therefore\ D = \rho \sum_{y_n} \sqrt{P(y_n/\hat{x}_n,1)P(y_n/x_n,1)} =$$

$$= \rho\left[2\sqrt{\frac{[1-\varepsilon(\rho)]\varepsilon(\rho)}{M-1}} + \frac{M-2}{M-1}\,\varepsilon(\rho)\right] \qquad (59)$$

It is easy to verify that $\rho = 1$ maximizes this bound, i.e., continuous jamming/broadband jamming is the worst case jamming for this receiver also. Substituting $\rho = 1$ in the bound above, we obtain:

$$D_{Wc} = 2\sqrt{\frac{\varepsilon(1-\varepsilon)}{M-1}} + \frac{M-2}{M-1}\,\varepsilon \qquad (60)$$

where ε denotes $\varepsilon(1)$.

D. Hard Decision Receiver With No J.S.I.

The input and output alphabets are:

$$x = Y\epsilon\{1,2,\ldots,M\}$$

but the conditional probability function is now:

$$P(y/x) = \begin{cases} 1 - \rho\varepsilon & ; \quad y = x \\ \dfrac{\rho\varepsilon}{M-1} & ; \quad y \neq x \end{cases} \qquad (61)$$

where

$$\varepsilon(\rho) = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1 + \dfrac{\rho\bar{E}_c}{N_J}\right)}$$

In this case we use the simple metric

$$m(y_n;x_n) = -\,W(y_n,x_n). \qquad (62)$$

This is in fact the M.L. metric, since it can be written in the form

$$m(y_n;x_n) = a\ln P(y_n/x_n) + b; \quad a > 0$$

hence, we can use the Bhattacharyya bound

$$P\,(\underline{x} \to \hat{\underline{x}}) \leq D^{W(\underline{x};\hat{x})}$$

where

$$D = \sum_{y} \sqrt{P(y/x_n)P(y/\hat{x}_n)} = \quad ; \quad x_n \neq \hat{x}_n$$

$$= 2\sqrt{\frac{\rho\varepsilon(\rho)[1-\rho\varepsilon(\rho)]}{M-1}} + \rho\frac{M-2}{M-1}\varepsilon(\rho) \tag{63}$$

It is easily verified that $\rho = 1$ maximizes D for this receiver also i.e.,

$$D_{Wc} = \max_{0<\rho\leq 1} D = 2\sqrt{\frac{\varepsilon(1-\varepsilon)}{M-1}} + \frac{M-2}{M-1}\varepsilon \tag{64}$$

where

$$\varepsilon \triangleq \varepsilon(1).$$

Clearly, the results of receivers C & D should coincide for $\rho = 1$, and indeed, we see that the corresponding bounds are also the same.

IV. NON-UNIFORM CHANNELS

In this section we assume the following: The jammer uses pulsed noise (duty cycle ρ) and non-uniform distribution over the "slotted channel." At the j^{th} sub-band we have background noise of one-sided spectral density N_j, and, in addition, jammer generated noise denoted N_{Jj}/ρ when the jammer is "on."

As defined earlier, the hopping sequence is: $\underline{L} = (\ell_1, \ell_2, \ldots, \ell_m)$, where ℓ_n is the index of the sub-band used in the n^{th} chip time, $\ell_n \epsilon \{1,2,\ldots,N\}$. We assume that the random variables ℓ_n, $n=1,\ldots,m$ are statistically independent discrete random variable, having the common probability distribution $P(\ell)$.

A. Soft Decision Receiver with J.S.I.

The conditional density function of $\underline{y}$, given $\underline{x}$, $\underline{L}$ and $\underline{Z}$ is:

$$P_{mM}(\underline{y}/\underline{x},\underline{L},\underline{Z}) = \prod_{n=1}^{m} P_M(\underline{y}^{(n)}/x^{(n)},\ell_n Z_n) \tag{65}$$

and

$$P_M(\underline{y}^{(n)}/x^{(n)},\ell_n,Z_n) = \prod_{k=1}^{M} P(y_k^{(n)}/x^{(n)},\ell_n,Z_n) \tag{66}$$

where

$$P(y_k^{(n)}/x^{(n)},\ell_n,Z_n=1) = \begin{cases} \dfrac{2y_k^{(n)}}{N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho}\right\}; & x^{(n)}=k \\ \dfrac{2y_k^{(n)}}{N_{\ell_n}+N_{J\ell_n}/\rho} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{\ell_n}+N_{J\ell_n}/\rho}\right\}; & x^{(n)}\neq k \end{cases} \tag{67}$$

and

$$P(y_k^{(n)}/x^{(n)},\ell_n,Z_n=0) = \begin{cases} \dfrac{2y_k^{(n)}}{N_{\ell_n}+\bar{E}_{\ell_n}} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{\ell_n}+\bar{E}_{\ell_n}}\right\}; & x^{(n)}=k \\ \dfrac{2y_k^{(n)}}{N_{\ell_n}} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{\ell_n}}\right\}; & x^{(n)}\neq k \end{cases} \tag{68}$$

Next defining:

$$G(\underline{y},\underline{L},\underline{Z}) \triangleq \prod_{\substack{n=1\\ n:Z_n=1}}^{m} \frac{N_{\ell_n}+N_{J\ell_n}/\rho}{N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho} \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_{\ell_n}+N_{J\ell_n}/\rho} \exp\left\{-\frac{y_k^{(n)^2}}{N_{\ell_n}+N_{J\ell_n}/\rho}\right\}$$

$$\Delta_1(\underline{y};\underline{x},\underline{L},\underline{Z}) \triangleq \sum_{\substack{n=1\\ n:Z_n=1}}^{m} \frac{\bar{E}_{\ell_n}}{\left(N_{\ell_n}+N_{J\ell_n}/\rho\right)\left(N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho\right)} y_{x^{(n)}}^{(n)^2} =$$

$$= \sum_{n=1}^{m} Z_n a_{\ell_n} y_{x^{(n)}}^{(n)^2} \tag{69}$$

where

$$a_{\ell_n} \triangleq \frac{\bar{E}_{\ell_n}}{\left(N_{\ell_n}+N_{J\ell_n}/\rho\right)\left(N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho\right)} \tag{70}$$

Also, let

$$F(\underline{y},\underline{L},\underline{Z}) = \prod_{\substack{n=1\\ n:Z_n=0}}^{m} \frac{N_{\ell_n}}{N_{\ell_n}+\bar{E}_{\ell_n}} \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_{\ell_n}} \exp\left\{-\frac{y_k^{(n)^2}}{N_{\ell_n}}\right\}$$

and

$$\Delta_0(\underline{y};\underline{x},\underline{L},\underline{Z}) = \sum_{\substack{n=1 \\ n:Z_n=0}}^{m} \frac{\bar{E}_{\ell_n}/N_{\ell_n}}{N_{\ell_n}+\bar{E}_{\ell_n}} \, y_{x^{(n)}}^{(n)^2} =$$

$$= \sum_{n=1}^{m} (1-Z_n) b_{\ell_n} y_{x^{(n)}}^{(n)^2} \tag{71}$$

where

$$b_{\ell_n} \triangleq \frac{\bar{E}_{\ell_n}/N_{\ell_n}}{N_{\ell_n}+\bar{E}_{\ell_n}} \tag{72}$$

Then

$$P_{mM}(\underline{y}/\underline{x},\underline{L},\underline{Z}) = G(\underline{y},\underline{L},\underline{Z})e^{\Delta_1(\underline{y};\underline{x},\underline{L},\underline{Z})} \; F(\underline{y},\underline{L},\underline{Z})e^{\Delta_0(\underline{y};\underline{x},\underline{L},\underline{Z})} \tag{73}$$

The ML receiver uses the total metric:

$$m(\underline{y};\underline{x}/\underline{L},\underline{Z}) = \ell n \; P_{mM}(\underline{y}/\underline{x},\underline{L},\underline{Z}) = \tag{74}$$

$$= \ell n \; G(\underline{y},\underline{L},\underline{Z}) + \Delta_1(\underline{y};\underline{x},\underline{L},\underline{Z}) + \ell n \; F(\underline{y},\underline{L},\underline{Z}) + \Delta_0(\underline{y},\underline{x},\underline{L},\underline{Z}).$$

But, since $G(\underline{y},\underline{L},\underline{Z})$ and $F(\underline{y},\underline{L},\underline{Z})$ do not depend on $\underline{x}$, it suffices to compute

$$\bar{m}(\underline{y};\underline{x}/\underline{L},\underline{Z}) = \Delta_1(\underline{y};\underline{x},\underline{L},\underline{Z}) + \Delta_0(\underline{y};\underline{x},\underline{L},\underline{Z}) =$$

$$= \sum_{n=1}^{m} \left[Z_n a_{\ell_n} + (1-Z_n) b_{\ell_n} \right] y_{x^{(n)}}^{(n)^2}$$

for each sequence $\underline{x} \epsilon C$ to determine the maximum likelihood sequence.

Again we use the Bhattacharyya bound to compute the performance of this receiver

$$P(\underline{x} \rightarrow \hat{x}) \leq D^{W(\underline{x};\hat{x})}$$

where

$$D \triangleq E\left\{ \int_0^\infty \cdots \int_0^\infty \sqrt{P\left(\underline{y}^{(n)}/x^{(n)},\ell_n,Z_n\right) P\left(\underline{y}^{(n)}/\hat{x}^{(n)},\ell_n,Z_n\right)} \; dy_1^{(n)} \cdots dy_M^{(n)} \Big/ x^{(n)} \right\}$$

$$= (1-\varepsilon) \; E\left\{ \int_0^\infty \cdots \int_0^\infty \sqrt{P\left(\underline{y}^{(n)}/x^{(n)},\ell_n,0\right) P\left(\underline{y}^{(n)}/\hat{x}^{(n)},\ell_n,0\right)} \;\; dy_1^{(n)} \cdots dy_M^{(n)} \Big/ x^{(n)} \right\} +$$

$$+\rho E\left\{\int_0^\infty\cdots\int_0^\infty\sqrt{P\left(\underline{y}^{(n)}/x^{(n)},\ell_n,1\right)P\left(\underline{y}^{(n)}/\hat{x}^{(n)},\ell_n,1\right)}\quad dy_1^{(n)}\ldots dy_M^{(n)}\Big/x^{(n)}\right\} \quad (76)$$

Substituting the corresponding conditional probabilities and integrating over $y_i^{(n)}$,

$$i=1,\ldots,M \quad ; \quad i\neq x^{(n)} \quad , \quad i\neq\hat{x}^{(n)}$$

we are left with

$$D=(1-\rho)\;E\left\{\left[\int_0^\infty\frac{2y_{x^{(n)}}^{(n)}}{\sqrt{N_{\ell_n}\left(N_{\ell_n}+\bar{E}_{\ell_n}\right)}}\exp\left\{-\frac{y_{x^{(n)}}^{(n)2}}{2}\left(\frac{1}{N_{\ell_n}}+\frac{1}{N_{\ell_n}+\bar{E}_{\ell_n}}\right)\right\}dy_{x^{(n)}}^{(n)}\right]^2\Big/x^{(n)}\right\}+$$

$$+\rho E\left\{\left[\int_0^\infty\frac{2y_{x^{(n)}}^{(n)}\quad dy_{x^{(n)}}^{(n)}}{\sqrt{\left(N_{\ell_n}+N_{J\ell_n}/\rho\right)\left(N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho\right)}}\exp\left\{-\frac{y_{x^{(n)}}^{(n)2}}{2}\left(\frac{1}{N_{\ell_n}+N_{J\ell_n}/\rho}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.+\frac{1}{N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho}\right)\right\}\right]^2\Big/x^{(n)}\right\} \quad (77)$$

$$=(1-\rho)E\left\{\left[\int_0^\infty\frac{2y\;dy}{\sqrt{N_{\ell_n}\left(N_{\ell_n}+\bar{E}_{\ell_n}\right)}}\exp\left\{-y^2\frac{N_{\ell_n}+\bar{E}_{\ell_n}/2}{N_{\ell_n}\left(N_{\ell_n}+\bar{E}_{\ell_n}\right)}\right\}\right]^2\Big/x^{(n)}\right\}+$$

$$+\rho E\left\{\left[\int_0^\infty\frac{2y\;dy}{\sqrt{\left(N_{\ell_n}+N_{J\ell_n}/\rho\right)\left(N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho\right)}}\exp\left\{-y^2\frac{N_{\ell_n}+\bar{E}_{\ell_n}/2+N_{J\ell_n}/\rho}{\left(N_{\ell_n}+N_{J\ell_n}/\rho\right)\left(N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho\right)}\right\}\right]^2\Big/x^{(n)}\right\}$$

$$=E\left\{(1-\rho)\frac{N_{\ell_n}\left(N_{\ell_n}+\bar{E}_{\ell_n}\right)}{\left(N_{\ell_n}+\bar{E}_{\ell_n}/2\right)^2}+\rho\frac{\left(N_{\ell_n}+N_{J\ell_n}/\rho\right)\left(N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho\right)}{\left(N_{\ell_n}+\bar{E}_{\ell_n}/2+\bar{E}_{J\ell_n}/\rho\right)^2}\Big/x^{(n)}\right\}=$$

$$=\sum_{j=1}^{N}P(j)\left[(1-\rho)\frac{4\left(1+\frac{\bar{E}_j}{N_j}\right)}{\left(2+\frac{\bar{E}_j}{N_j}\right)^2}+\rho\frac{4\left(1+\frac{\bar{E}_j}{N_j+N_{Jj}/\rho}\right)}{\left(2+\frac{\bar{E}_j}{N_j+N_{Jj}/\rho}\right)^2}\right] \quad (78)$$

$$= \sum_{j=1}^{N} P(j)\left[\frac{4\left(1+\frac{\bar{E}_j}{N_j}\right)}{\left(2+\frac{\bar{E}_j}{N_j}\right)^2} + \frac{4\bar{E}_j^2 N_j\left(2N_j+\bar{E}_j+N_{Jj}/\rho\right)}{\left(2N_j+\bar{E}_j\right)^2\left(2N_j+\bar{E}_j+2N_{Jj}/\rho\right)^2}\right]$$

$$= \sum_{j=1}^{N} P(j)\left[\frac{4\left(1+\frac{\bar{E}_j}{N_j}\right)}{\left(2+\frac{\bar{E}_j}{N_j}\right)^2} + \frac{4\bar{E}_j^2 N_{Jj}}{\left(2N_j+\bar{E}_j\right)^3}\,\rho\,\frac{\left(\rho+\frac{N_{Jj}}{2N_j+\bar{E}_j}\right)}{\left(\rho+\frac{2N_{Jj}}{2N_j+\bar{E}_j}\right)^2}\right] \tag{79}$$

It can be easily shown that $\rho = 1$ maximizes this bound over the interval $0 < \rho \leq 1$. Hence, continuous jamming is the worst case jamming in this case also.

$$\therefore D_{Wc} \triangleq \max_{0<\rho\leq 1} D = \sum_{j=1}^{N} P(j)\,\frac{4\left(1+\frac{\bar{E}_j}{N_j+N_{Jj}}\right)}{\left(2+\frac{\bar{E}_j}{N_j+N_{Jj}}\right)^2} \tag{80}$$

B. Soft Decision Receiver with no J.S.I.

The jammer uses pulsed noise (duty cycle ρ) and nonuniform distribution over the slotted channel (Figure 1).

We have shown before that when the background noise is negligible and the channel is uniform, a receiver using the simple total metric:

$$m(\underline{y};\underline{x}) = \sum_{n=1}^{m} {y_{x^{(n)}}^{(n)}}^2$$

results in an unacceptable performance under low duty cycle jamming. Having the choice between this receiver and the Hard Decision receiver, the latter should of course be preferred. This is also the case when the channel is not uniform and the background noise not negligible. Still, as discussed above for the special case, the receiver, not being able to detect the presence of

the jammer for each chip time, may still be able to make a reasonable measurement of ρ. In this case the ML metric can be used. We follow this idea below. The conditional probability of the channel output is now

$$P_{mM}(\underline{y}/\underline{x},\underline{L}) = \prod_{n=1}^{m} P_M\left(\underline{y}^{(n)}/x^{(n)},\ell_n\right) \tag{81}$$

where

$$P_M\left(\underline{y}^{(n)}/x^{(n)},\ell_n\right) = \sum_{z=0}^{1} P_M\left(\underline{y}^{(n)}/x^{(n)},\ell_n,Z_n=z\right) P_{Z_n}(z) =$$

$$= P_M\left(\underline{y}^{(n)}/x^{(n)},\ell_n,Z_n=0\right)(1-\rho) + P_M\left(\underline{y}^{(n)}/x^{(n)},\ell_n,Z_n=1\right)\rho \tag{82}$$

and

$$P_M\left(y_k^{(n)}/x^{(n)},\ell_n,Z_n=0\right) = \begin{cases} \dfrac{2y_k^{(n)}}{N_{\ell_n}+\bar{E}_{\ell_n}} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{\ell_n}+\bar{E}_{\ell_n}}\right\} & ; \quad x^{(n)}=k \\[2ex] \dfrac{2y_k^{(n)}}{N_{\ell_n}} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{\ell_n}}\right\} & ; \quad x^{(n)}\neq k \end{cases} \tag{83}$$

$$P_M\left(y_k^{(n)}/x^{(n)},\ell_n,Z_n=1\right) = \begin{cases} \dfrac{2y_k^{(n)}}{N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho}\right\} & ; \quad x^{(n)}=k \\[2ex] \dfrac{2y_k^{(n)}}{N_{\ell_n}+N_{J\ell_n}/\rho} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{\ell_n}+N_{J\ell_n}/\rho}\right\} & ; \quad x^{(n)}\neq k \end{cases} \tag{84}$$

Hence

$$P_{mM}(\underline{y}/\underline{x},\underline{L}) = \prod_{n=1}^{m} \left[(1-\rho)P_M\left(\underline{y}^{(n)}/x^{(n)},\ell_n,Z_n=0\right)+\rho P_M\left(\underline{y}^{(n)}/x^{(n)},\ell_n,Z_n=1\right)\right]$$

$$= \prod_{n=1}^{m}\left[(1-\rho)\,\frac{2y_{x^{(n)}}^{(n)}}{N_{\ell_n}+\bar{E}_{\ell_n}} \exp\left\{-\frac{y_{x^{(n)}}^{(n)^2}}{N_{\ell_n}+\bar{E}_{\ell_n}}\right\} \prod_{\substack{k=1 \\ k\neq x^{(n)}}}^{M} \frac{2y_k^{(n)}}{N_{\ell_n}} \exp\left\{-\frac{y_k^{(n)^2}}{N_{\ell_n}}\right\} + \right.$$

$$+ \rho \frac{2y_{x^{(n)}}^{(n)}}{N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho} \exp\left\{-\frac{y_{x^{(n)}}^{(n)^2}}{N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho}\right\} \prod_{\substack{k=1 \\ k\neq x^{(n)}}}^{M} \frac{2y_k^{(n)}}{N_{\ell_n}+N_{J\ell_n}/\rho} \exp\left\{-\frac{y_k^{(n)^2}}{N_{\ell_n}+N_{J\ell_n}/\rho}\right\}\Bigg]$$

$$= \prod_{n=1}^{m}\left[(1-\rho)\exp\left\{\frac{y_{x^{(n)}}^{(n)^2}\bar{E}_{\ell_n}}{N_{\ell_n}\left(N_{\ell_n}+\bar{E}_{\ell_n}\right)}\right\} G_{\ell_n}^1\left(\underline{y}^{(n)}\right)+\right.$$

$$\left.+\rho\exp\left\{\frac{y_{x^{(n)}}^{(n)^2}\bar{E}_{\ell_n}}{\left(N_{\ell_n}+N_{J\ell_n}/\rho\right)\left(N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho\right)}\right\} G_{\ell_n}^2\left(\underline{y}^{(n)}\right)\right] \tag{85}$$

where

$$G_{\ell_n}^1\left(\underline{y}^{(n)}\right) \triangleq \frac{N_{\ell_n}}{N_{\ell_n}+\bar{E}_{\ell_n}} \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_{\ell_n}} \exp\left\{-\frac{y_k^{(n)^2}}{N_{\ell_n}}\right\} \tag{86}$$

and

$$G_{\ell_n}^2\left(y^{(n)}\right) \triangleq \frac{N_{\ell_n}+N_{J\ell_n}/\rho}{N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho} \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_{\ell_n}+N_{J\ell_n}/\rho} \exp\left\{-\frac{y_k^{(n)^2}}{N_{\ell_n}+N_{J\ell_n}/\rho}\right\} \tag{87}$$

To use the ML metric we take

$$m(\underline{y};\underline{x}/\underline{L}) = \ln \underset{mM}{P}(\underline{y}/\underline{x},\underline{L}) = \tag{88}$$

$$= \sum_{n=1}^{m} \ln\left[(1-\rho)G_{\ell_n}^1\left(\underline{y}^{(n)}\right)\exp\left\{\frac{y_{x^{(n)}}^{(n)^2}\bar{E}_{\ell_n}}{N_{\ell_n}\left(N_{\ell_n}+\bar{E}_{\ell_n}\right)}\right\}+\right.$$

$$\left.+\rho G_{\ell_n}^2\left(\underline{y}^{(n)}\right)\exp\left\{\frac{y_{x^{(n)}}^{(n)^2}\bar{E}_{\ell_n}}{\left(N_{\ell_n}+\bar{E}_{\ell_n}+N_{J\ell_n}/\rho\right)\left(N_{\ell_n}+N_{J\ell_n}/\rho\right)}\right\}\right] \tag{89}$$

Obviously, this can hardly be considered a practical metric for a receiver. Nevertheless we will proceed, trying to find $P(\underline{x} \to \underline{\hat{x}})$.

Since the receiver uses a ML metric we can use the Bhattacharyya bound

$$D = E\left\{\int_0^{\infty}\cdots\int_0^{\infty}\sqrt{P_M\left(\underline{y}^{(n)}/\hat{x}^{(n)},\ell_n\right)P_M\left(\underline{y}^{(n)}/x^{(n)},\ell_n\right)}\; dy_1^{(n)}\cdots dy_M^{(n)}\Big/x^{(n)}\right\} =$$

$$=\sum_{j=1}^{N}P(j)\int_0^{\infty}\cdots\int_0^{\infty}\sqrt{P_M\left(\underline{y}^{(n)}/\hat{x}^{(n)},j\right)P_M\left(\underline{y}^{(n)}/x^{(n)},j\right)}\; dy_1^{(n)}\cdots dy_M^{(n)} \qquad (90)$$

and

$$D_{Wc} = \max_{0<\rho\leq 1} D.$$

It is difficult to proceed analytically to compute D, but, it can be easily verified that

$$\lim_{\rho\to 0} D(\rho) = \sum_{j=1}^{N} P(j)\,\frac{\left(1+\frac{\bar{E}_j}{N_j}\right)}{\left(1+\frac{1}{2}\frac{\bar{E}_j}{N_j}\right)^2} \qquad (91)$$

which is exactly what we have with no jammer, i.e., the jammer has no effect at all. For $\rho = 1$, we obtain

$$D(1) = \sum_{j=1}^{N}P(j)\,\frac{1+\frac{\bar{E}_j}{N_j+N_{Jj}}}{\left(1+\frac{\bar{E}_j}{2(N_j+N_{Jj})}\right)^2}$$

which is what we obtained before for the receiver in A above. This is what we would expect, since when $\rho = 1$ both receivers are the same. However, for intermediate values of ρ, i.e., $0 < \rho < 1$, the performance differs.

Note that for the same receiver over a uniform channel, with no background noise, we have found that $\rho = 1$ generates the worst case jamming. The same result seems to hold in the general case also.

C. Hard Decision Receiver with J.S.I.

The jammer uses pulsed noise (duty cycle ρ) and nonuniform distribution over the slotted channel.

The input and output alphabets of the channel are

$$x = Y \in \{1,2,\ldots,M\}$$

and the conditional probability, when using the ℓ^{th}; $\ell = 1,\ldots,N$ subchannel is

$$P(y/x,\ell,Z) = \begin{cases} 1-\epsilon_\ell & ; \quad y=x \;, \quad Z=0 \\ \dfrac{\epsilon_\ell}{M-1} & ; \quad y \neq x \;, \quad Z=0 \\ 1-\epsilon_{J\ell} & ; \quad y=x \;, \quad Z=1 \\ \dfrac{\epsilon_{J\ell}}{M-1} & ; \quad y \neq x \;, \quad Z=1 \end{cases} \tag{92}$$

where

$$\epsilon_{J\ell}(\rho) \triangleq \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1+\dfrac{\bar{E}_\ell}{N_\ell+N_{J\ell}/\rho}\right)} \tag{93}$$

$$\epsilon_\ell \triangleq \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1+\dfrac{\bar{E}_\ell}{N_\ell}\right)} \tag{94}$$

The conditional probability of the channel is

$$P_m(\underline{y}/\underline{x},\underline{L},\underline{Z}) = \prod_{n=1}^{m} P\left(y_n/x_n,\ell_n,Z_n\right)$$

Using the ML metric

$$m(y_n;x_n/\ell_n,Z_n) = \ln P(y_n/x_n,\ell_n,Z_n).$$

Hence, using the Bhattacharyya bound

$$P(\underline{x} \to \hat{x}) \leq D^{W(\underline{x};\hat{\underline{x}})}$$

where

$$D = E\left\{\sum_{y_n} \sqrt{P(y_n/\hat{x}_n,\ell_n,Z_n)P(y_n/x_n,\ell_n,Z_n)} \Big/ x_n \right\}_{\hat{x}_n \neq x_n} = \tag{95}$$

$$= \sum_{j=1}^{N} P(j) \sum_{z=0}^{1} P_{Z_n}(z) \sum_{y} \sqrt{P(y/\hat{x}_n,j,z)P(y/x_n,j,z)} =$$

$$= \sum_{j=0}^{N} P(j)\left\{(1-\rho)\left[2\sqrt{\frac{\epsilon_j(1-\epsilon_j)}{M-1}} + \epsilon_j \frac{M-2}{M-1}\right] + \rho\left[2\sqrt{\frac{\epsilon_{Jj}(1-\epsilon_{Jj})}{M-1}} + \frac{M-2}{M-1}\epsilon_{Jj}\right]\right\} \tag{96}$$

Here, again $\rho = 1$, maximizer D

$$\therefore D_{Wc} \triangleq \max_{0<\rho\leq 1} D = \sum_{j=1}^{N} P(j)\left[2\sqrt{\frac{\epsilon_{Jj}(1)[1-\epsilon_{Jj}(1)}{M-1}} + \frac{M-2}{M-1}\epsilon_{Jj}(1)\right] \tag{97}$$

D. Hard Decision Receiver with no J.S.I.

The jammer uses pulsed noise (duty cycle ρ) and nonuniform distribution over the slotted channel.

The input and output alphabets are

$$X = Y \epsilon\{1,2,\ldots,M\}$$

The conditional probability density functions, when using the ℓ^{th}, $\ell = 1,\ldots,N$ subchannel are

$$P(y/x,\ell) = \begin{cases} 1-\epsilon_\ell(1-\rho)-\epsilon_{J\ell}\rho & ; \quad y=x \\ \dfrac{\epsilon_\ell(1-\rho)+\epsilon_{J\ell}\rho}{M-1} & ; \quad y\neq x \end{cases} \tag{98}$$

where

$$\epsilon_\ell \triangleq \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1+\frac{E_\ell}{N_\ell}\right)}$$

$$\epsilon_{J\ell}(\rho) \triangleq \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1+\frac{E_\ell}{N_\ell+N_{J\ell}/\rho}\right)}$$

The metric we use is again the ML metric

$$m(\underline{y};\underline{x}/\underline{L}) = \ln P(\underline{y}/\underline{x},\underline{L}).$$

Hence, we have the Bhattacharyya bound

$$P(\underline{x} \to \hat{\underline{x}}) \le D^{W(\underline{x};\hat{\underline{x}})}$$

where

$$D = E\left\{\sum_{y_n}\sqrt{P(y_n/\hat{x}_n,\ell_n)P(y_n/x_n,\ell_n)}\Big/ x^{(n)}\right\} \quad ; \quad \hat{x}_n \neq x_n$$

$$= \sum_{j=1}^{N} P(j)\sum_{y}\sqrt{P(y/\hat{x}_n j)P(y/x_n,j)} =$$

$$= \sum_{j=1}^{N} P(j)\left[2\sqrt{\frac{[\varepsilon_j(1-\rho)+\varepsilon_{Jj}\rho][1-\varepsilon_j(1-\rho)-\varepsilon_{Jj}\rho]}{M-1}} + \frac{M-2}{M-1}\left[\varepsilon_j(1-\rho)+\varepsilon_{Jj}\rho\right]\right]. \tag{99}$$

But

$$\max_{0<\rho\le 1}\{\varepsilon_j(1-\rho)+\varepsilon_{Jj}\rho\} = \left|\varepsilon_j(1-\rho)+\varepsilon_{Jj}\rho\right|_{\rho=1} = \varepsilon_{Jj}(1)$$

It is therefore clear that

$$D_{Wc} \triangleq \max_{0<\rho\le 1} D = \sum_{j=1}^{N} P(j)\left[2\sqrt{\frac{\varepsilon_{Jj}(1-\varepsilon_{Jj})}{M-1}} + \frac{M-2}{M-1}\varepsilon_{Jj}\right] \tag{100}$$

where

$$\varepsilon_{Jj} \triangleq \varepsilon_{Jj}(1).$$

Note, that if the receiver had no Channel State Information then

$$\bar{P}(\underline{y}/\underline{x}) = \begin{cases} 1-p \triangleq \sum_{\ell} P(\ell)[1-\varepsilon_\ell(1-\rho)+\varepsilon_{J\ell}\rho] & ; \quad y=x \\ \frac{p}{M-1} \triangleq \sum_{\ell} P(\ell)\left[\frac{\varepsilon_\ell(1-\rho)+\varepsilon_{J\ell}\rho}{M-1}\right] & ; \quad y\neq x \end{cases} \tag{101}$$

and using the ML metric, we would have obtained

$$\bar{D} = 2\sqrt{\frac{p(1-p)}{M-1}} + \frac{M-2}{M-1}\,p \tag{102}$$

when the bar is used to discriminate between the two receivers. Since $D(\underline{P})$ is convex $\cap$, it is clear that $\bar{D} \geq D$.

V. R_o EVALUATION AND SIMPLE APPLICATIONS

A. As shown in Note #3, the cutoff rate for each coded bit in the worst case jamming environment is given by

$$R_o = \log_2 M - \log_2[1+(M-1)D_{Wc}] \quad \text{bits/ch.use} \tag{103}$$

It is now a trivial matter to derive R_o for all the situations analyzed above. In particular, we have derived D_{Wc} for the Soft Decision receiver with J.S.I. over a uniform channel. Since $\rho = 1$ is the worst case jamming, we now let N_o represent the total uniform noise spectral density, which includes the background noise and the effect of the jamming.

D_{Wc} is in this case given by

$$D_{Wc} = \frac{4\left(1 + \frac{\bar{E}_c}{N_o}\right)}{\left(2 + \frac{\bar{E}}{N_o}\right)^2} \tag{104}$$

Using equations 103 & 104 we have computed R_o for M = 2,4,8,16,32 as a function of $\bar{E}_c/N_o$. These results are shown in Figure 4.

Since the Soft Decision receiver having knowledge of ρ only, achieves the same performance for $\rho = 1$, as the receiver having J.S.I., the same Figure applies to this receiver as well. Figure 4 shows also the cutoff rate of the two Hard Decision receivers considered above. For both receivers over the uniform channel D_{Wc} is given by

$$D_{Wc} = 2\sqrt{\frac{\varepsilon(1-\varepsilon)}{M-1}} + \frac{M-2}{M-1}\,\varepsilon \tag{105}$$

where

$$\epsilon = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1 + \frac{\bar{\bar{E}}_c}{N_o}\right)} \tag{106}$$

In general, D_{Wc} is a function of $\underline{\bar{\bar{E}}_c}$, $\underline{J}$, $\underline{N_t}$ & $\underline{P}$, but for a uniform channel D_{Wc} can be written as a function of $\bar{\bar{E}}_c/N_o$ only. To emphasize this fact we write

$$D = D\left(\frac{\bar{\bar{E}}_c}{N_o}\right)$$

and

$$R_o = R_o\left(\frac{\bar{\bar{E}}_c}{N_o}\right).$$

B. MFSK

Conventional MFSK modulation has the symbol error probability bound

$$P_S \leq \frac{1}{2} (M-1)D \tag{107}$$

and bit error bound

$$P_b = \frac{M/2}{M-1} P_S. \tag{108}$$

Now, since for MFSK

$$\bar{\bar{E}}_c = K\bar{\bar{E}}_b , \tag{109}$$

where

$$K \triangleq \log_2 M \tag{110}$$

we have for the uniform channel

$$P_b \leq 2^{K-2} D\left(\frac{K\bar{\bar{E}}_b}{N_o}\right). \tag{111}$$

Hence, the bound of the Soft Decision receiver is in this case

$$P_b \le 2^{K-2} \frac{4\left(1 + \frac{K\bar{E}_b}{N_o}\right)}{\left(2 + \frac{K\bar{E}_b}{N_o}\right)^2} = $$

$$= 2^K \frac{\left(1 + \frac{K\bar{E}_b}{N_o}\right)}{\left(2 + \frac{K\bar{E}_b}{N_o}\right)^2} \qquad (112)$$

whereas the exact bit error probability is

$$P_b = \frac{\frac{1}{2} M}{M-1} P_S =$$

$$= \frac{2^{K-1}}{2^K-1} \sum_{k=1}^{2^K-1} \binom{2^K-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1 + \frac{K\bar{E}_b}{N_o}\right)} . \qquad (113)$$

For m diversity MFSK we have the symbol error bound

$$P_S \le \frac{1}{2} (M-1) \; D\left(\frac{\bar{E}_c}{N_o}\right)^m \qquad (114)$$

where

$$\bar{E}_c = \frac{K}{m} \bar{E}_b \; . \qquad (115)$$

Hence

$$P_b = \frac{\frac{1}{2} M}{M-1} P_S =$$

$$= \frac{M}{4} D\left(\frac{\bar{E}_c}{N_o}\right)^m \qquad (116)$$

$$= \frac{M}{4} D\left(\frac{K}{m} \frac{\bar{E}_b}{N_o}\right)^m$$

For the Soft Decision receiver we then obtain

$$P_b \leq \frac{M}{4}\left[\frac{4\left(1+\frac{K}{m}\frac{\bar{E}_b}{N_o}\right)}{\left(2+\frac{K}{m}\frac{\bar{E}_b}{N_o}\right)^2}\right]^m \tag{117}$$

We can compare this result with the exact bit error probability obtained by a ML receiver, which, for $M = 2$ is known tc be [4]

$$P_{be} = P^m \sum_{j=0}^{m-1} \binom{m+j-1}{j} (1-P)^j \tag{118}$$

where

$$P \triangleq \frac{1}{2 + \frac{1}{m}\left(\frac{\bar{E}_b}{N_o}\right)} \tag{119}$$

Figure 5 shows both curves for several values of m. Given $\frac{\bar{E}_b}{N_o}$ we can find the optimal value of m and the resultant P_b bound from equation 117. Figures 6a,6b shows the P_b bound as a function of E_b/N_o using optimal diversity for $M = 2,4,8,16,32$. The figure shows also the value of the optimal m used to derive each calculated point. Since m can only assume integer values, the smooth curves shown are only approximations to the actual results. As well known [4], for $M = 2$ the optimal diversity is given by

$$m_{opt} \approx \frac{1}{3}\left(\frac{E_b}{N_o}\right)$$

For the same value of E_b/N_o but higher M, m_{opt} is also higher.

Even when moderate values of signal-to-noise ratio $\left(\text{say } \frac{E_b}{N_o} = 20 \text{ db}\right)$ are expected, the optimum value of m may well be unrealistically high. A variety of "practical" reasons may preclude the use of high m values. If, for instance, the information rate is such that the "instantaneous" bandwidth of the transmitted chip is nearly equal to the "coherence bandwidth" of the HF channel,

then, a substantial degradation in performance, not accounted for in our analysis, may appear, when the bit interval is chopped to shorter chips. Moreover, changing the chip rate so as to follow changes in $\bar{E}_b/N_o$ is usually undesirable in practice. In such cases it seems reasonable to choose low value of m, so that optimal performance will be achieved when the signal is weak. Figures 7& 8 may be of interest in such a situation. In this Figure we compare the performance of systems using $M= 2,4,8,16,32$, for $m=K$, i.e., the chip time T_c is equal to T_b for all curves. It can be seen that under such a constraint, high M systems have a profound advantage.

D. Orthogonal Convolutional Codes.

Conventional MFSK with m diversity is merely a block code containing M code words of blocklength m. We can consider more general codes using M-ary alphabets. An orthogonal convolutional code, for instance, generates one $2^K=M$-ary symbols per bit. When used with m diversity, each symbol is "chopped" into m chips and the bit error bound is [3] :

$$P_b \le \frac{D\left[\frac{\bar{E}_b}{mN_o}\right]^{mK}}{2\left[1-2D\left[\frac{\bar{E}_b}{mN_o}\right]^{m}\right]^{2}} \tag{120}$$

E. Example .

Consider a Soft Decision receiver with JSI and a uniform channel. The information rates are:

a. $R_H = 2400$ bits/sec.

b. $R_L = 75$ bits/sec.

and suppose that at the higher information rate $\bar{E}_b/N_o = 16$ dB.

a. For a binary receiver with no diversity, i.e., $M=2$, $K=m=1$, we obtain from

equations 104 and 107:

$$D\left[\frac{\bar{E}_b}{N_o}\right] = \frac{4(1+39.8)}{(2+39.8)^2} = 0.934\ 10^{-1}$$

and

$$P_b \leq \frac{1}{2} D = 0.467\ 10^{-1}$$

The convolutional code from 105 yields in this case:

$$P_b \leq 0.706\ 10^{-1}$$

Recalling that diversity may help, we see in figure 6a that for this binary receiver at $\bar{E}_b/N_o$ = 16 dB, optimum diversity is $m = 13$. Using this value we obtain:

$$P_b \leq \frac{M}{4} D^m$$

$$= \frac{2}{4}\ \frac{4(1+39.8/13)^{13}}{(2+39.8/13)^2} = 0.134\ 10^{-2}$$

The chip rate is then :

$$R_c = R_H m = 2400 \times 13 = 31{,}200 \text{ chips/second.}$$

If 2400 chips/sec. is the highest permissible chip rate, we can try to use a higher M. For $M=2^K=8$ and $m=K=3$, the chip rate remains R_c=2400 chips/sec. and $\bar{E}_c=\bar{E}_b$.

Hence:

$$P_b \leq \frac{M}{4} D\left[\frac{\bar{E}_b}{N_o}\right]^m = \frac{8}{4}\ 0.0934^3 = 0.162\ 10^{-2}$$

Which is almost as good as optimal diversity for $M=2$.

The orthogonal convolutional code with $M=8$, $m=3$ yields in this case:

$$D\left[\frac{\bar{E}_b}{mN_o}\right] = \frac{4(1+39.8/3)}{(2+39.8/3)^2} = 0.2448$$

$$\therefore\quad P_b \leq 0.168\ 10^{-5}$$

b. For the low data rate, $R_L = 75$ bits/sec. :

$$\left[\frac{\bar{E}_b}{N_o}\right]_L = \frac{2400}{75}\left[\frac{\bar{E}_b}{N_o}\right]_H = 32 \times 38.9 = 1.244\ 10^3$$

Hence, for the binary receiver with no diversity we obtain from equations 104& 111 :

$$D = 3.208\ 10^{-3}$$

and

$$P_b \lesssim 0.1604\ 10^{-3}$$

There is no need to use high m in this case. Suppose we take $m=4$.

Then:

$$P_b \lesssim 0.1317\ 10^{-8}$$

and the chip rate:

$$R_c = mR_L = 4 \times 75 = 300 \text{ chips/sec.}$$

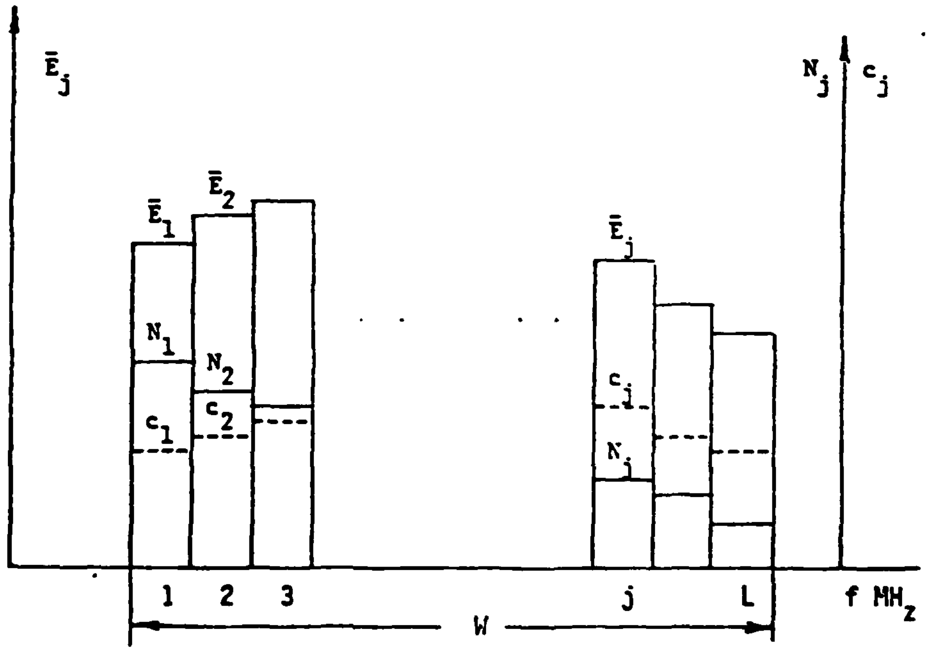


Figure 1 : The Slotted Channel

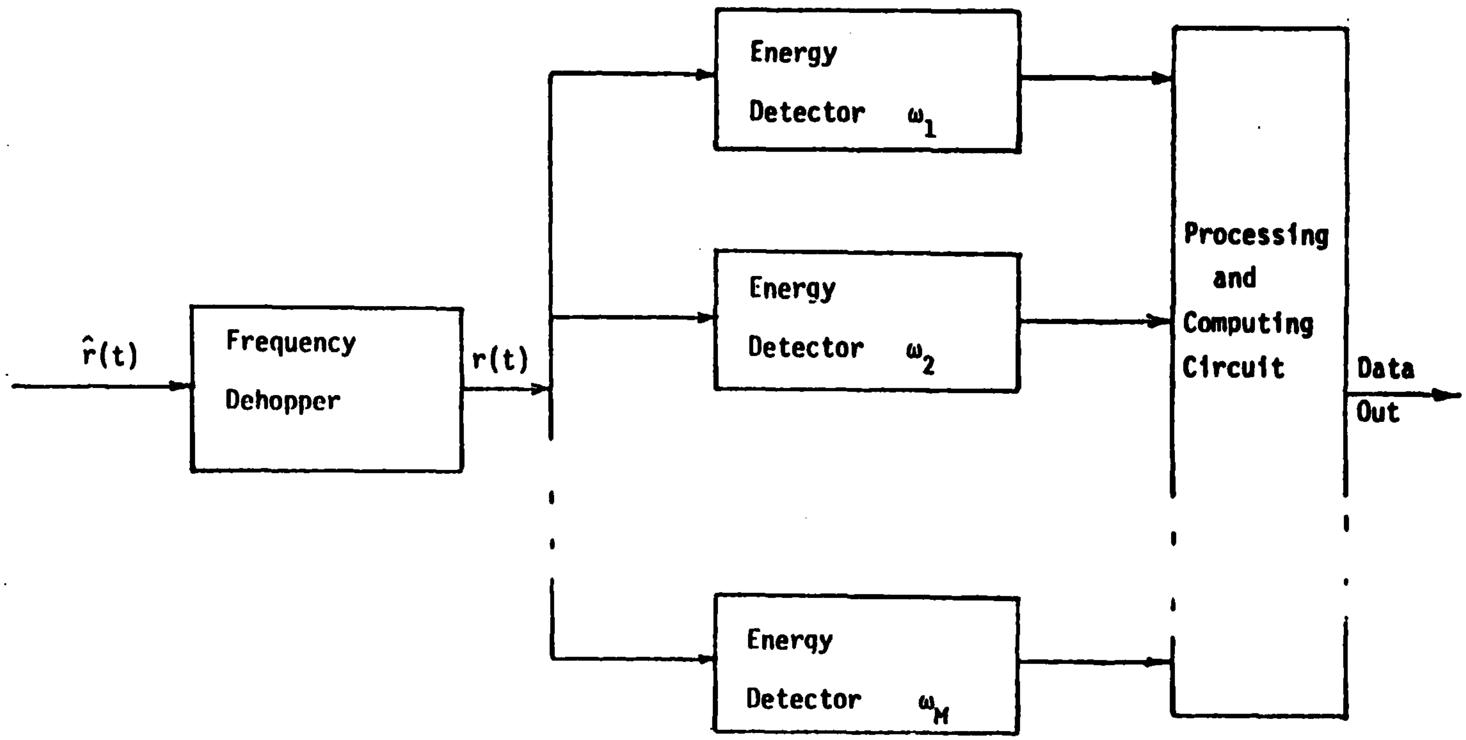
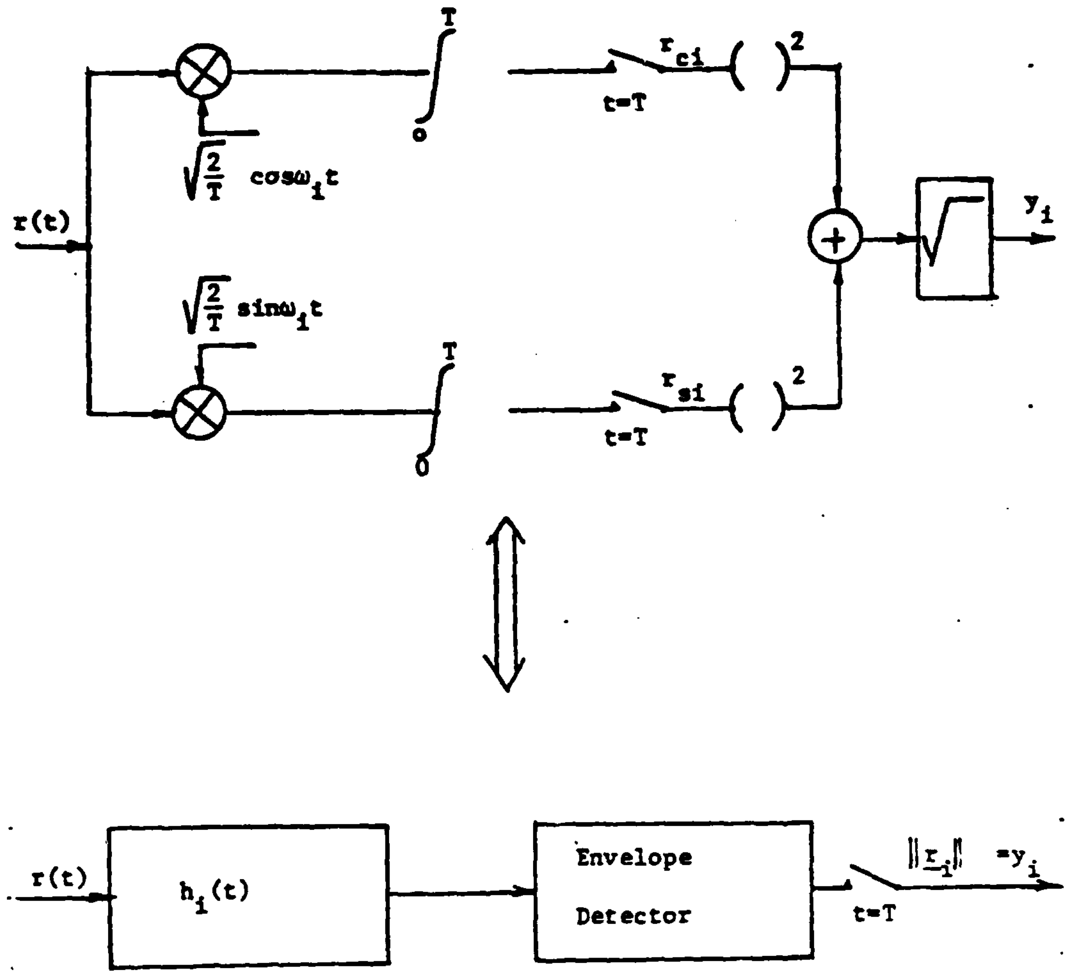


Figure 2 The Basic Receiver



$$h_i(t) = \sqrt{\frac{2}{T}}\cos\omega_i(T-t)$$

Figure 3 An Energy Detector

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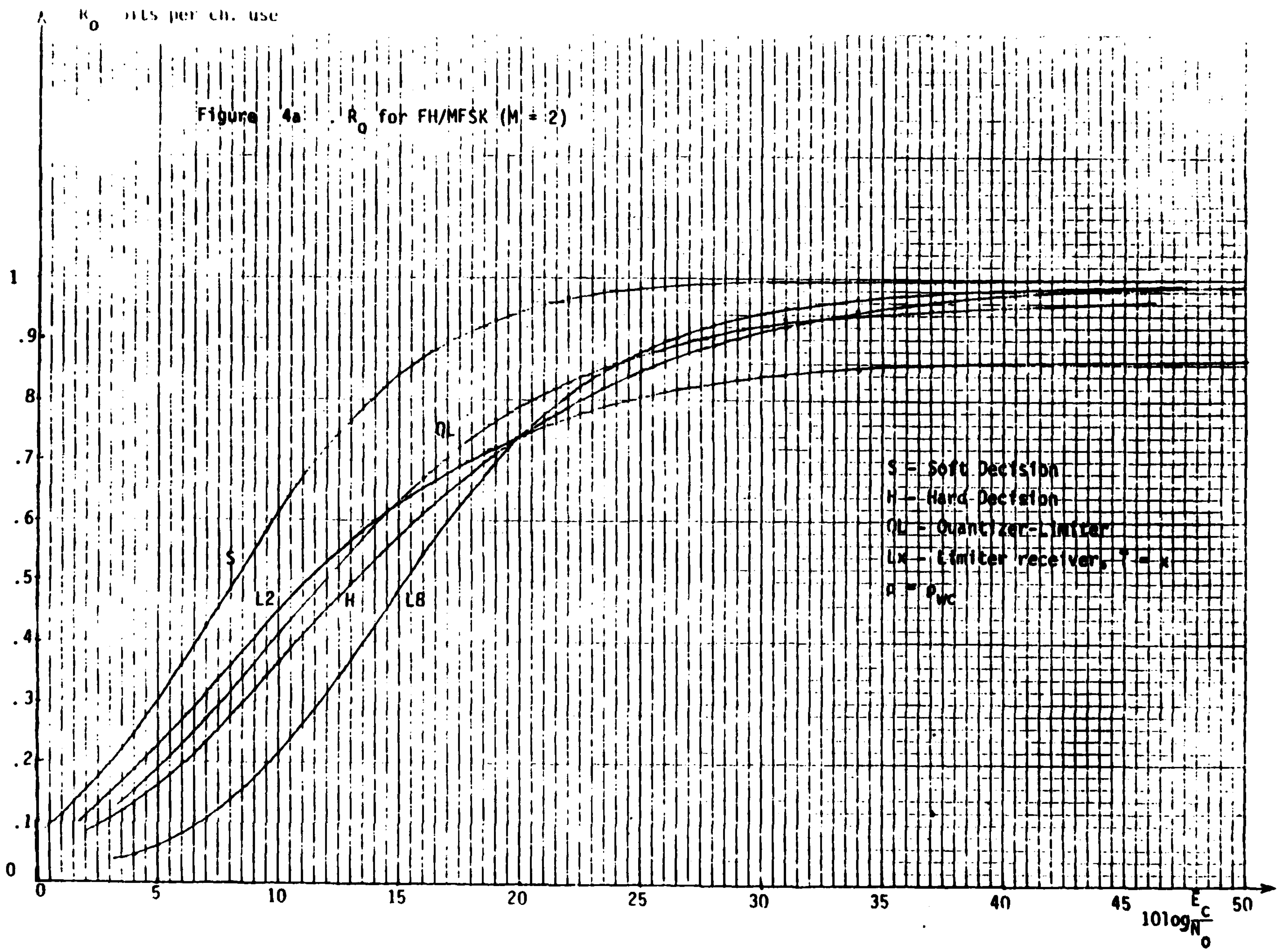
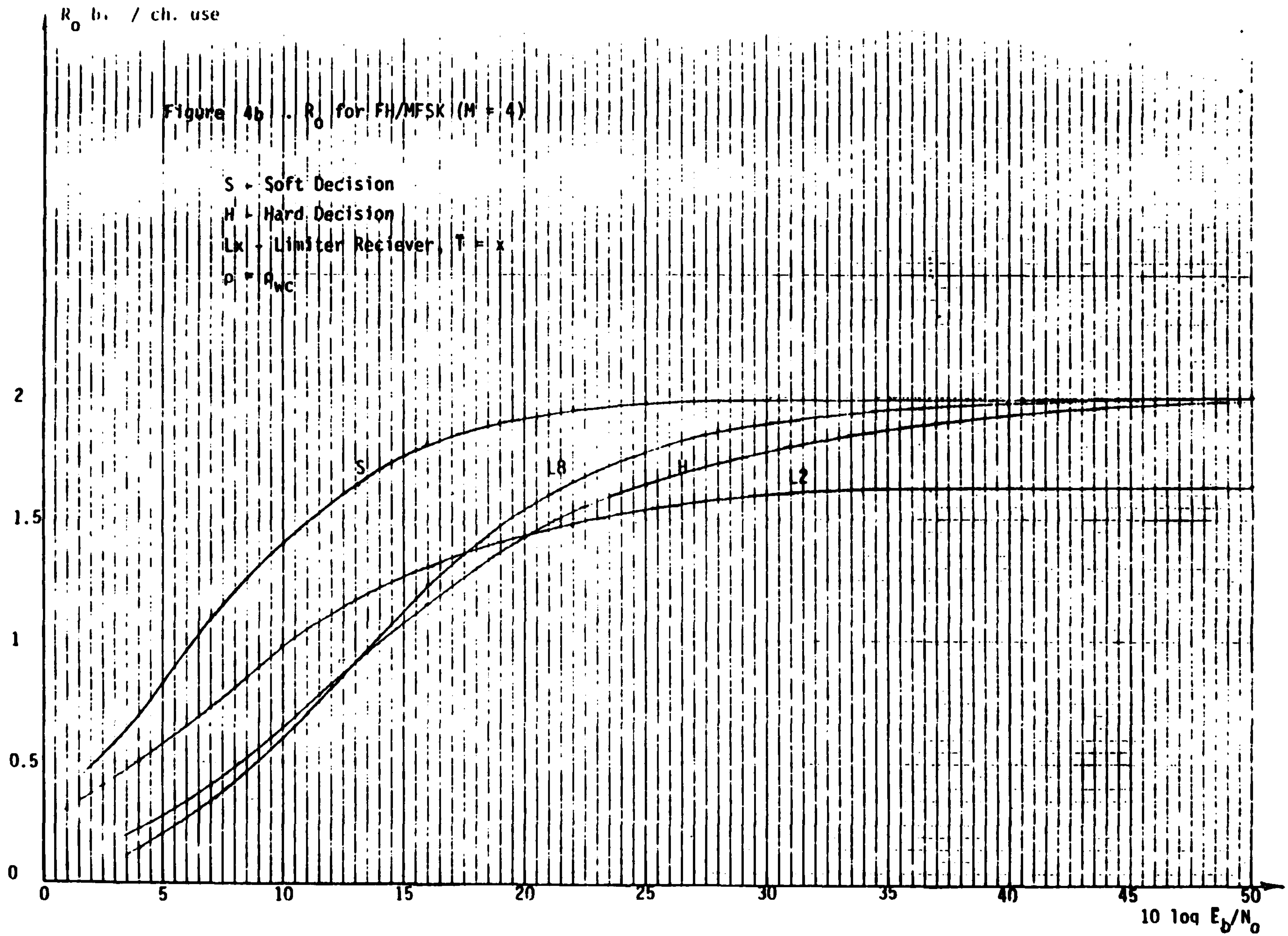
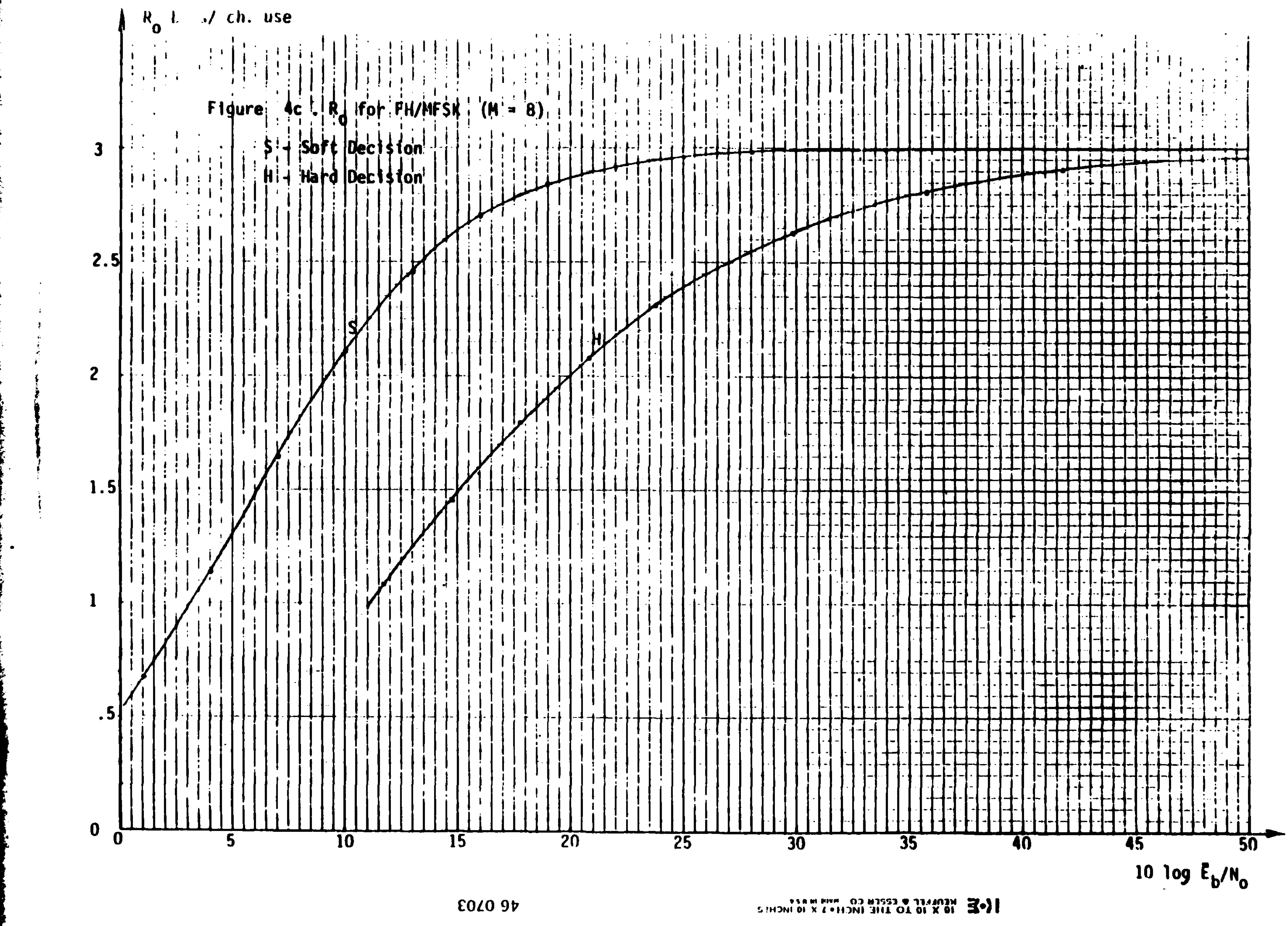


Figure 4a. R_0 for FH/MFSK (M = 2)



R_o b. / ch. use
Figure 4b . R_o for FH/MFSK (M = 4)
S - Soft Decision
H - Hard Decision
Lx - Limiter Reciever, T = x
$\rho = \rho_{WC}$
S
L8
H
L2
2
1.5
1
0.5
0
0
5
10
15
20
25
30
35
40
45
50
10 log E_b/N_o


R_o b/ ch. use
Figure 4c. R_o for FH/MFSK (M = 8)
S - Soft Decision
H - Hard Decision
S
H
3
2.5
2
1.5
1
.5
0
0
5
10
15
20
25
30
35
40
45
50
10 log E_b/N_o
46 0703
10 X 10 TO THE INCH • 7 X 10 INCHES
KEUFFEL & ESSER CO.

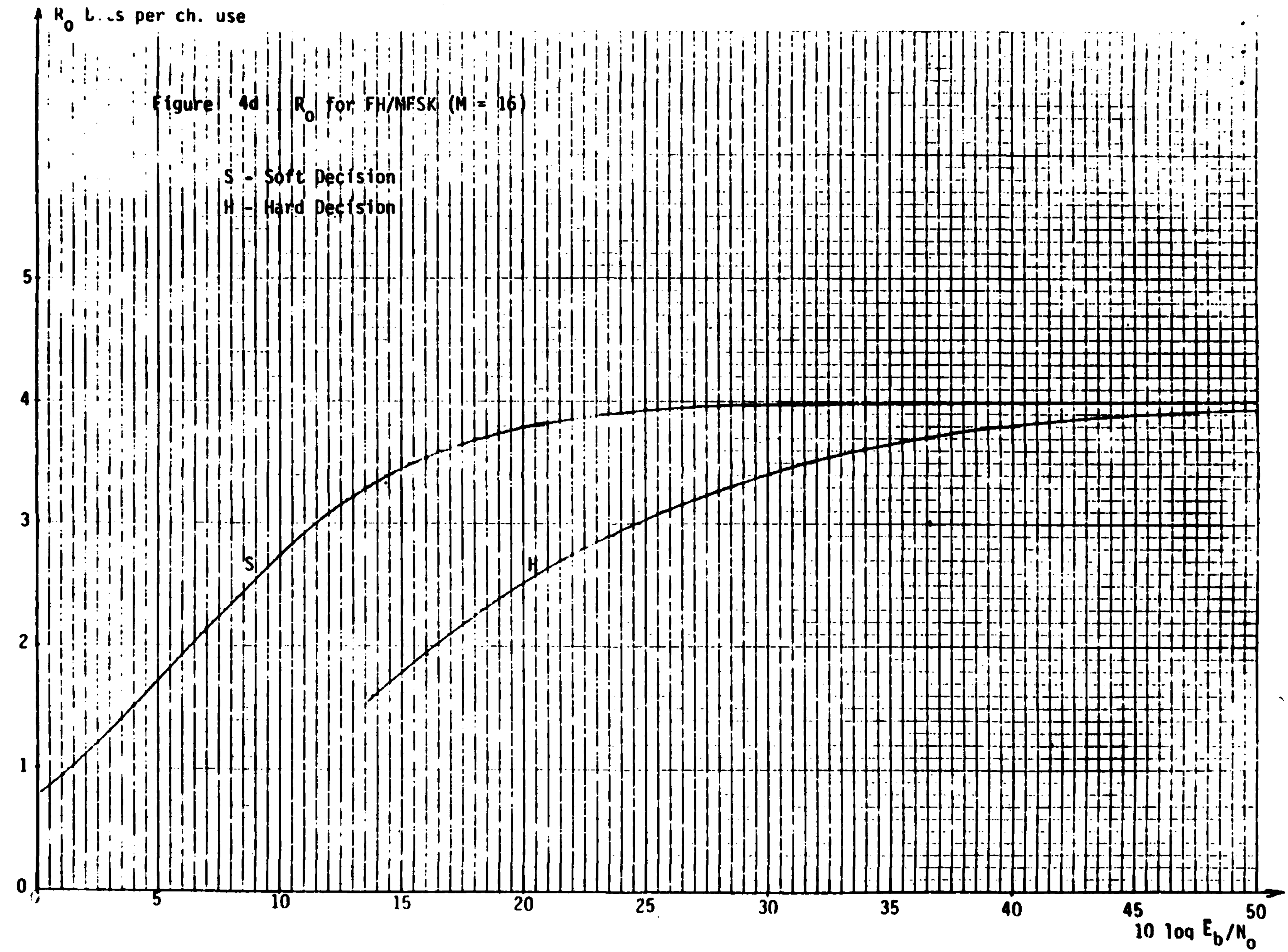
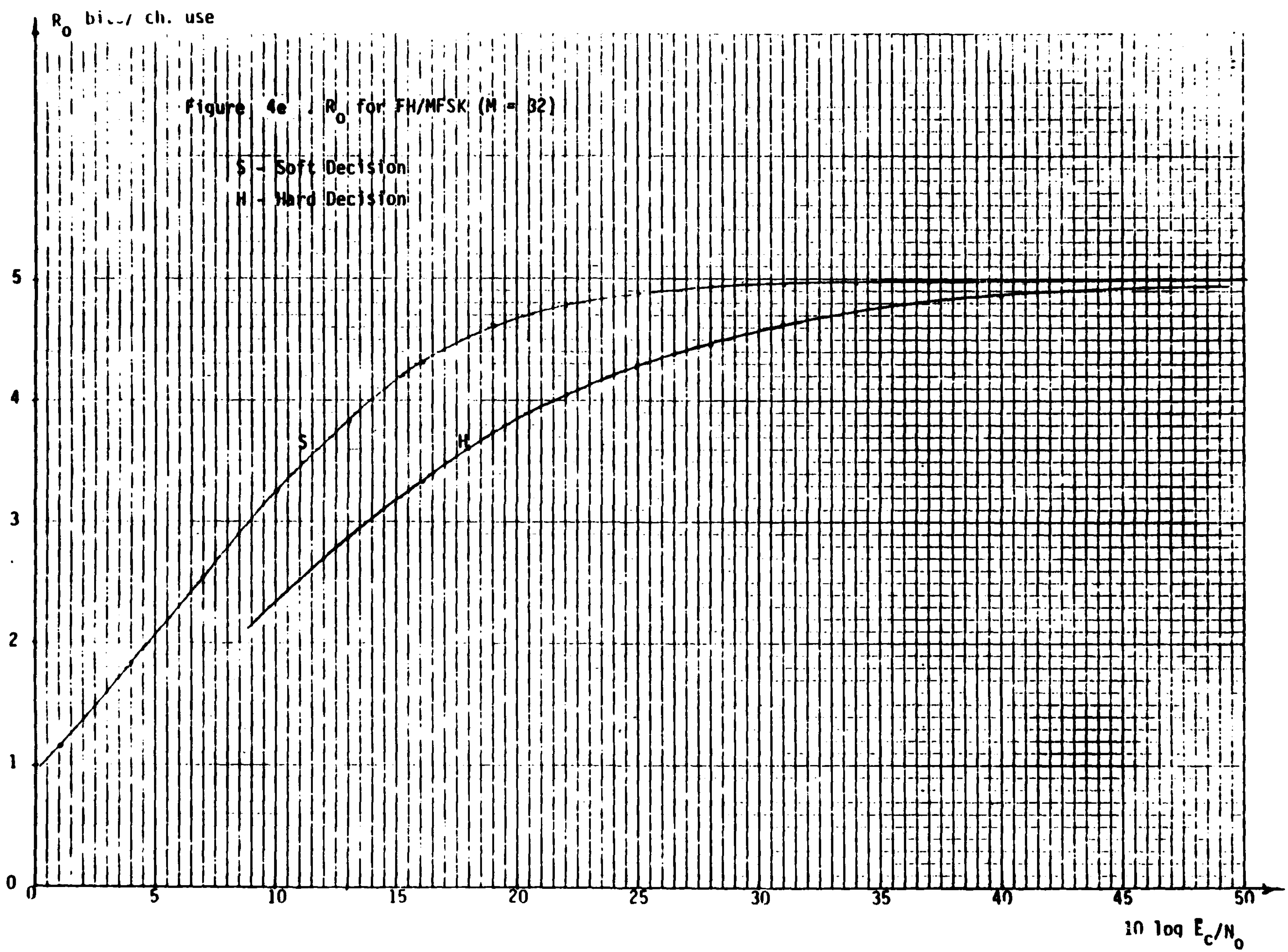


Figure 4d R_o for FH/MFSK (M = 16)

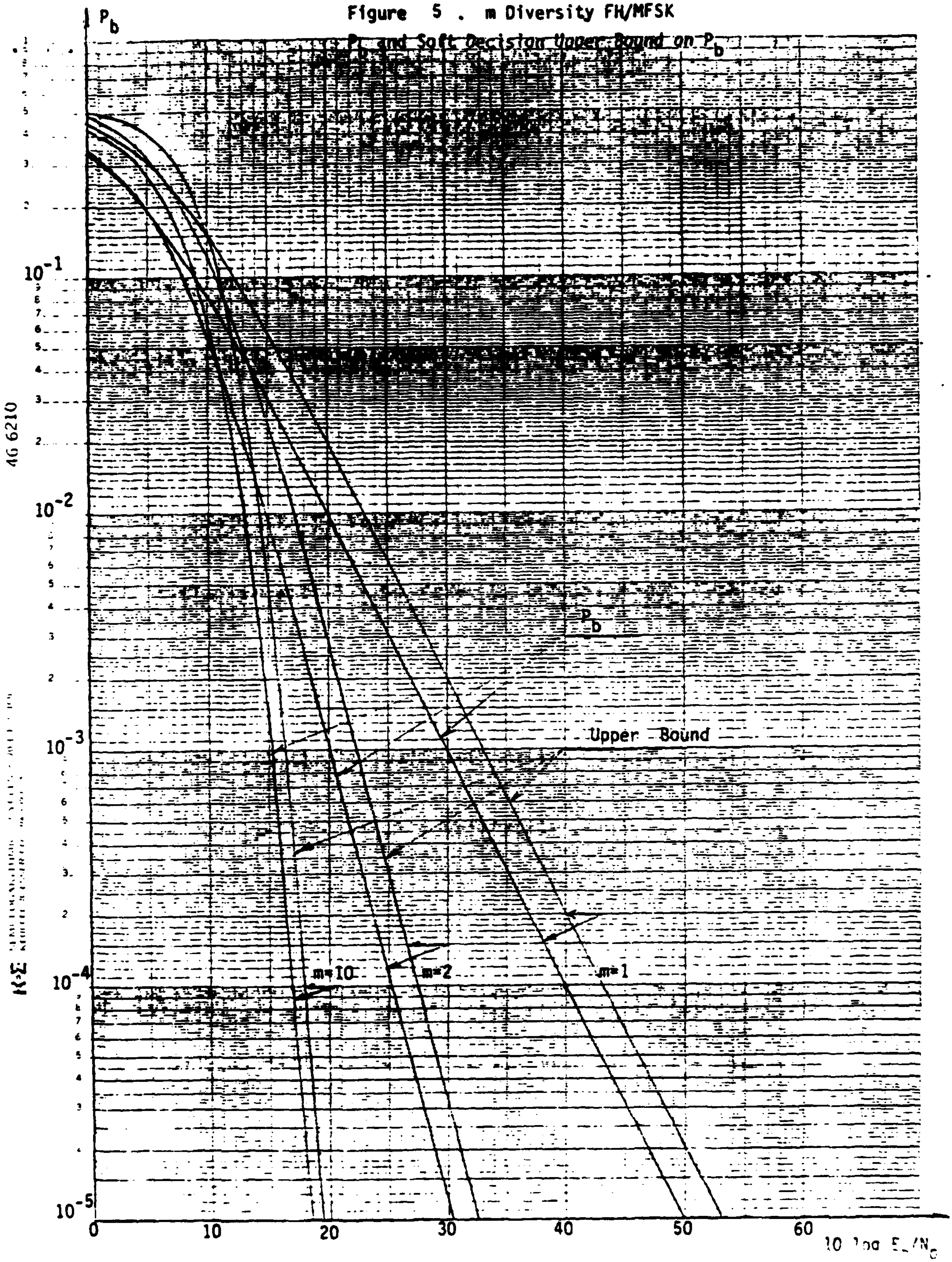


R_o bi../ ch. use
Figure 4e . R_o for FH/MFSK (M = 32)
S - Soft Decision
H - Hard Decision
S
H
0
1
2
3
4
5
0
5
10
15
20
25
30
35
40
45
50
10 log E_c/N_o

46 0703

Figure 5 . m Diversity FH/MFSK

P_b and Soft Decision Upper Bound on P_b



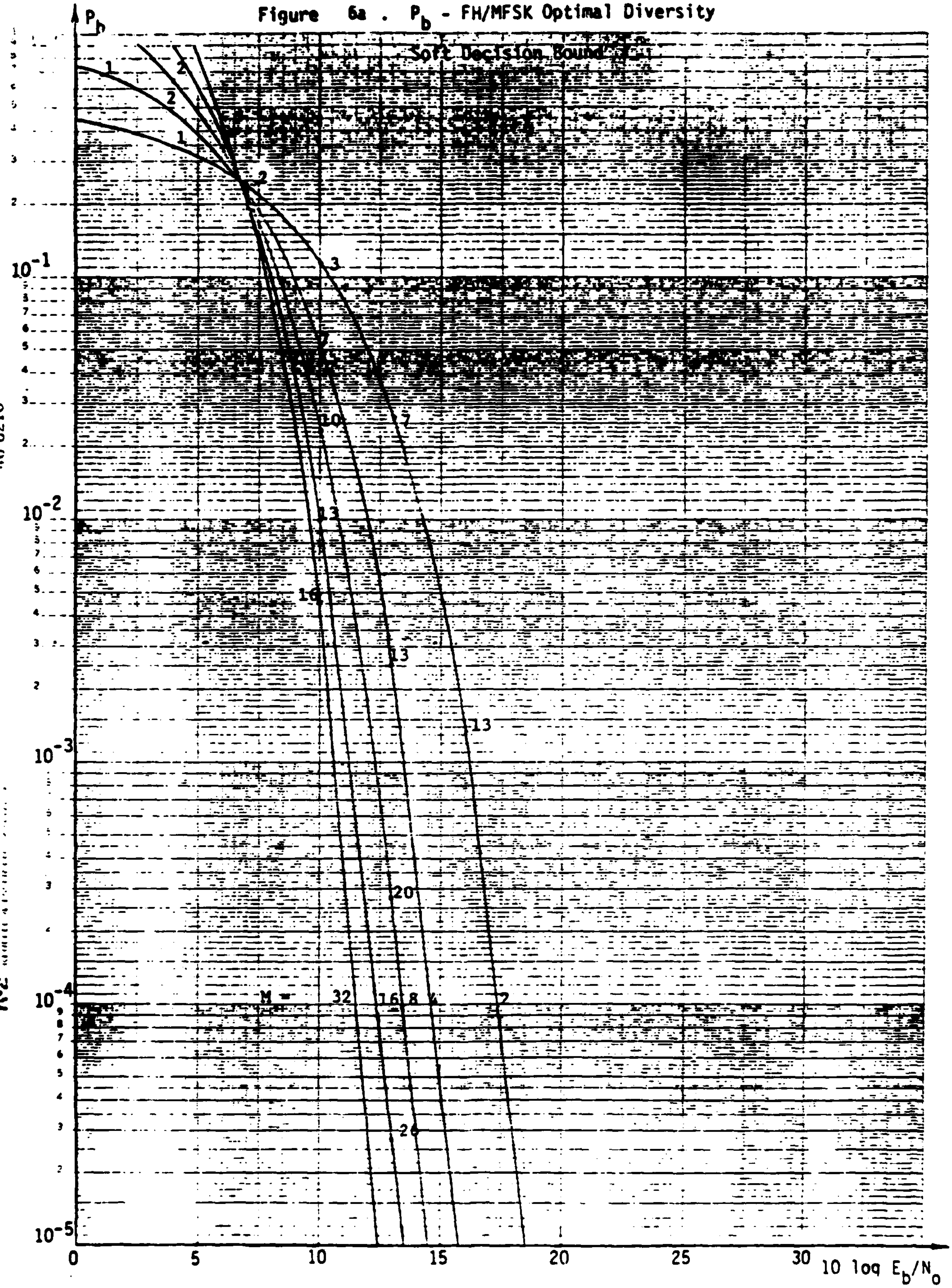


Figure 6a . P_b - FH/MFSK Optimal Diversity
Soft Decision Bound
P_b
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
M = 32 16 8 4 2
0 5 10 15 20 25 30
10 log E_b/N_o

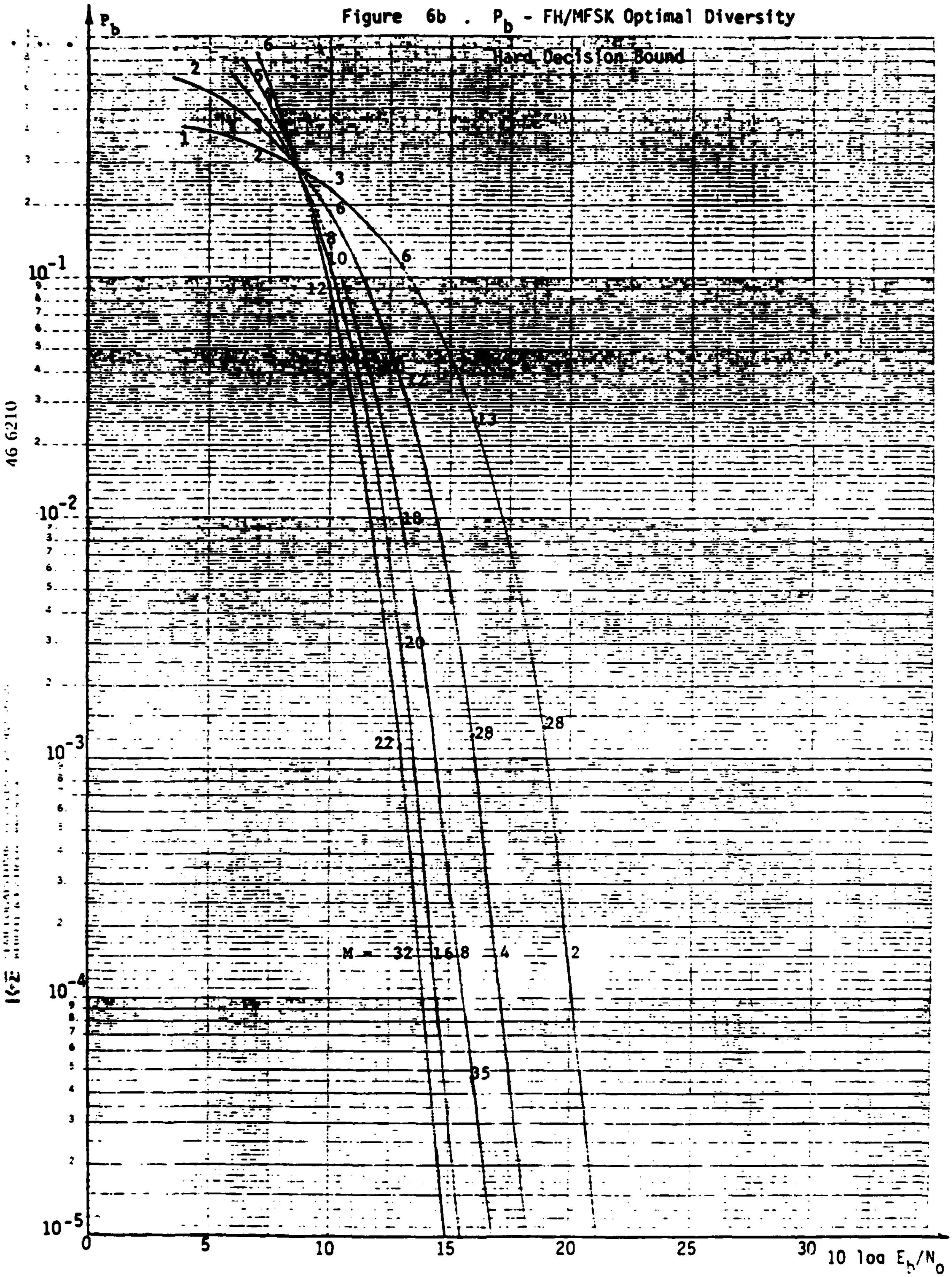


Figure 6b . P_b - FH/MFSK Optimal Diversity
Hard Decision Bound
P_b
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
5
10
15
20
25
30
10 log E_b/N_0
M = 32 16 8 4 2

Figure 7 . P_b - FH/MFSK , m = K Diversity

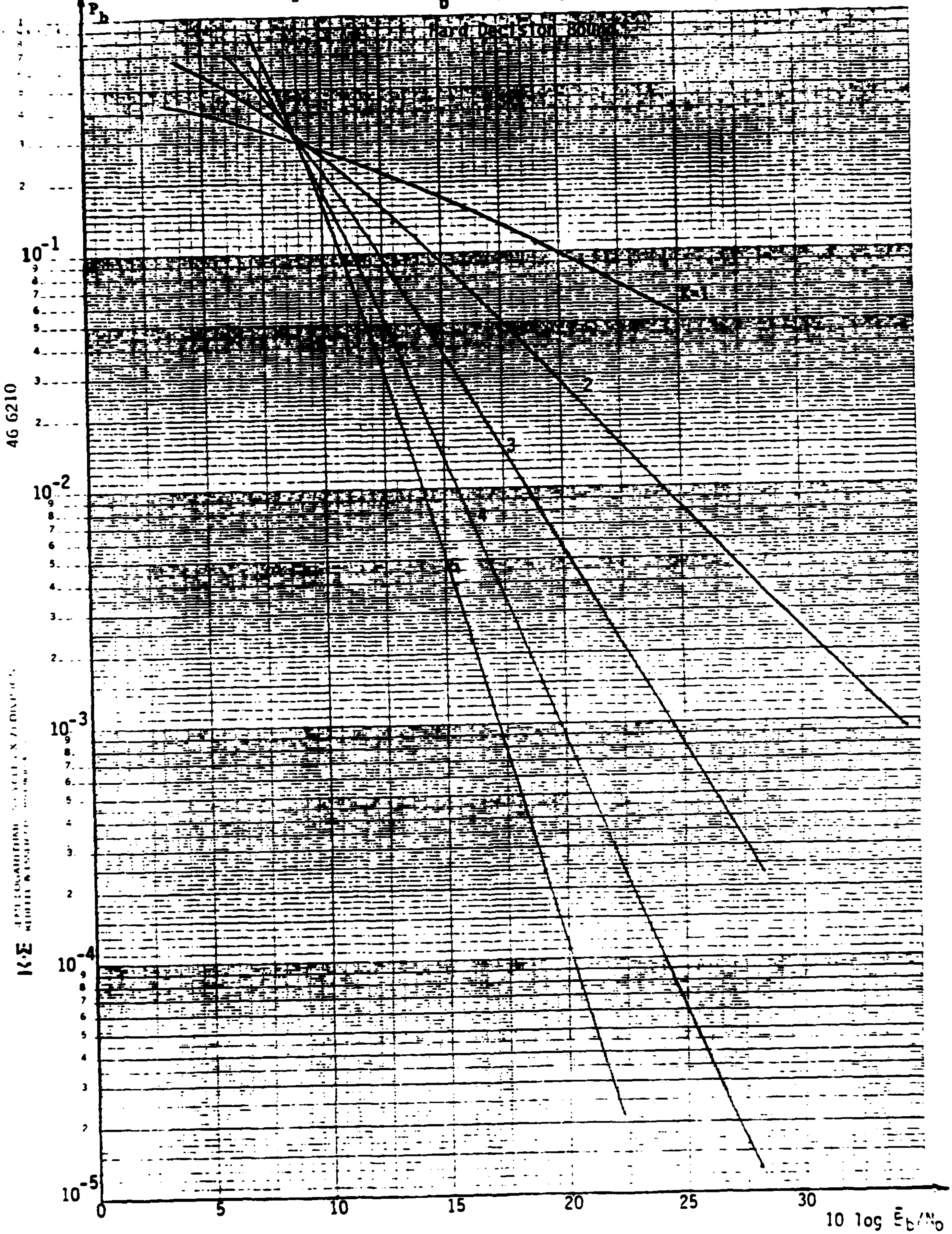
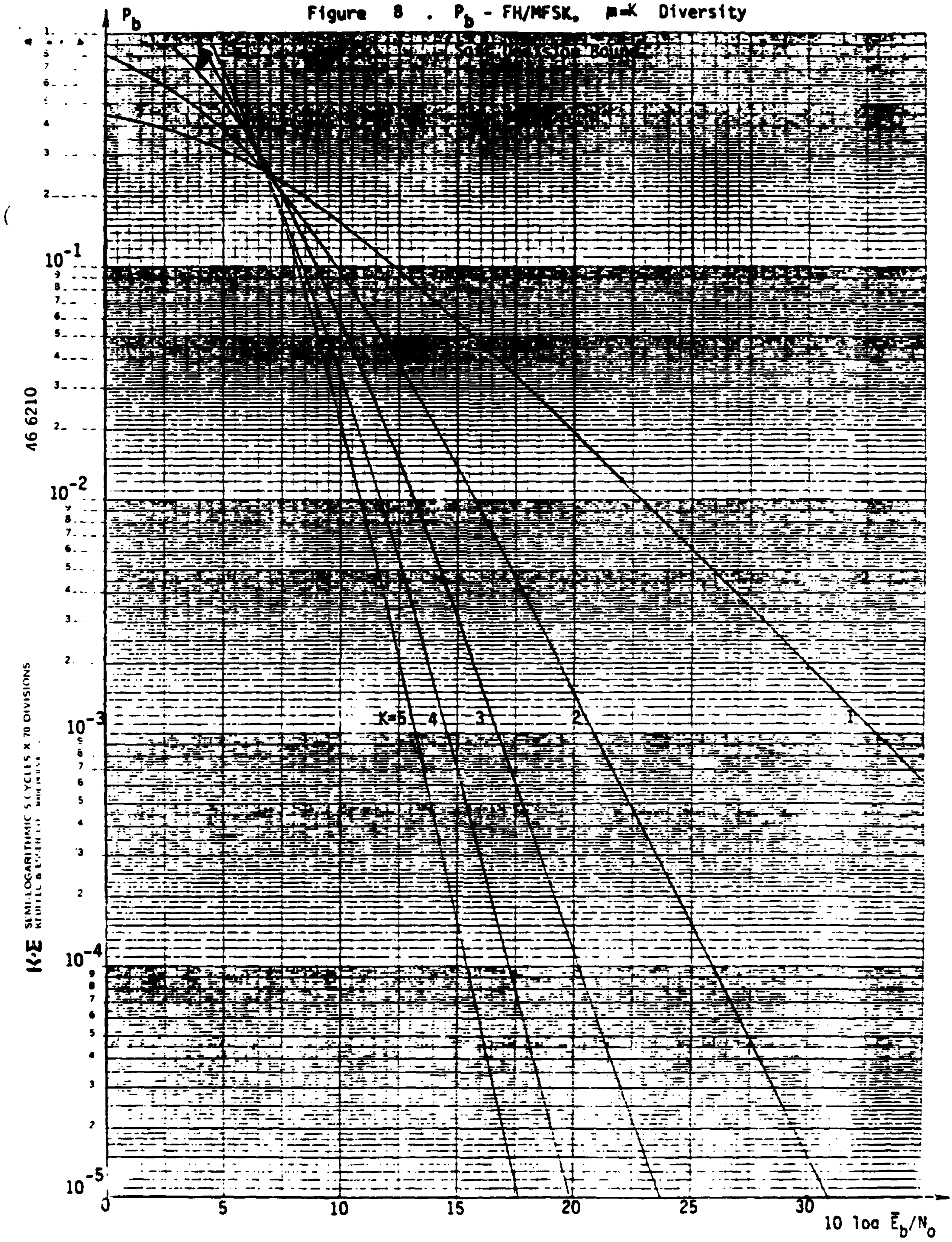


Figure 8 . P_b - FH/MFSK, m=K Diversity



Note No. 7

VARIABLE DATA BIT RATES WITH A FIXED HOP RATE

NONCOHERENT FH/MFSK SYSTEM

to

NAVAL RESEARCH LABORATORY

(Contract Award No. N00014-80-K-0935)

for

HF COMMUNICATION NETWORK SIGNALS

USING CHANNEL EVALUATION DATA

Principal Investigator

Jim K. Omura
Professor
System Science Department
University of California
Los Angeles, California

May, 1981

I. INTRODUCTION

In this note we examine the impact of variable data bit rates on a fixed hop rate noncoherent FH/MFSK system with diversity. This points out the advantage of some diversity and the impact of noncoherent combining losses associated with too much diversity.

To combat jamming effectively FH/MFSK systems must use diversity.* This means that each MFSK signal is hopped m times and at the receiver the m "chips" are noncoherently combined to provide diversity. For the uncoded* case where $M = 2^K$ we examine here the parameter

$$L = \frac{m}{K} \quad \text{hops/data bit}$$

Rather than consider the optimum choice of L we examine the bit error bounds for a wide range of L values.

Our motivation for examining various values of the diversity is that in many systems the hop rate, R_h hops/second, is fixed and the data rate, R bits/second, can change to accommodate various types of users. Also the processing gain

$$PG = \frac{W}{R}$$

where W is the fixed total spread spectrum bandwidth can be increased by lowering the data rate R. This may be necessary to combat strong jamming where we have the effective energy per bit to noise ratio

$$\frac{E_b}{N_o} \equiv \frac{PG}{(J/S)} = \frac{SW}{JR}$$

where

S = signal power

J = jammer power

---

*Diversity can be viewed as a special case of coding. MFSK modulation can also be viewed as a form of coding.

at the intended receiver. Usually total bandwidth W and hop rate R_h are fixed in a spread spectrum system while data rates may be varied.

In general, some form of diversity can improve performance by up to 40 dB at bit error rates near $P_b = 10^{-6}$ in both a worst case partial jamming and a Rayleigh fading channel. However, noncoherent combining losses begin to dominate with too much diversity. Here we examine the bit error probabilities of noncoherent FH/MFSK systems for a wide range of diversity values, L, measured in diversity chips per data bit. This is done for fading and non-fading channels as well as for the constraint length K = 7, rate $r = \frac{1}{2}$, binary convolutional code.

The usual diversity combining loss associated with broadband jamming (or additive white Gaussian noise channels) in a non-fading channel shows considerable loss associated with too much diversity. The relative impact of large diversity is somewhat less against worst case partial band jamming since this type of jammer can degrade performance at low diversity values more than at higher diversities. Indeed, with no diversity there can be up to 40 dB loss at $P_b = 10^{-6}$ with the worst case partial band jammer compared to broadband jamming. At higher diversity values the worst case partial band jammer becomes the broadband jammer.

In a Rayleigh fading channel the worst case partial band jammer is the broadband jammer. In general, the bit error probabilities of the Rayleigh fading channel upper bounds the bit error probabilities of the non-fading channel in a worst case partial band jamming channel. The difference in these bit error probabilities become smaller with increasing diversity values. In the limit of large diversity values the performance of fading and non-fading channels are the same.

We also have the observation that as diversity increases the relative

coding gains decrease. Like diversity, coding is more effective against worst case partial band noise at smaller diversity values.

II. WORST CASE PARTIAL BAND NOISE JAMMING WITH NO CHANNEL FADING

Consider first the case where we have no diversity and each MFSK signal is hopped once for each MFSK signal. Assuming partial band noise jamming where ρ is the fraction of the band jammed with noise of spectral density N_o/ρ where $N_o = \frac{J}{W}$. We have ρ as the probability that any MFSK signal is in the jammed part of the total spread bandwidth W and $1 - \rho$ as the probability it is hopped outside the jammed frequencies. Then the symbol error probability is

$$P_E = \rho\, P_E(N_o/\rho) + (1-\rho)\, P_E(0)$$

where $P_E(\alpha)$ is the MFSK error probability in white Gaussian noise of spectral density α. We assume negligible channel noise other than the jamming noise so that $P_E(0) = 0$. Using the union bound we have,

$$P_E(N_o/\rho) \le (M-1)\, \frac{1}{2}\, e^{-\rho \frac{E_c}{2N_o}}$$

where E_c is the energy per MFSK signal. Here $E_c = KE_b$ where $M = 2^K$. Also using the relationship between bit error and symbol error,

$$P_b = \frac{\frac{1}{2} M}{M-1} P_E$$

we have the no diversity bit error union bound

$$P_b \le 2^{K-2}\, \rho e^{-\rho \frac{KE_b}{2N_o}}$$

The worst case ρ that maximizes this bound is given by

$$\rho^* = \left(\frac{KE_b}{2N_o}\right)^{-1}$$

provided $\frac{E_b}{N_o} > \frac{2}{K}$ which is the case of interest. The bound for worst case partial band jammer with no diversity is thus

$$P_b \leq \frac{2^{K-1}}{Ke(E_b/N_o)}$$

and here we have effectively

$$L = \frac{1}{K} \quad \text{hops/data bit}$$

or one hop per MFSK symbol (no diversity).

In summary, with no diversity we have the bit error bounds

$$P_b \leq \begin{cases} 2^{K-2} e^{-K \frac{E_b}{2N_o}} & \text{, broadband jamming} \\ \dfrac{2^{K-1}}{Ke(E_b/N_o} & \text{, worst case partial band jamming} \end{cases}$$

These bounds are generally quite tight below 10^{-2} bit error probabilities. In Figures 1-4 we show these bounds as the exact bit error probabilities. (See dashed lines.)

With each MFSK signal hopped m times and using noncoherent combining of the m energy detector outputs we require a looser Chernoff bound to evaluate the bit error probabilities. Assuming soft decision combining where the receiver knows when a chip is jammed or not,* we have the bit error bound (see note 2)

$$P_b \leq 2^{K-2} \left[\frac{\rho}{1-\lambda^2} e^{-\frac{\lambda}{1+\lambda} \rho \left(\frac{E_c}{N_o} \right)} \right]^m$$

where λ is a Chernoff bound parameter and $E_c = KE_b/m$ is the energy per chip.

*This means all m chips must be jammed in order to cause any symbol error.

Figure 1. No Fading M=2

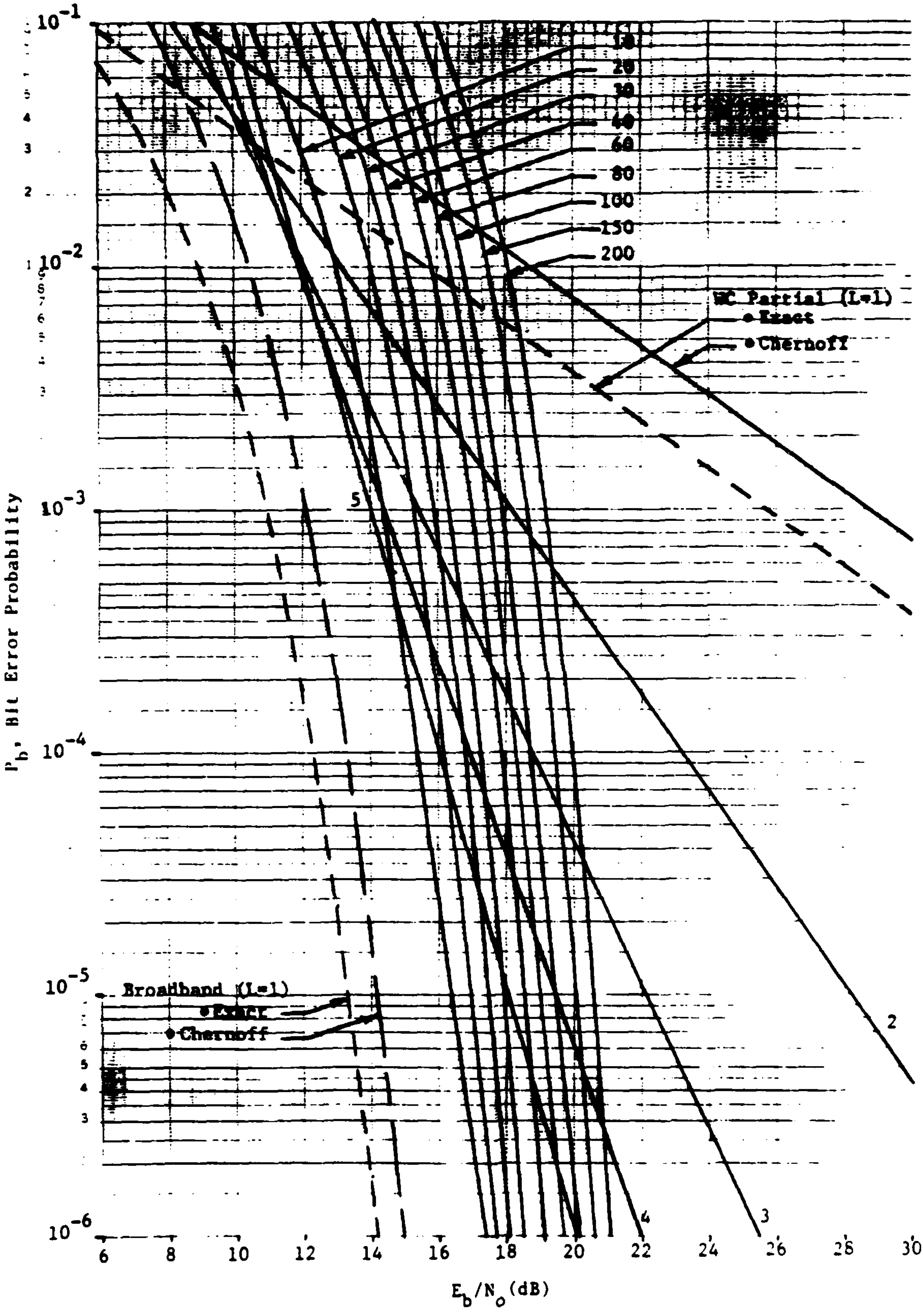


Figure 2. No Fading M=4

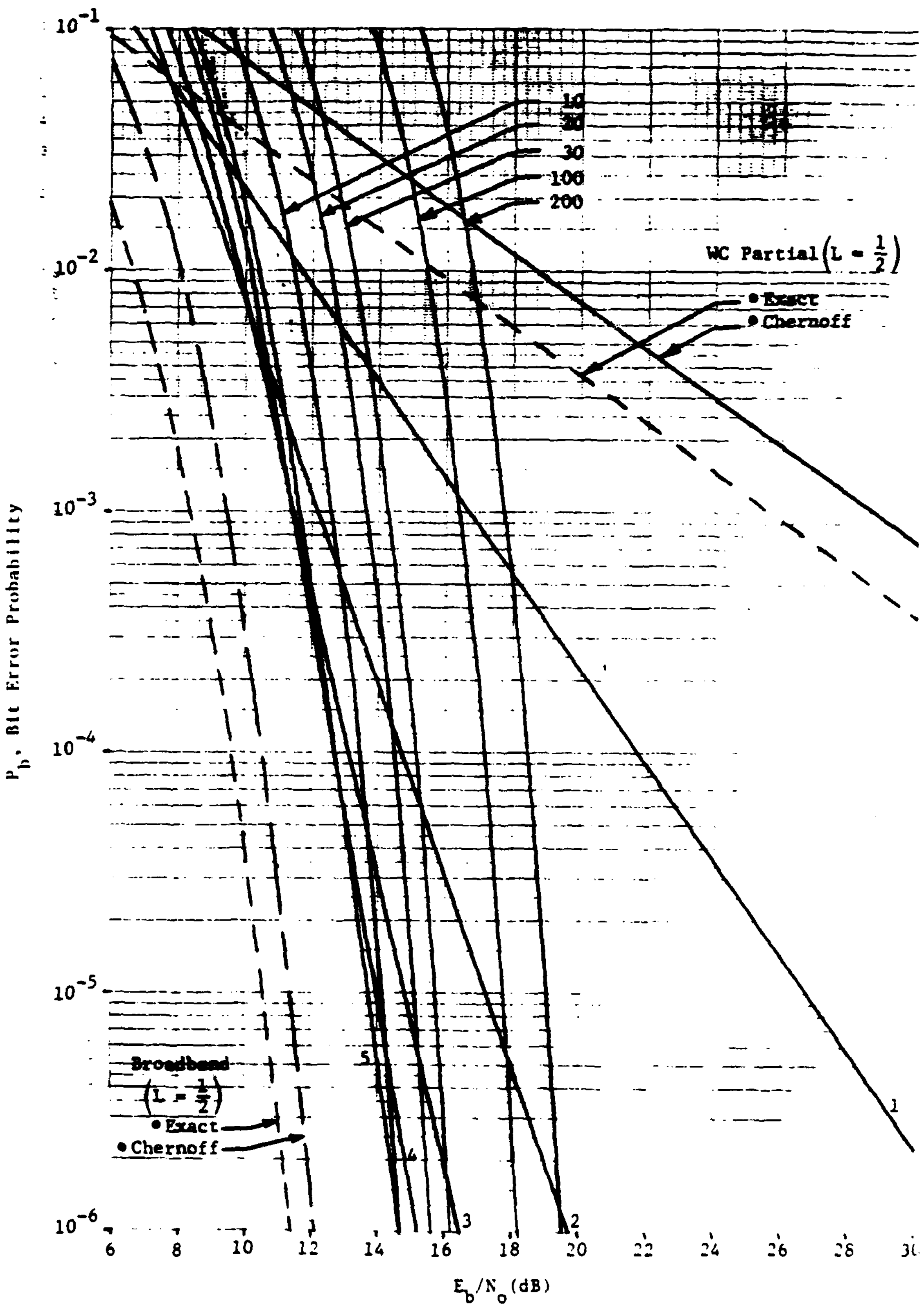


Figure 3. No Fading M=8

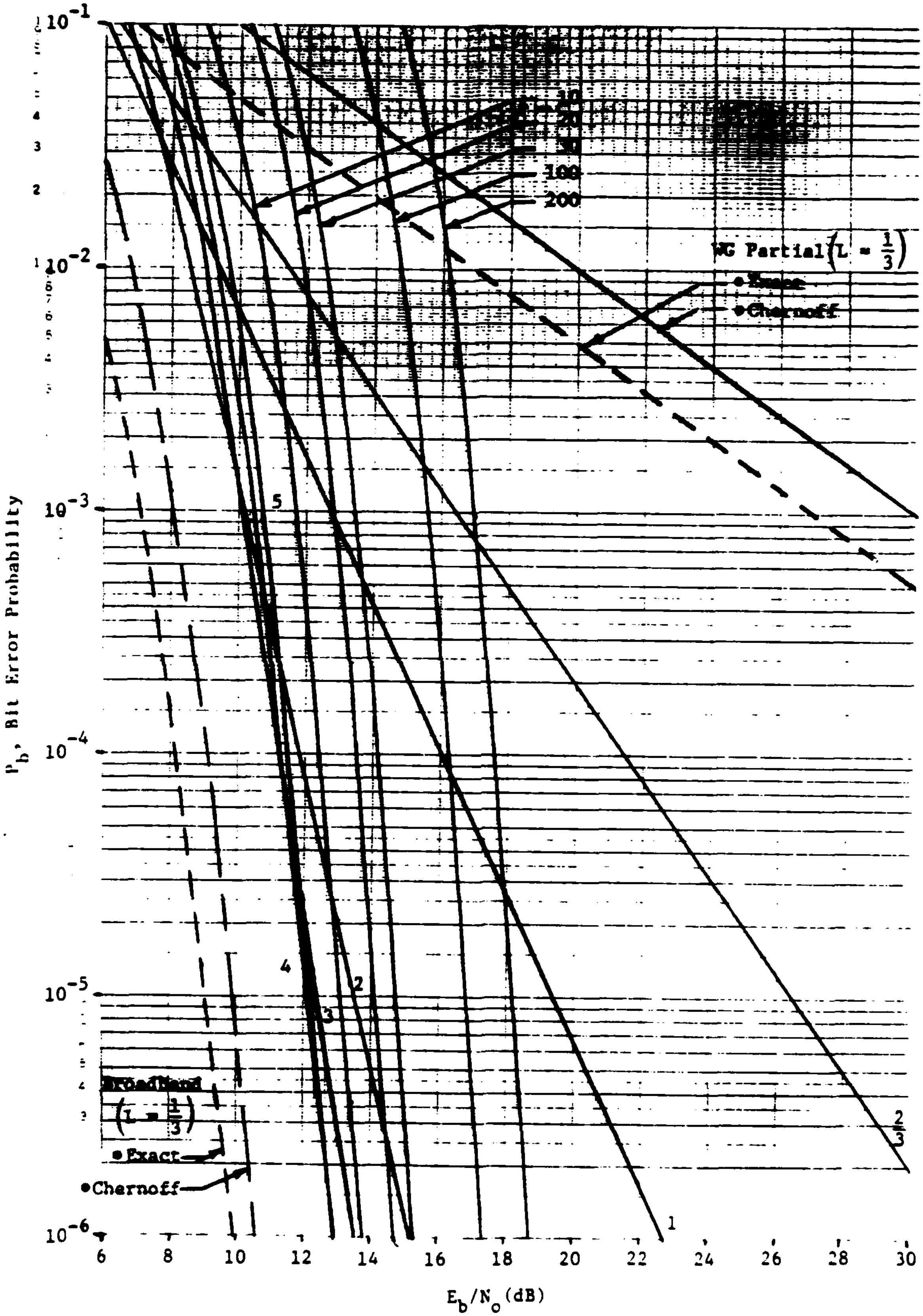
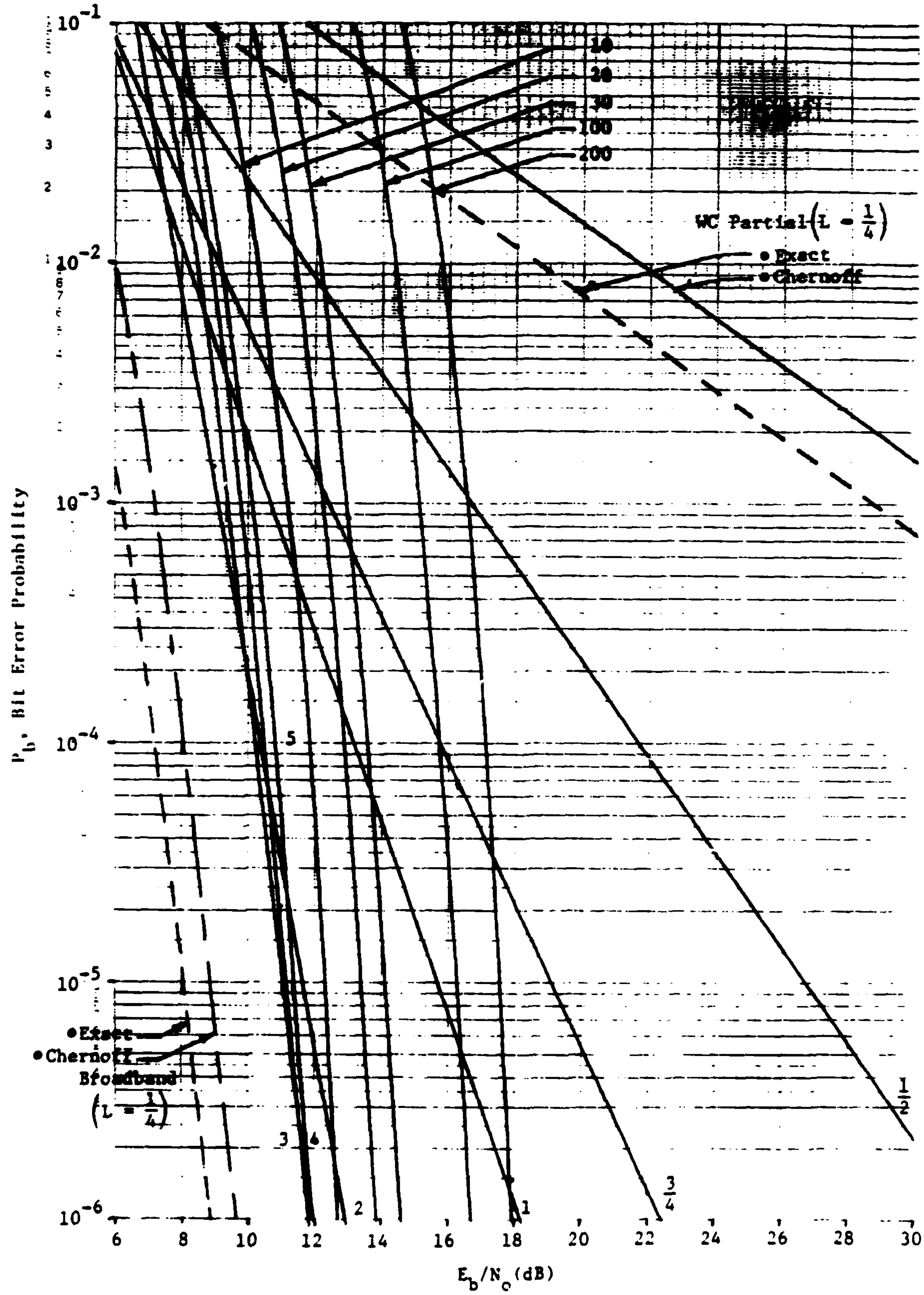


Figure 4. No Fading M=16



For the case where $E_c/N_o \geq 3$ we have the worst case ρ value that maximizes this bound and the best λ value that minimizes this bound given by

$$\lambda = \frac{1}{2}$$

$$\rho^* = 3\left(\frac{E_c}{N_o}\right)^{-1}$$

This results in the bound

$$P_b \leq 2^{K-2}\left[\frac{4m}{Ke(E_b/N_o)}\right]^m$$

or in terms of

$$L = \frac{m}{K} \quad \text{hops/ data bits}$$

we have

$$P_b \leq 2^{K-2}\left[\frac{4L}{e(E_b/N_o}\right]^{LK}$$

When $E_c/N_o < 3$ the worst case ρ that maximizes the bound is simply

$$\rho^* = 1 \quad \text{(broadband jamming)}$$

while the minimizing choice of the Chernoff bound parameter is

$$\lambda = \frac{1}{2}\left[\sqrt{1+6a+a^2} -1-a\right]$$

where

$$a = \left(\frac{1}{2L}\right)\left(\frac{E_b}{N_o}\right).$$

This is used in

$$P_b \leq 2^{K-2}\left[\frac{1}{1-\lambda^2}\, e^{-\left(\frac{\lambda}{1+\lambda}\right)\left(\frac{1}{L}\right)\left(\frac{E_b}{N_o}\right)}\right]^{KL}.$$

In summary, for noncoherent MFSK with $M = 2^K$ and L hops per data bits we have

$$P_b \le \begin{cases} 2^{K-2}\left[\dfrac{1}{1-\lambda^2} e^{-\frac{\lambda}{1+\lambda}\left(\frac{1}{L}\right)\left(\frac{E_b}{N_o}\right)}\right]^{KL} & ; \dfrac{E_b}{N_o} < 3L \\ 2^{K-2}\left[\dfrac{4L}{e(E_b/N_o)}\right]^{KL} & ; \dfrac{E_b}{N_o} \ge 3L \end{cases}$$

where

$$\lambda = \frac{1}{2}\left[\sqrt{1+6a+a^2} \ -1-a\right]$$

$$a = \left(\frac{1}{2L}\right)\left(\frac{E_b}{N_o}\right).$$

Figures 1-4 show these bounds for various values of L. For $L = 1/K$ we have no diversity and so we can compare these Chernoff bounds with the tighter bounds derived for this special case which we label as exact.

III. WORST CASE PARTIAL BAND NOISE JAMMING WITH CHANNEL FADING

We next consider a fading channel where each MFSK hop band has independent "flat-flat" Rayleigh fading with the same characteristics. Thus, each MFSK "chip" is assumed to experience independent Rayleigh fading with the same Rayleigh fading probability density function

$$f(r) = 2re^{-r^2} \qquad r \ge 0$$

where we have normalized to

$$E\{R^2\} = \int_0^\infty r^2 f(r)dr = 1.$$

This means that for a random fading envelope R the received energy per chip is R^2E_c where E_c is the energy with no fading. Then the average received energy per chip is

$$
\begin{aligned}
\bar{E}_c &= E\left\{R^2 E_c\right\} \\
&= E\left\{R^2\right\} E_c \\
&= E_c .
\end{aligned}
$$

As discussed earlier, we assume a partial band noise jammer with parameter ρ and noise density N_o/ρ for ρ fraction of the band where $N_o = J/W$. With no diversity each MFSK symbol experiences a single fade R. Defining $P_E(\alpha;R)$ as the symbol error probability in white Gaussian noise of spectral density α and fade R we have the symbol error probability

$$P_E(R) = \rho P_E(N_o/\rho;R) + (1-\rho)P_E(0;R).$$

Here $P_E(0;R) = 0$ for any R and we use the union bound

$$P_E(N_o/\rho;R) \leq \frac{1}{2}(M-1)e^{-\rho R^2 \frac{E_c}{2N_o}}$$

The bit error bound for fixed R is thus

$$P_b(R) \leq \frac{1}{4} M\rho e^{-\rho R^2 \frac{E_c}{2N_o}}$$

Averaging this over the fading random variable R gives the bound on the average bit error probability

$$
\begin{aligned}
P_b &= \int_0^\infty P_b(r)f(r)dr \\
&\leq \frac{1}{4} M\rho \frac{1}{1+(\bar{E}_c/N_o)\rho/2} \\
&= \frac{\rho 2^{K-1}}{2+K\rho(\bar{E}_b/N_o)}
\end{aligned}
$$

where $\bar{E}_b$ is the average received energy per bit. Since

$$\frac{d}{d\rho}\left[\frac{\rho}{1+a\rho}\right] = \left(\frac{1}{1+a\rho}\right)^2$$

is positive, the largest possible value of ρ maximizes this bound.

Thus

$$\rho^* = 1 \quad \text{(broadband jamming)}$$

and

$$P_b \leq \frac{2^{K-1}}{2+K(\bar{E}_b/N_o)}$$

In Figures 5-8 we show this union bound as a dotted line for M = 2, 4, 8, and 16.

If we now hop each MFSK symbol m times then an error occurs only when all m chips fall in the jammed band. If

$$\underline{R} = (R_1, R_2, \ldots, R_m)$$

are the independent Rayleigh Fading variables for the m chips, then the conditional symbol error probability is

$$P_E(\underline{R}) = \rho^m P_E(N_o/\rho;\underline{R})$$

where $P_E(N_o/\rho;\underline{R})$ is the error probability with white Gaussian noise of spectral density N_o/ρ with fixed fade variables $\underline{R}$. If we average this over $\underline{R}$ we know that the maximum likelihood metric is indeed the noncoherent combining of the m energy detector outputs for the M possible symbols. We can thus apply the union Bhattacharyya bound on the average to get,

$$\int P_E(N_o/\rho;\underline{r})f(\underline{r})d\underline{r}$$

$$\leq \frac{1}{2}(M-1)\left\{\frac{4[1+\rho(\bar{E}_c/N_o)]}{2+\rho(\bar{E}_c/N_o)]^2}\right\}^m$$

yielding the symbol error bound

$$P_E = \int P_E(\underline{r})f(\underline{r})d\underline{r}$$

Figure 5. Fading M=2

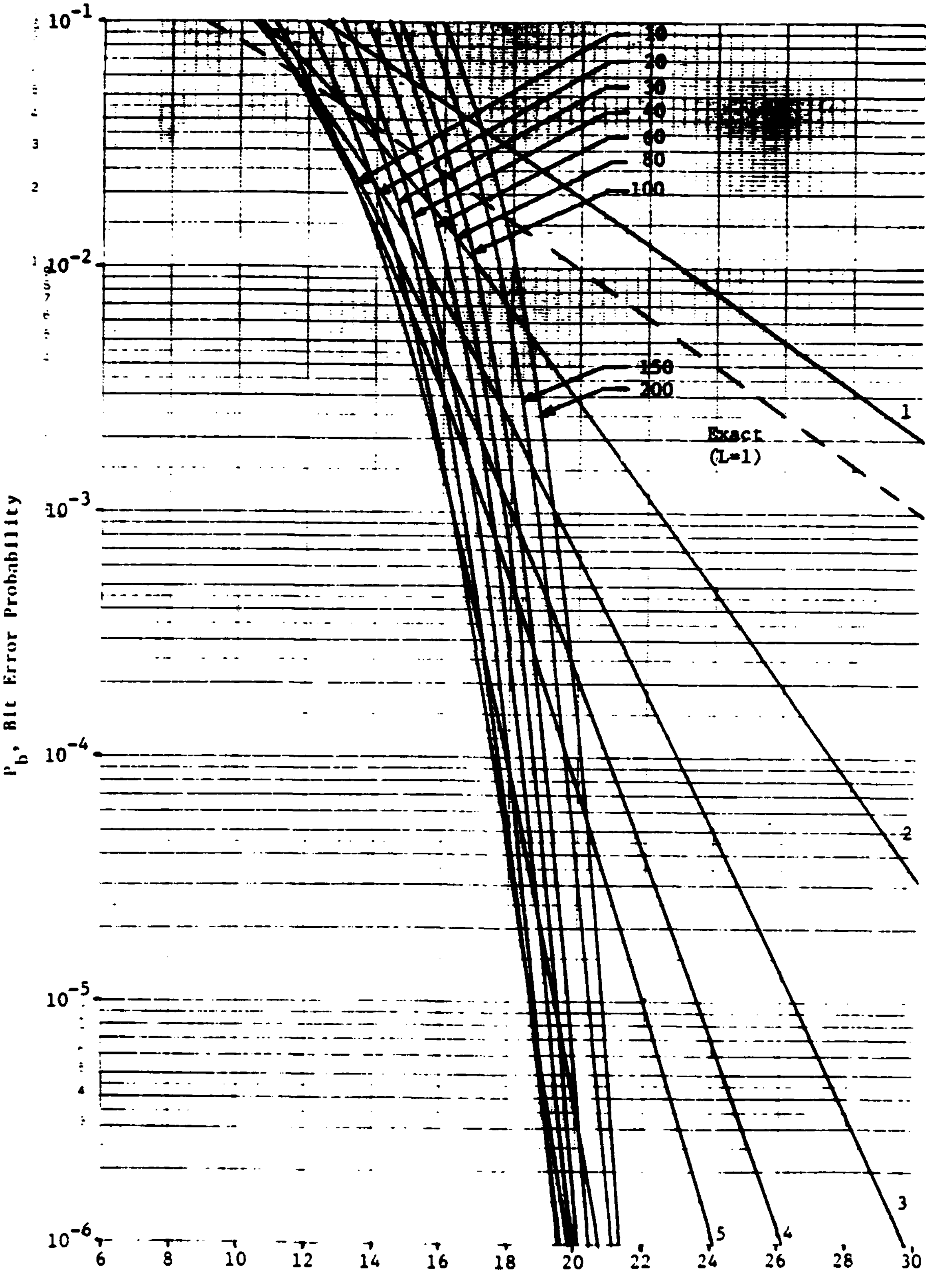


Figure 6. Fading M=4

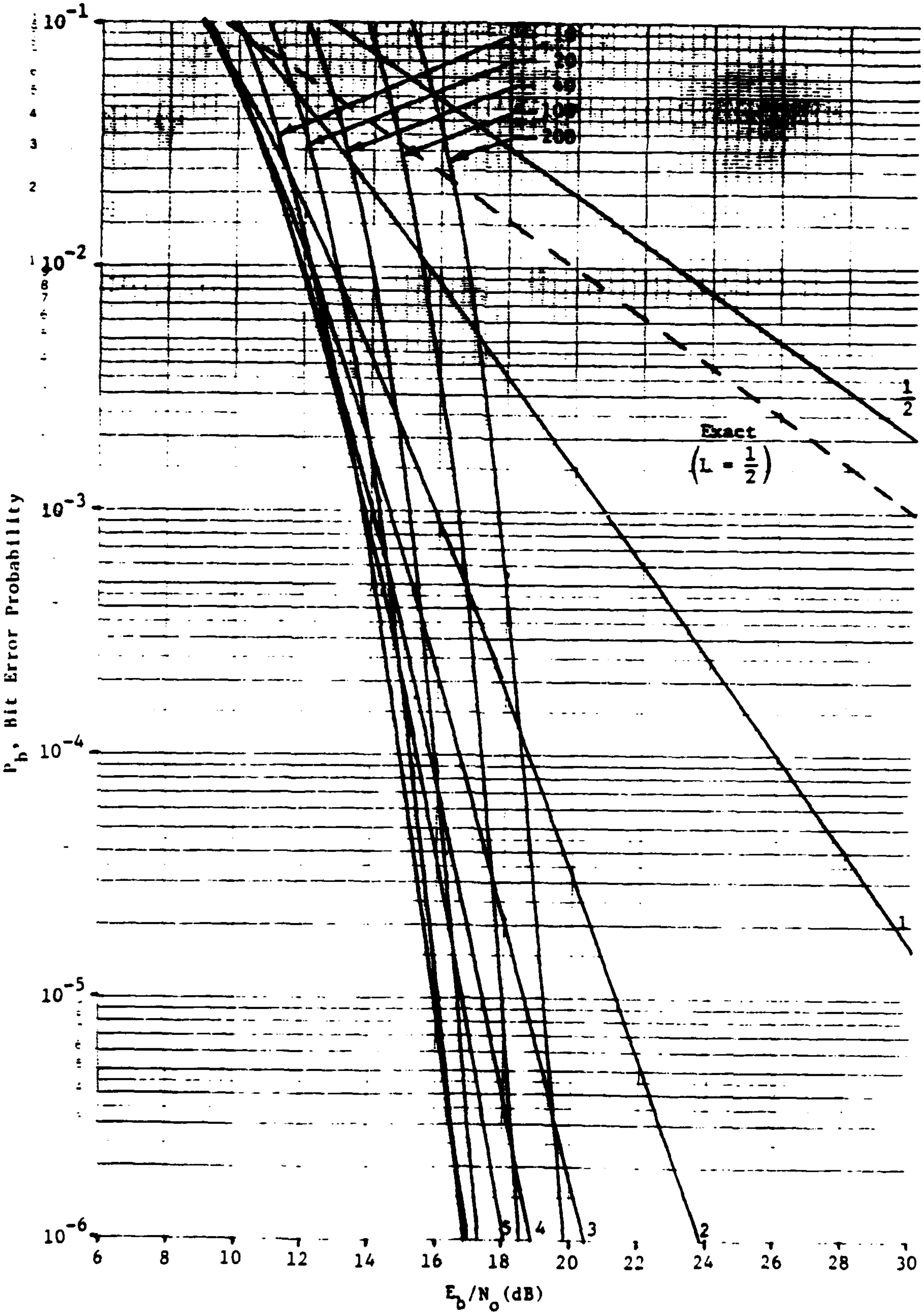


Figure 7. Fading M=8

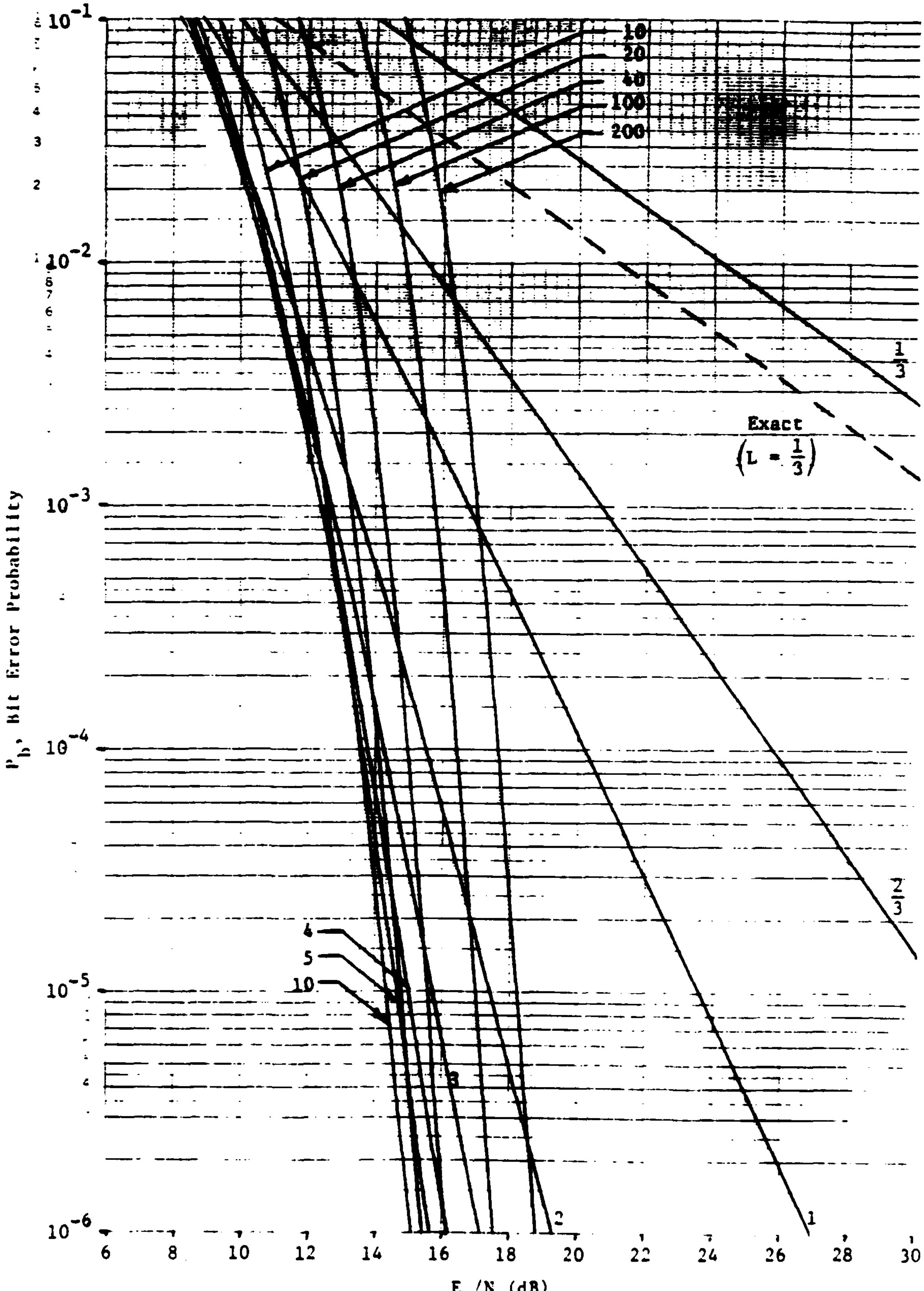
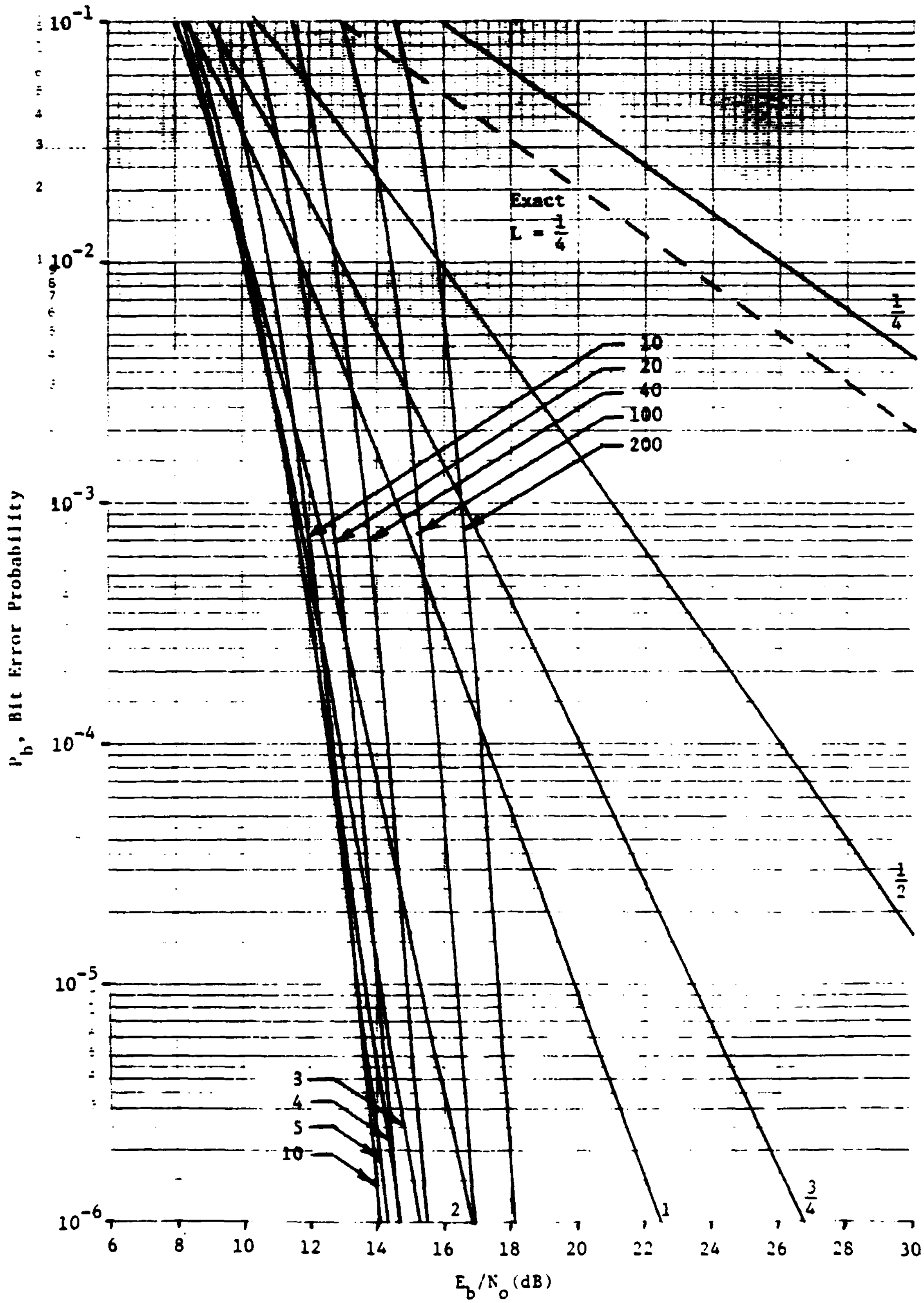


Figure 8. Fading M=16



$$\leq \frac{1}{2}\,(M-1)\rho^{m} \left\{ \frac{4[1+\rho(\bar{E}_c/N_o)]}{[2+\rho(\bar{E}_c/N_o)]^2} \right\}^{m}$$

Again noting that

$$\frac{d}{d\rho} \left\{ \frac{\rho(1+2a\rho}{(1+a\rho)^2} \right\}$$

is positive the largest possible value of ρ maximizes this bound.

Thus

$$\rho^* = 1 \qquad \text{(broadband jammer)}$$

and the bit error bound becomes,

$$P_b \leq \frac{1}{4}\,M \left\{ \frac{4[1+(\bar{E}_c/N_o)]}{[2+(E_c/N_o)]^2} \right\}^{m}$$

Substituting the relationships

$$M = 2^K$$

$$\bar{E}_c = K\bar{E}_b/m$$

$$m = LK$$

we have the final form of this bound

$$P_b \leq 2^{K-2} \left\{ \frac{4L^2+4L(\bar{E}_c/N_o)}{[2L+(\bar{E}_b/N_o)]^2} \right\}^{LK}$$

Figures 5-8 show this bound for various values of L hops per data bit with $M = 2, 4, 8$, and 16.

IV. USE OF CODES — NO FADING

When considering the use of codes we can regard each MFSK chip as a code symbol. As shown in Note 4 when we have two sequences $\underline{x}$ and $\hat{\underline{x}}$ the pairwise error probability is given by

$$P(\underline{x} \to \hat{\underline{x}}) \leq D^{w(\underline{x},\hat{\underline{x}})}$$

where $w(\underline{x},\hat{\underline{x}})$ is the number of places where $\underline{x}$ and $\hat{\underline{x}}$ differ. For the non-fading channel with worst case partial band jamming we have (see Note 4)

$$D = \min_{0\le\lambda\le 1} \max_{0\le\rho\le 1} \left\{ \frac{\rho}{1-\lambda^2} e^{-\left(\frac{\lambda}{1+\lambda}\right)\rho\left(\frac{E_c}{N_o}\right)} \right\}$$

$$= \begin{cases} \dfrac{1}{1-\lambda^2} e^{-\left(\frac{\lambda}{1+\lambda}\right)\left(\frac{E_c}{N_o}\right)} & , \quad \dfrac{E_c}{N_o} \le 3 \\[2ex] \dfrac{4}{e(E_c/N_o)} & , \quad \dfrac{E_c}{N_o} > 3 \; . \end{cases}$$

where

$$\lambda = \frac{1}{2}\left[\sqrt{1+6a+a^2} -1-a\right]$$

$$a = \frac{1}{2}\left(\frac{E_c}{N_o}\right).$$

Note that the symbol error probability for MFSK with m diversity chips per symbol is union bounded by

$$P_E \le \frac{1}{2}\,(M-1)D^m$$

since $w(\underline{x},\hat{\underline{x}}) = m$. Using the relationship $KE_b = mE_c$, $L = m/K$, and

$$P_b = \frac{\frac{1}{2}M}{M-1} P_E$$

gives us the same bound derived in Section II. In this sense, diversity can be thought of as a form of coding.

For the conventional constraint length $K = 7$ rate, $r = \frac{1}{2}$ binary convolutional code using BFSK with m diversity we have the bit error bound

$$P_b \leq 36D^{10m} + 211D^{12m} + 1404D^{14m}$$

$$+ 11633D^{16m} + \ldots.$$

where

$$E_b = 2E_s$$

$$= 2mE_c$$

$$E_s = \text{energy per coded BFSK symbol.}$$

Here we assume m diversity per coded BFSK symbol. Hence, the diversity per data bit is

$$L = 2m$$

Figure 9 shows the bit error bounds for the binary convolutional code using BFSK and diversity of L hops per data bit.

V. <u>USE OF CODES-FADING</u>

For independent Rayleigh fading on each MFSK chip we have the same metric and the only change in the form of D is an average over the independent fading statistics. Thus

$$D = \min_{0\leq\lambda\leq 1} \max_{0\leq\rho\leq 1} E\left\{\frac{\rho}{1-\lambda^2} e^{-\left(\frac{\lambda}{1+\lambda}\right)\rho R^2\left(\frac{E_c}{N_o}\right)}\right\}$$

$$= \min_{0<\lambda<1} \max_{0<\rho<1} \left\{\frac{\rho}{1-\lambda^2+\lambda(1-\lambda)\rho(E_c/N_o)}\right\}$$

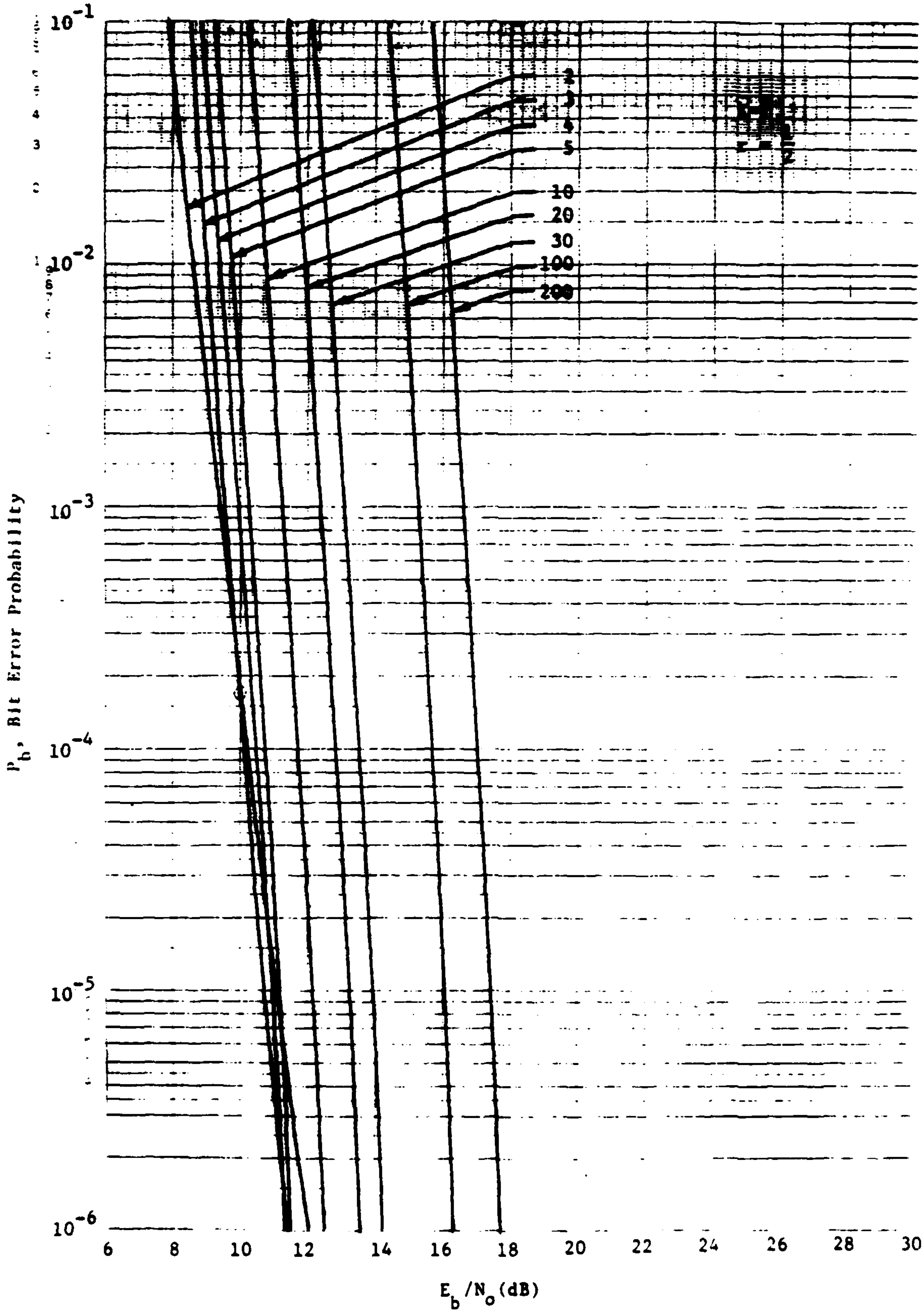
$$= \min_{0\leq\lambda\leq 1} \left\{\frac{1}{1-\lambda^2+\lambda(1-\lambda)(E_c/N_o)}\right\}$$

$$= \frac{4[1+(E_c/N_o)]}{[2+(E_c/N_o)]^2}.$$

where the maximizing ρ is

$$\rho^* = 1 \quad \text{(broadband jamming)}$$

Figure 9. No Fading Convolutional Code with BFSK



and the minimizing Chernoff bound parameter is (see Notes 5 and 6)

$$\lambda = \frac{E_c/N_o}{2[1+(E_c/N_o)]} .$$

Note that the symbol error probability for MFSK with m diversity chips per symbol is union bounded by

$$P_E \leq \frac{1}{2} (M-1)D^m$$

since $w(\underline{x},\hat{\underline{x}}) = m$. Using the relationships $KE_b = mE_c$, $L = m/K$ and

$$P_b = \frac{\frac{1}{2} M}{M-1} P_E$$

gives us the same bound derived in Section III.

In general, the bound for the fading channel is always an upper bound for the non-fading channel. This follows from the fact that for a convex $\cup$ function f(x) and any random variable X we have Jensen's inequality

$$f(E\{X\}) \leq E\{f(X)\}.$$

Note that

$$e^{-\left(\frac{\lambda}{1+\lambda}\right) \rho R^2 \left(\frac{E_c}{N_o}\right)}$$

is a convex $\cup$ function of R^2 and $E\{R^2\} = 1$. Thus

$$\frac{\rho}{1-\lambda^2} e^{-\left(\frac{\lambda}{1+\lambda}\right) \rho \left(\frac{E_c}{N_o}\right)} \leq E\left\{\frac{\rho}{1-\lambda^2} e^{-\left(\frac{\lambda}{1+\lambda}\right) \rho R^2 \left(\frac{E_c}{N_o}\right)}\right\}$$

$$= \frac{\rho}{1-\lambda^2+\lambda(1-\lambda)\rho(E_c/N_o)} .$$

For the conventional constraint length K = 7, rate $r = \frac{1}{2}$ binary convolutional code using BFSK with m diversity we have the bit error bound

$$P_b \leq 36D^{10m} + 211D^{12m} + 1404D^{14m}$$

$$+ 11633D^{16}m + \ldots$$

where

$$E_b = 2E_s$$

$$= 2mE_c$$

$$L = 2m \ .$$

Figure 10 shows the bit error bounds for the binary convolutional code using BFSK and diversity of L hops per data bit.

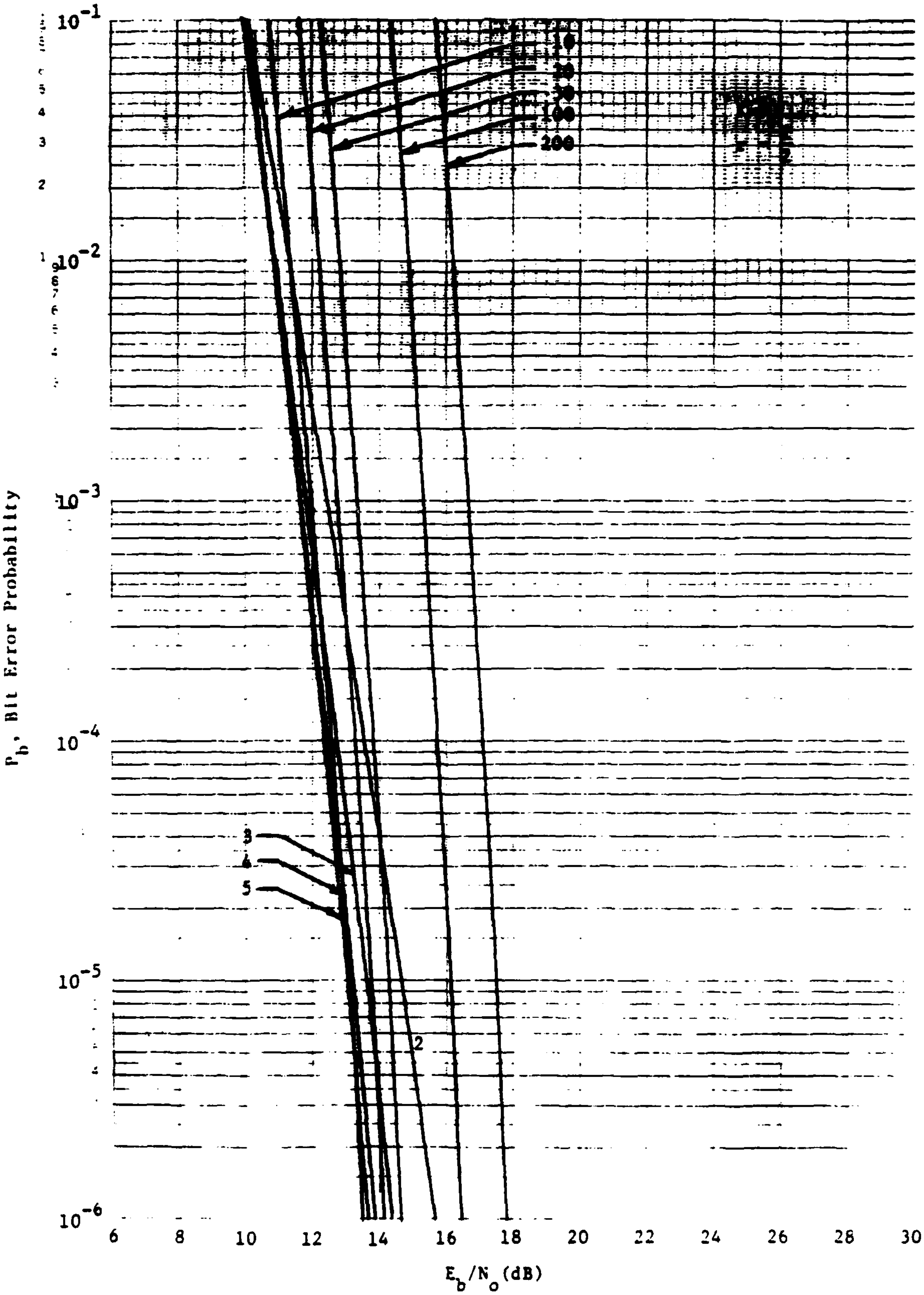
VI. DISCUSSION

Figures 1-8 show clearly that with too little diversity worst case partial band jamming or Rayleigh fading can cause a large loss in performance relative to the ideal additive white Gaussian noise channel (broadband jamming with no fading). Excessive diversity, however, means considerably less loss in performance. At $P_b = 10^{-6}$ and $M = 2$ (see Figure 1) there is about a 4dB loss from optimum diversity ($L \tilde{=} 10$) to the diversity $L = 200$. This difference increases to about 6dB with $M = 16$. This is primarily due to the fact that the performance for larger alphabet size M improves with the smaller diversity values. This is essentially a coding gain since MFSK modulation is a form of block orthogonal codes. The BFSK convolutional code performance of Figure 9 shows this same characteristic.

For the Rayleigh fading channels shown in Figures 5-8, the loss due to excessive diversity is less than corresponding losses in the non-fading case. This is partly due to the fact that the noncoherent combining of the chip energies is indeed the maximum likelihood metric for the Rayleigh fading channel. More important, however, is the fact that here the worst case partial band jammer is always the broadband jammer.

The Rayleigh fading case always results in poorer performance or larger bit error probabilities than the non-fading case. For large diversity,

Figure 10. Fading Convolutional Code with BFSK



however, the difference between these two cases disappear.

In the following table we show the values of E_b/N_o at $P_b = 10^{-6}$ for all the cases shown in Figures 1-8. This shows that the loss due to excessive diversity is greater for larger alphabet size M even though the overall performance is better. This is most likely true for convolutional codes with large constraing lengths.

Table $P_b = 10^{-6}$

		M=2	M=4	M=8	M=16
Non-Fading	L_{opt}	17.3	14.6	12.9	11.8
	L=200	21.0	19.5	18.7	18.0
Fading	L_{opt}	19.5	16.9	15.1	14.1
	L=200	21.3	19.8	18.8	18.1

Note No. 8

PERFORMANCE OF CONVOLUTIONALLY CODED NONCOHERENT FH/MFSK SYSTEMS

to

NAVAL RESEARCH LABORATORY

(Contract Award No. N00014-80-K-0935)

for

HF COMMUNICATION NETWORK SIGNALS

USING CHANNEL EVALUATION DATA

Principal Investigator

Jim K. Omura
Professor
System Science Department
University of California
Los Angeles, California

July, 1981

I. INTRODUCTION

Optimum binary alphabet convolutional codes commonly used with coherent BPSK and QPSK modulations were found by Odenwalder [1] for constraint lengths up to $K = 9$. These binary codes were selected on the basis of the Hamming distance between coded binary sequences. For symmetric M$\sim$ary input coding channels such as those resulting from MFSK modulation, code selection should be based on the Hamming distance between coded M$\sim$ary sequences. Lyon [2] and Trumpis [3] found some optimum convolutional codes for $M = 4$ and $M = 8$. These codes are rate $r = 1$ codes where there is one coded M$\sim$ary symbol for every data binary symbol. For $M = 2^n$ they are equivalent to binary rate $1/n$ codes where the n coded binary symbols are used to form a single $2^n$$\sim$ary symbol.

In this note we show the bit error bounds for noncoherent FH/MFSK signals with various optimum convolutional codes for constraint length $K = 7$ and alphabet size $M = 2$, 4, and 8. This is done for a wide range of diversity values with the worst case partial band jammer. Non-fading and Rayleigh fading channels are included in this note.

II. BIT ERROR BOUNDS

Our basic MFSK modulation is frequency hopped once every T_c seconds. The MFSK pulse during each hop is sometimes referred to as a "chip." These form the basic M$\sim$ary input coding channel. As shown in Note 4, the resulting coding channel has a cutoff rate

$$R_0 = \log_2 M - \log_2[1+(M-1)D] \quad \text{bits}$$

where

$$D = D(E_c/N_o)$$

depends on the channel noise, jammer, receiver characteristics, and the equivalent chip energy to noise ratio denoted E_c/N_o. We assume worst case partial band jamming

where

$$N_o = \frac{J}{W}$$

and

J = jammer power

W = total spread bandwidth.

For the non-fading channel, soft decision additive chip energy metric, jammer state known at the receiver, and worst case partial band jamming we have

$$D = \min_{0\le\lambda\le 1} \max_{0\le\rho\le 1} \left\{ \frac{\rho}{1-\lambda^2} e^{-\left(\frac{\lambda}{1+\lambda}\right)\rho\left(\frac{E_c}{N_o}\right)} \right\}$$

$$= \begin{cases} \dfrac{1}{1-\lambda^2} e^{-\left(\frac{\lambda}{1+\lambda}\right)\left(\frac{E_c}{N_o}\right)} & , \quad \dfrac{E_c}{N_o} \le 3 \\[2ex] \dfrac{4}{e(E_c/N_o)} & , \quad \dfrac{E_c}{N_o} > 3 \end{cases}$$

where

$$\lambda = \frac{1}{2}\left[\sqrt{1+6a+a^2} \;\; -1-a\right]$$

$$a = \frac{1}{2}\left(\frac{E_c}{N_o}\right).$$

For the same channel with Rayleigh fading we have

$$D = \frac{4[1+(E_c/N_o)]}{[2+(E_c/N_o)]^2} .$$

The number of chips or hops per data bit is denoted

L = chips/bit .

For uncoded MFSK where $M = 2^n$ we have

$$E_b = LE_c$$

as the energy per data bit. Here the union-Chernoff bit error bound is (see Note 4)

$$P_b \leq 2^{n-2} D^{nL}.$$

For a code of rate

$$r = \text{bits/M}\sim\text{ary symbol}$$

we have as before

$$E_b = LE_c$$

as the energy per bit. The bit error union-Chernoff bound is

$$P_b \leq \frac{1}{2} \sum_{k=d_{min}}^{\infty} N_k D^{rLk}$$

where $\{N_k\}$ and d_{min} are parameters of the specific convolutional code.

For constraint length K = 7 parameters for good codes found by Odenwalder are

$M = 2$	$r = \frac{1}{2}$	$d_{min} = 10$
$N_{10} = 36$		$N_{14} = 1404$
$N_{11} = 0$		$N_{15} = 0$
$N_{12} = 211$		$N_{16} = 11633$
$N_{13} = 0$		$N_{17} = 0$

$M = 2$	$r = \frac{1}{3}$	$d_{min} = 14$
$N_{14} = 1$		$N_{18} = 53$
$N_{15} = 0$		$N_{19} = 0$
$N_{16} = 20$		$N_{20} = 184$
$N_{17} = 0$		$N_{21} = 0$

Rate r = 1 codes of Trumpis are:

$M = 4$	$r = 1$	$d_{min} = 7$
$N_7 = 7$		$N_9 = 134$
$N_8 = 39$		$N_{10} = 352$
		$N_{11} = 1348$

$M = 8$	$r = 1$	$d_{min} = 7$
$N_7 = 1$		$N_9 = 8$
$N_8 = 4$		$N_{10} = 49$
		$N_{11} = 92$

III. DISCUSSION

For M = 2 and the rate $r = \frac{1}{2}$ and $r = \frac{1}{3}$ codes found by Odenwalder, we have the curves in Figures 1 and 2 showing the bit error bounds discussed above for various values of diversity L. There is little difference between these two code rates. The codes found by Trumpis for M = 4 and M = 8 are shown in Figures 3 and 4 which show 1 to 2 dB improvement. Figures 5 through 8 show the same situation with the Rayleigh fading channel.

For the extreme case where the data rate and hop rate are equal (L=1), we can compare the relative performance of the various codes as shown in Figures 9 and 10. For M = 4 and 8 we assume there is one hop per coded M~ary symbol. For the M = 2 case with rate r = 1/n we assume n coded symbols are hopped together. For the cases $r = \frac{1}{2}$ and $r = \frac{1}{3}$ we approximate the L = 1 case shown here by using the bound derived in Section II. Although this is an approximation, we feel it is close to the true bit error probabilities. Based on Figures 9 and 10, M = 4 or M = 8 is preferred over M = 2 with optimum convolutional codes of constraint length K = 7. For HF channels where there is a limited number of MFSK subchannels, M = 4 with the K = 7 convolutional code appears to be a good choice.

IV. REFERENCES

[1] J. P. Odenwalder, Optimal Decoding of Convolutional Codes, Ph.D. dissertation, System Science Department, University of California, Los Angeles, California, 1970.

[2] R. F. Lyon, "Convolutional Codes for M~ary Orthogonal and Simplex Channels," Jet Propulsion Laboratory, Pasadena, California, Deep Space Network Progress Report 42-24, pp. 60-77, 1974.

[3] B. D. Trumpis, Convolutional Codes for M~ary Channels, Ph.D. dissertation, System Science Department, University of California, Los Angeles, California, 1975.

Figure 1 $M = 2,\ r = \frac{1}{2}$

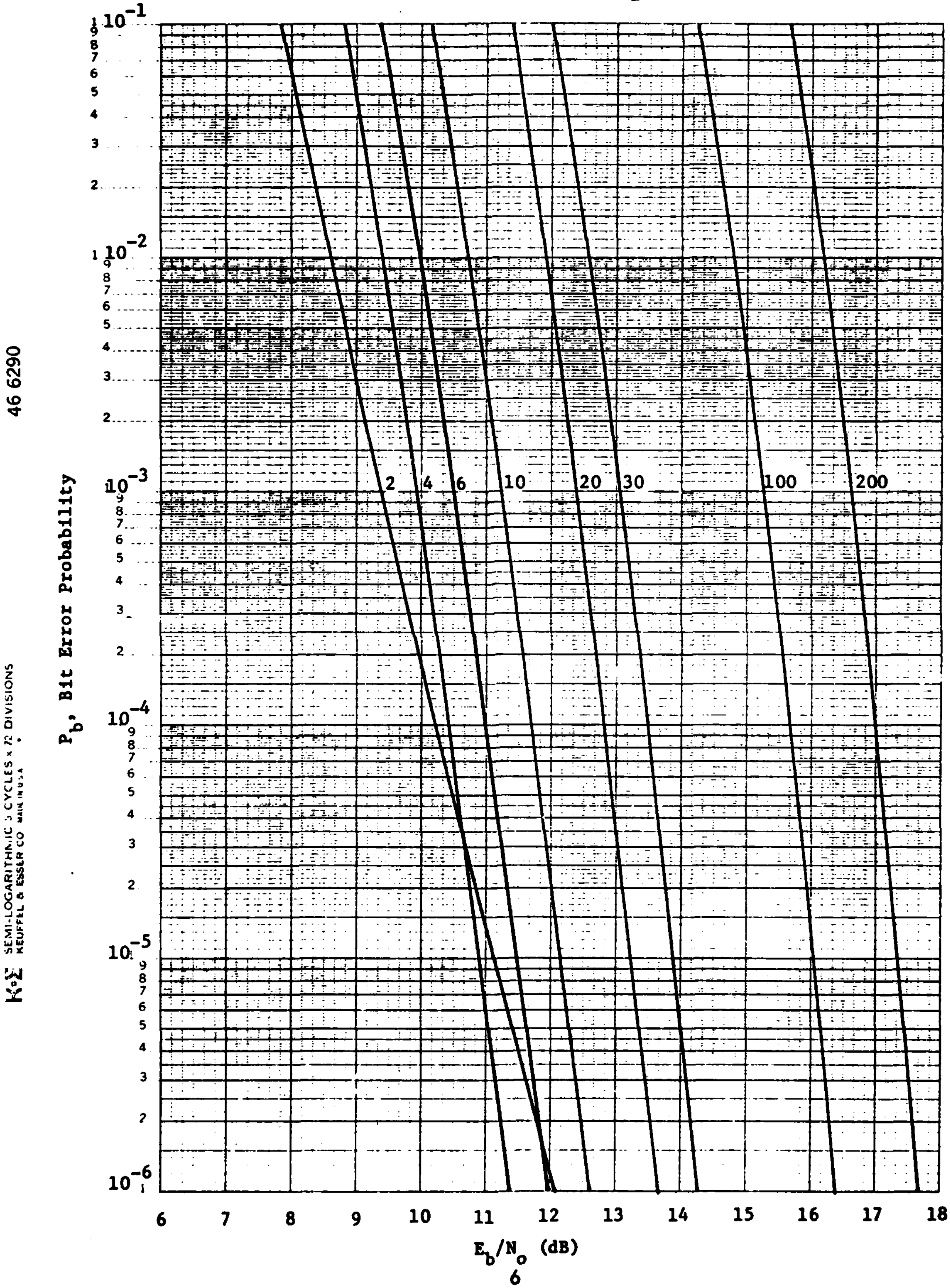


Figure 2 $M = 2,\ r = \frac{1}{3}$

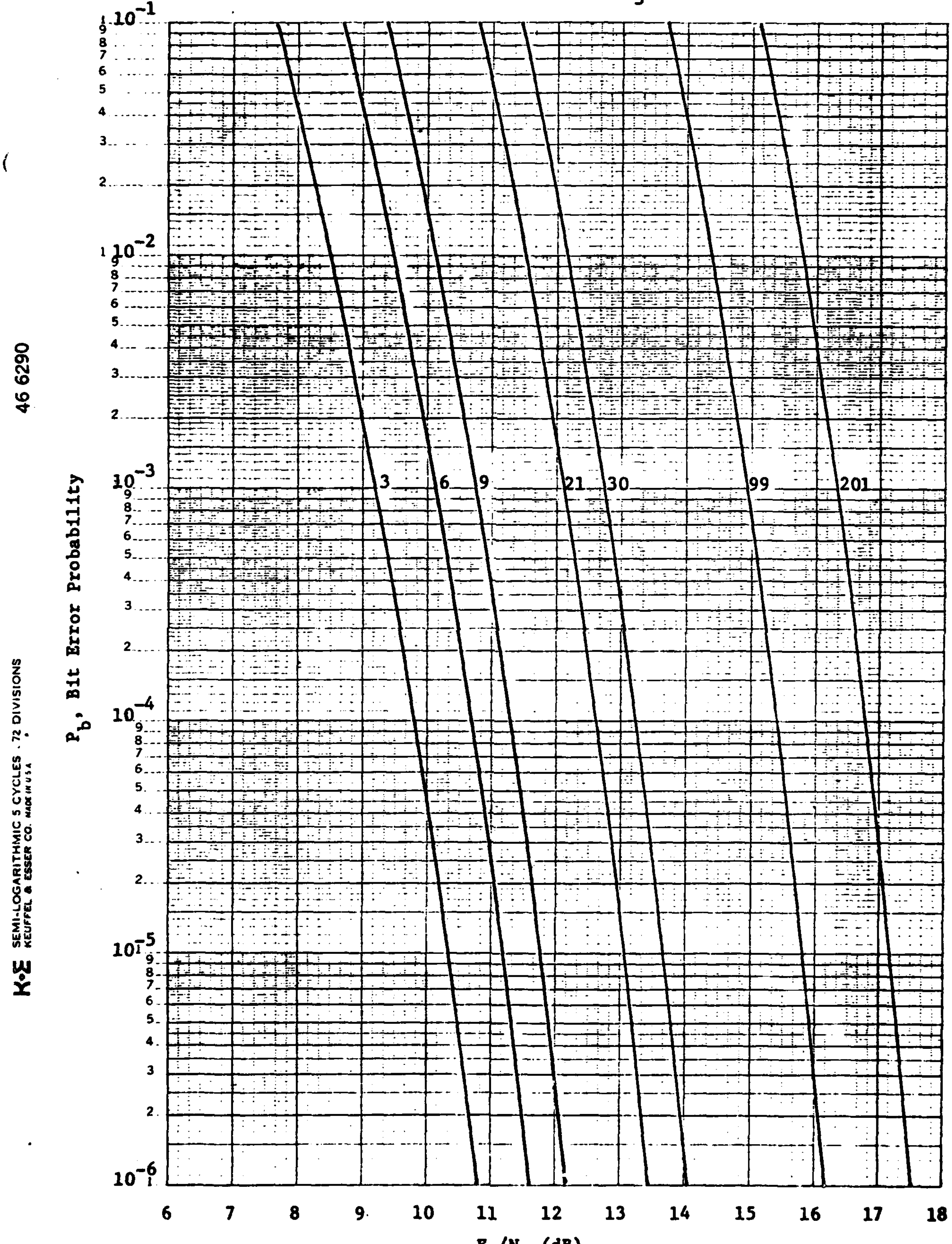


Figure 3 M = 4, r = 1

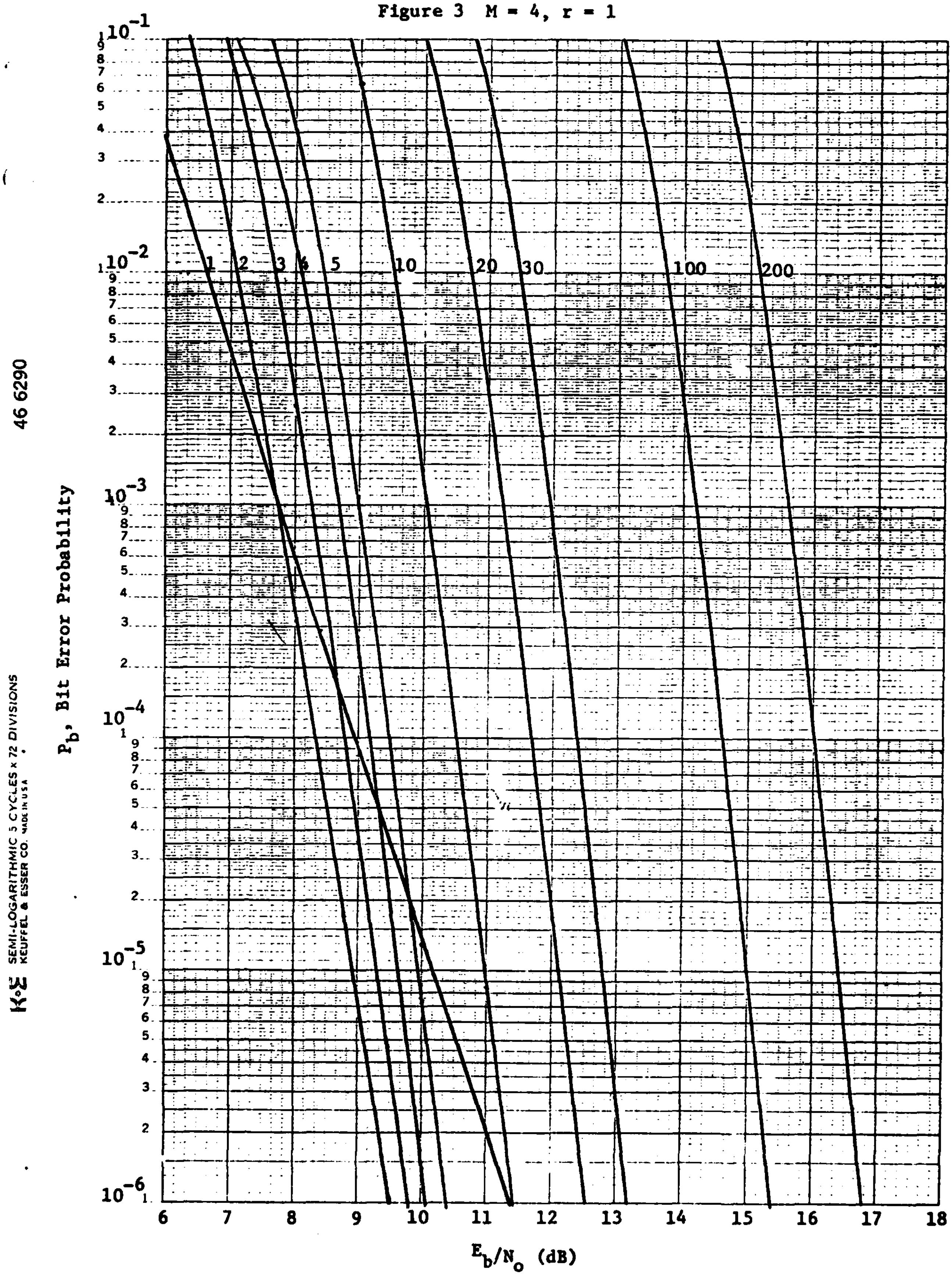


Figure 4 M = 8, r = 1

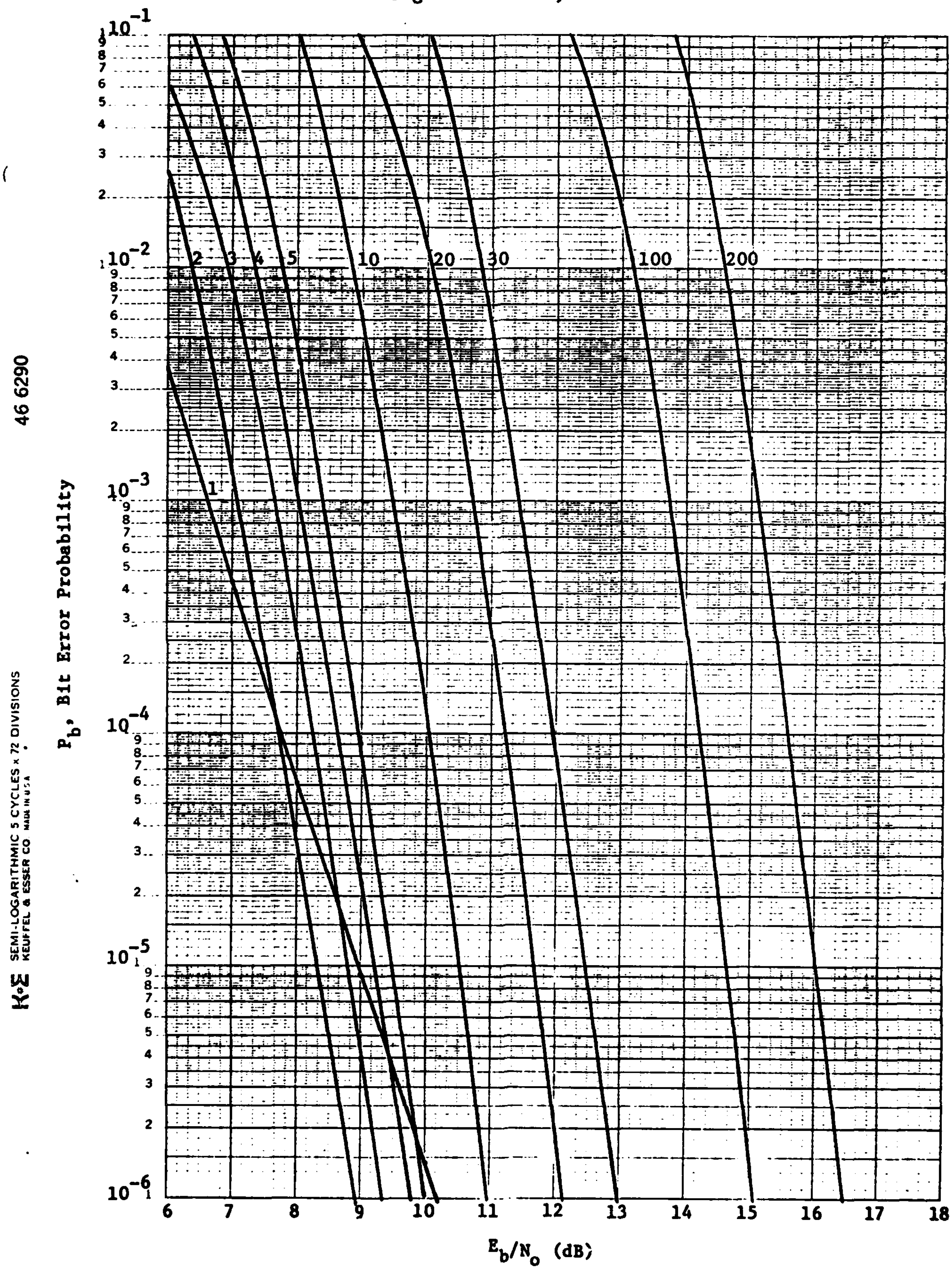


Figure 5 M = 2, $r = \frac{1}{2}$, Fading

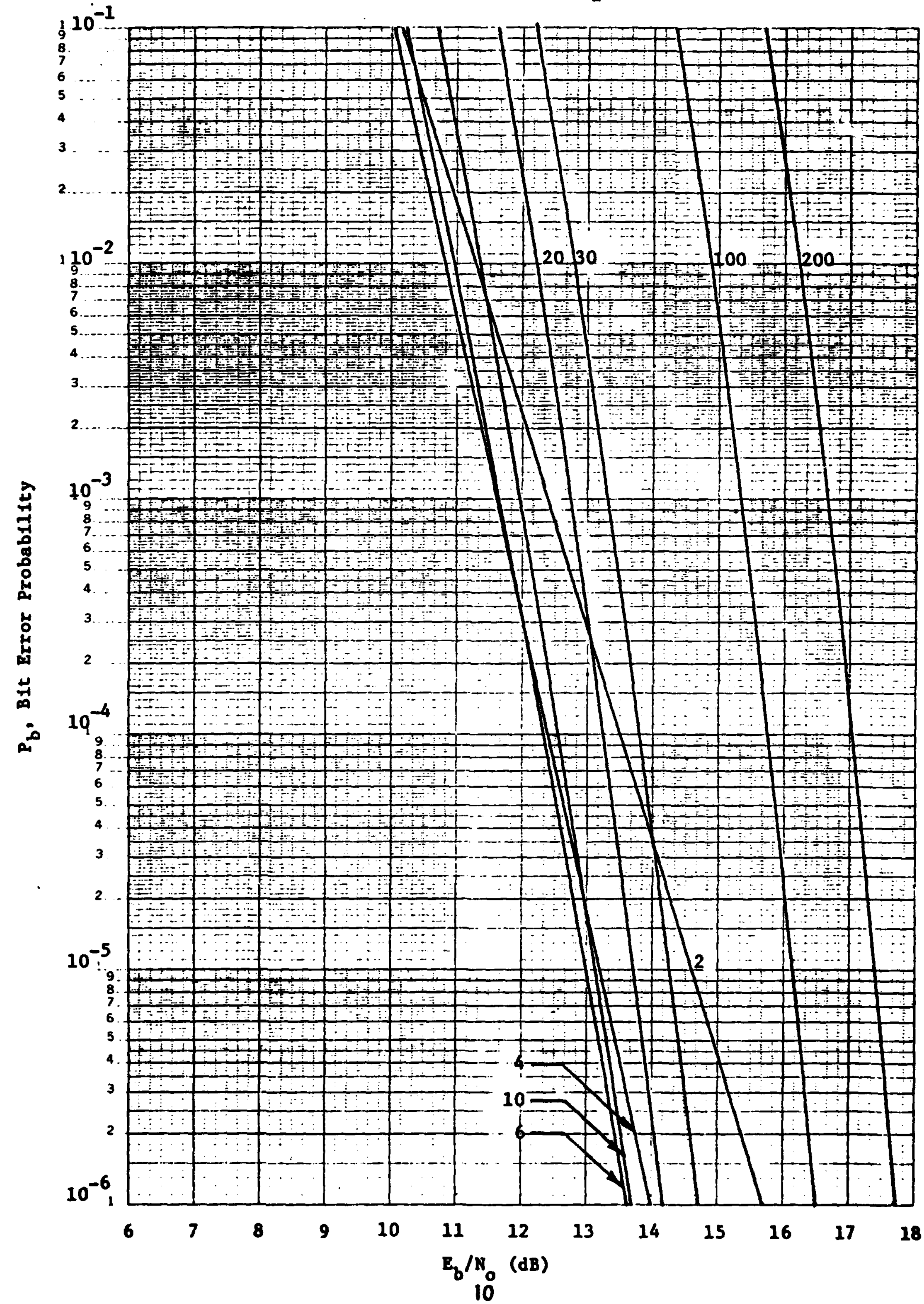


Figure 6 $M = 2,\ r = \frac{1}{3}$, Fading

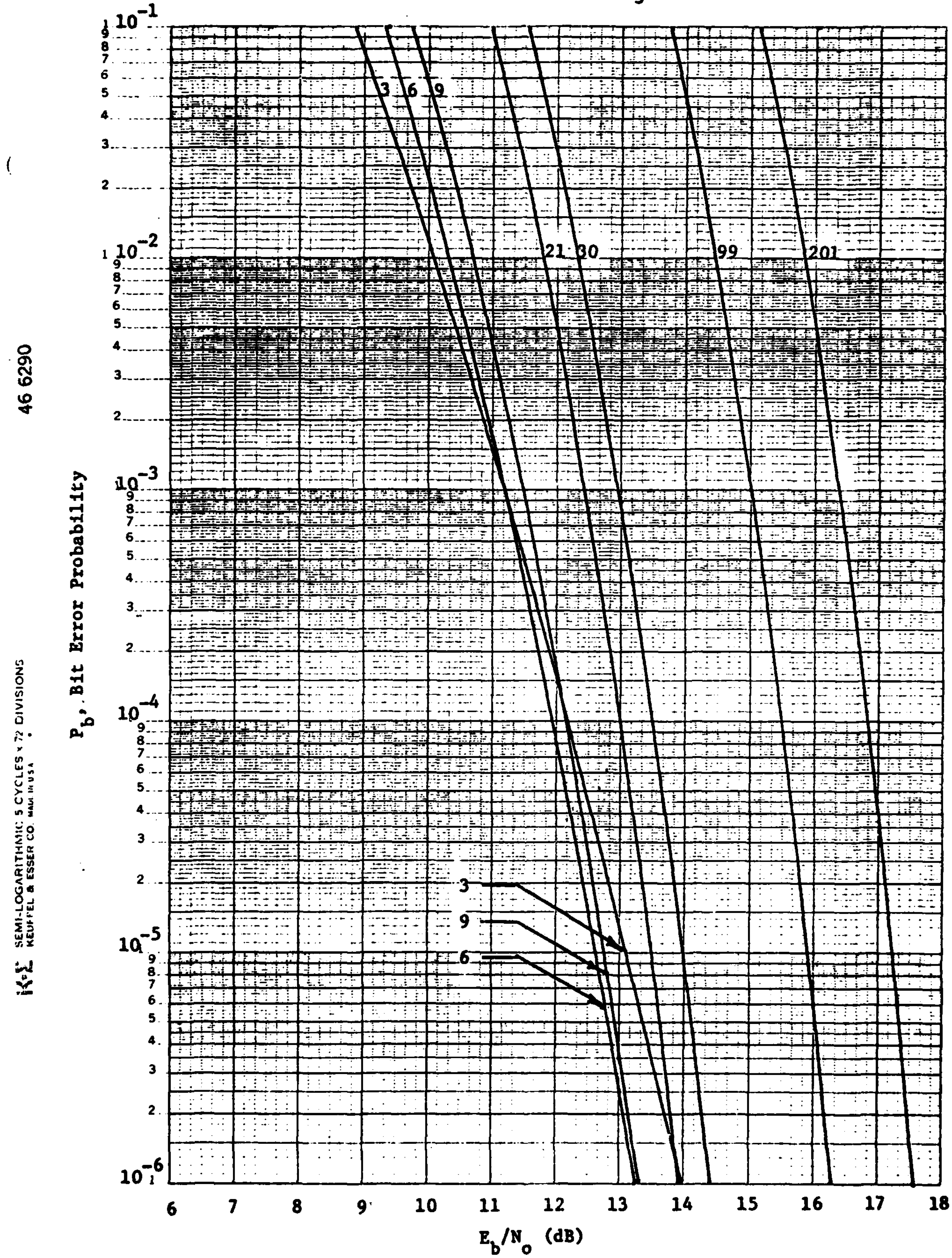
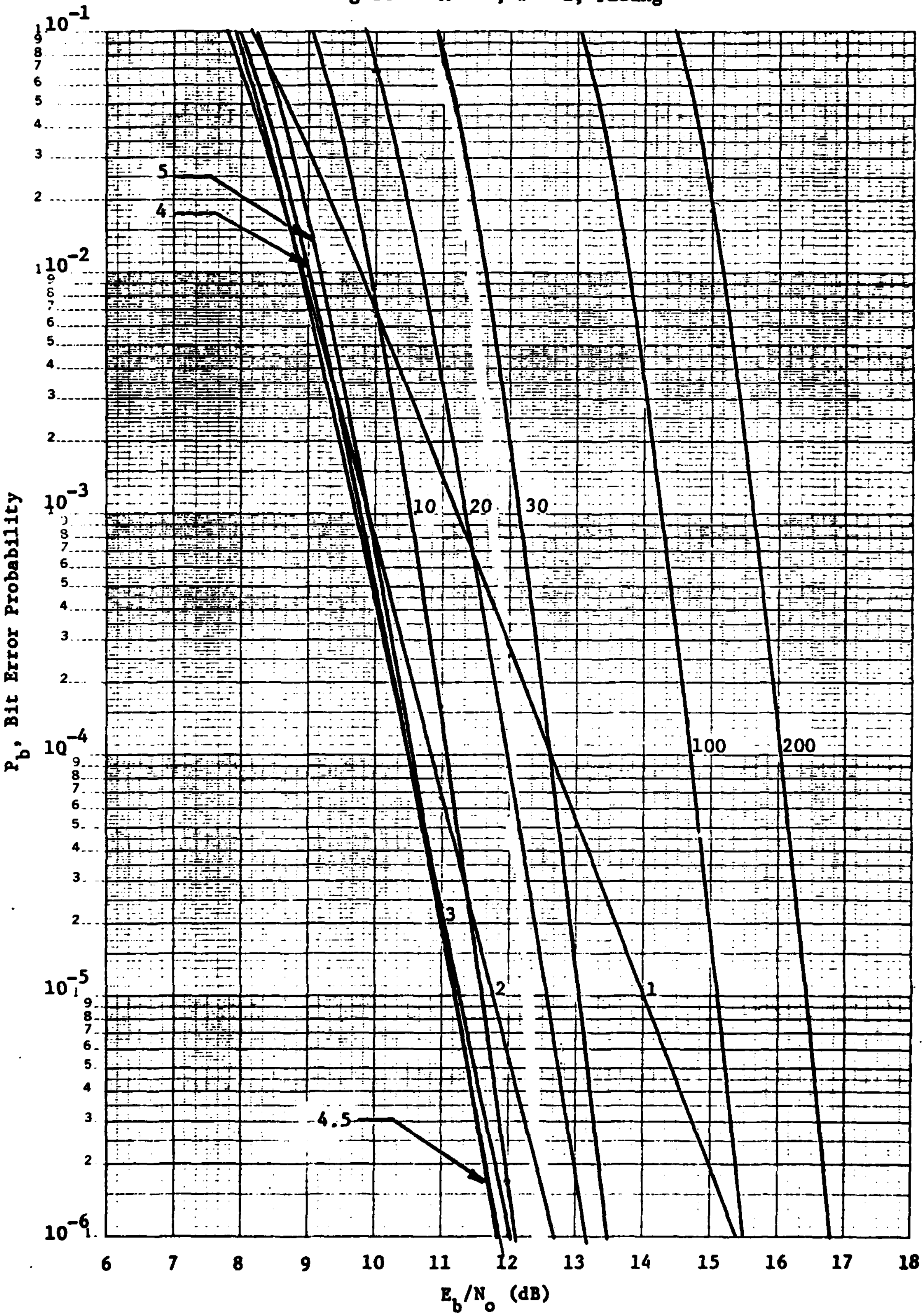


Figure 7 M = 4, r = 1, Fading



46 6290

K+E SEMI-LOGARITHMIC 5 CYCLES x 72 DIVISIONS
KEUFFEL & ESSER CO. MADE IN U.S.A.

Figure 8   M = 8, r = 1, Fading

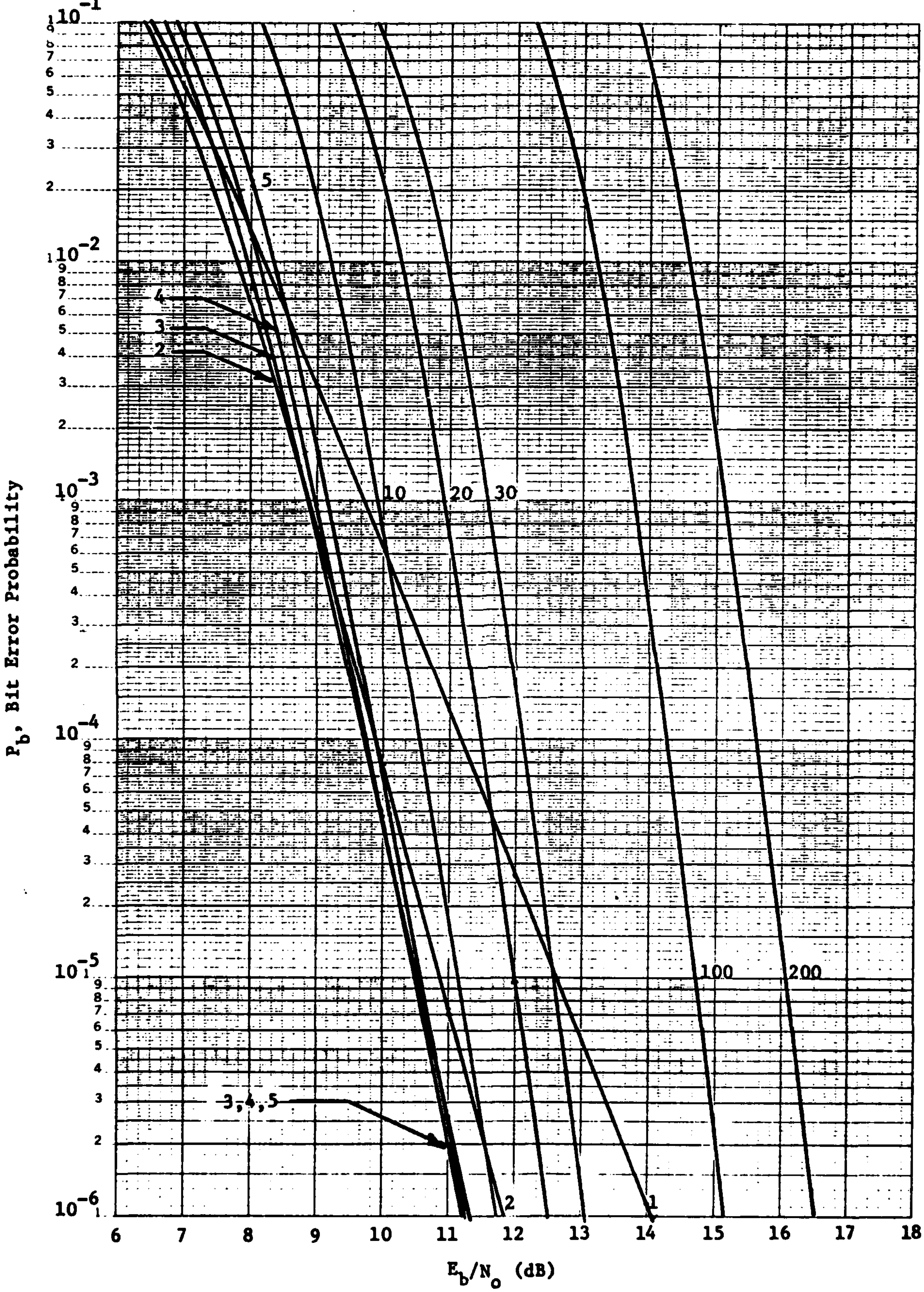
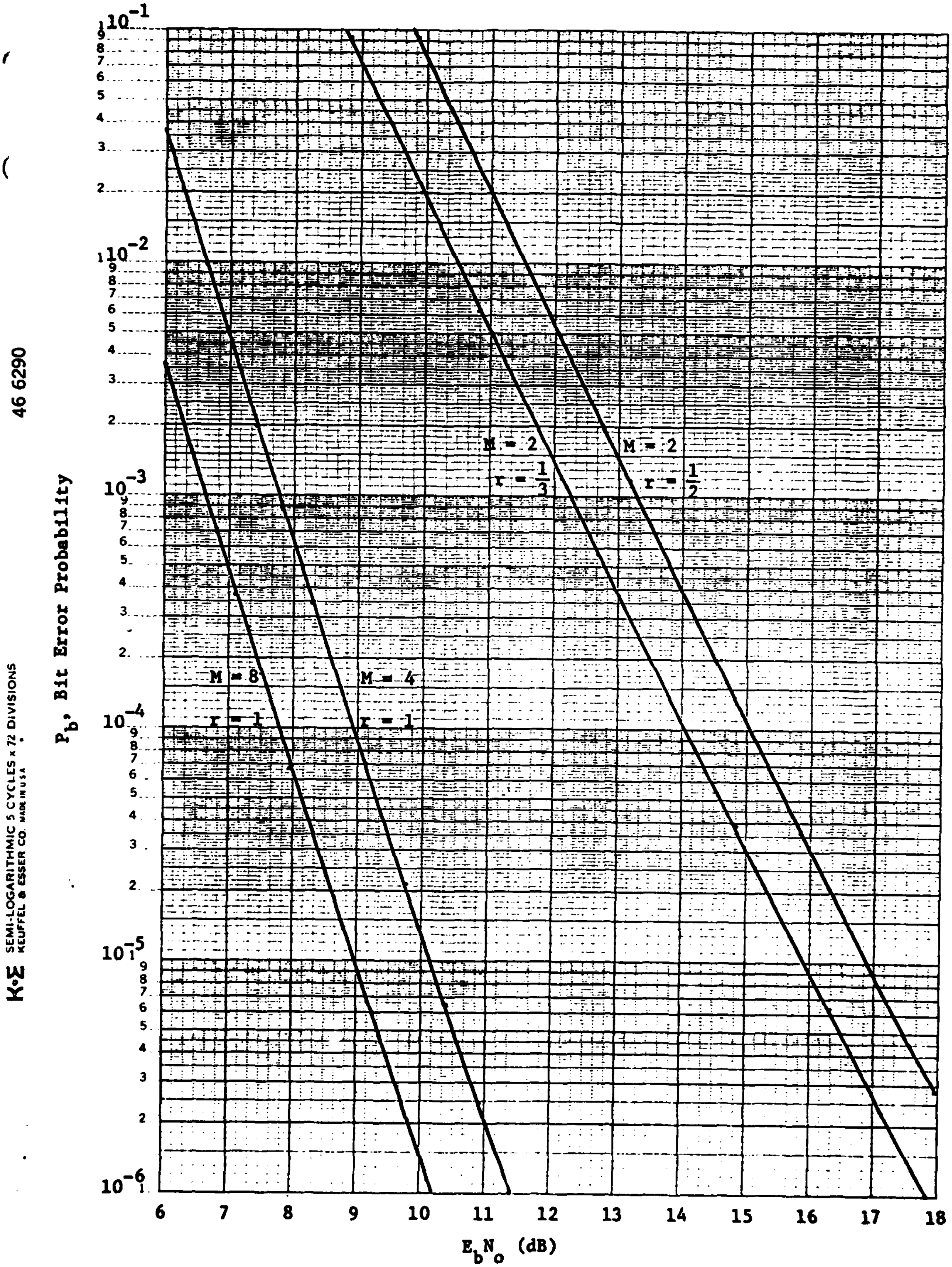
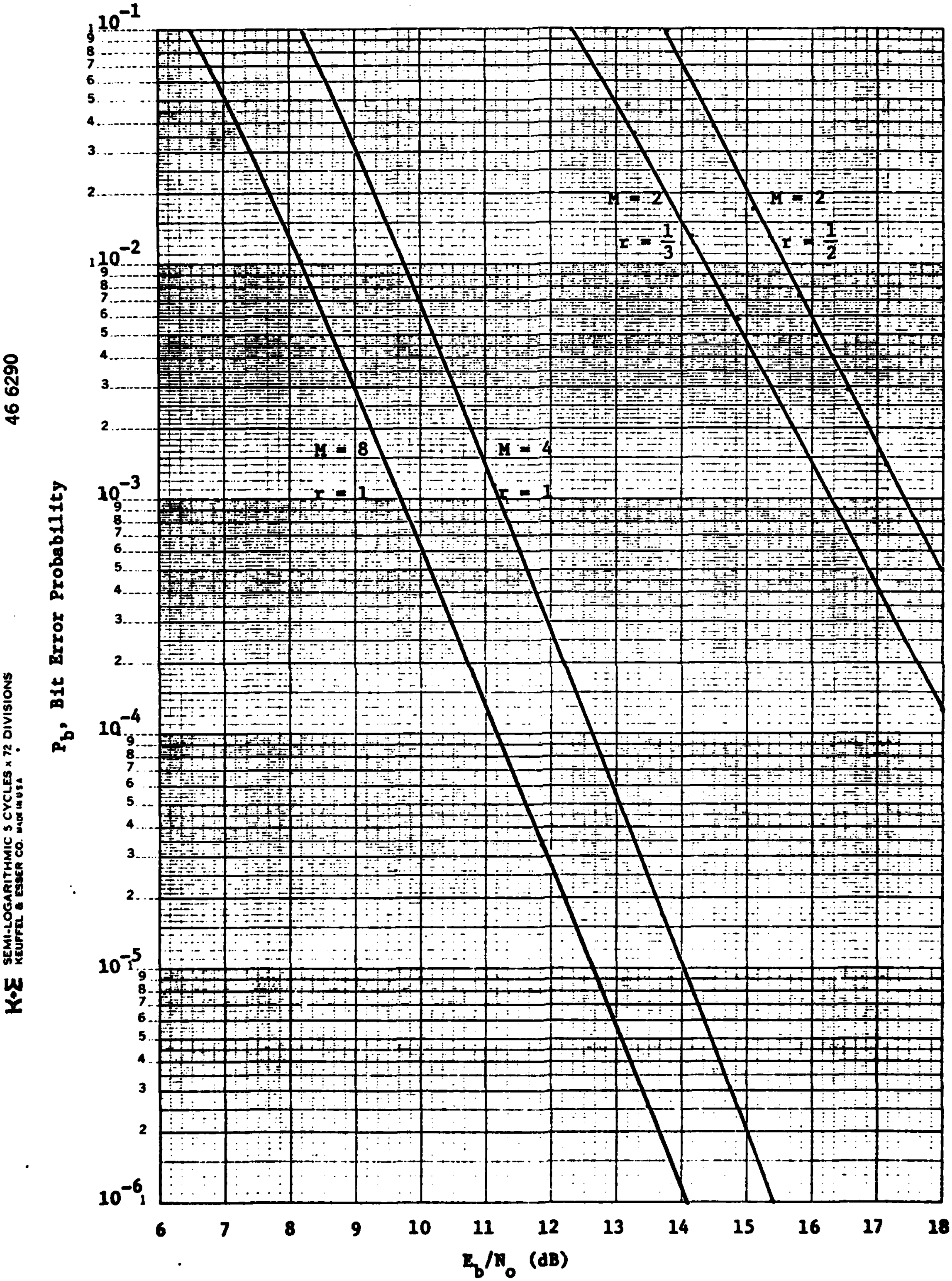


Figure 9 L = 1



10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P_b, Bit Error Probability
M = 2
$r = \frac{1}{3}$
M = 2
$r = \frac{1}{2}$
M = 8
r = 1
M = 4
r = 1
6 7 8 9 10 11 12 13 14 15 16 17 18
E_bN_o (dB)
46 6290
K•E SEMI-LOGARITHMIC 5 CYCLES x 72 DIVISIONS
KEUFFEL & ESSER CO. MADE IN U.S.A.

Figure 10 L = 1, Fading



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